5.2. Analysis of the conversion ratio $M(D,K)$

Only $M(D,7)$ for CCM, but for DCM

Formula:

$$k = \frac{2L}{R}$$

Analysis techniques for the discontinuous conduction mode:

Critical set by topology chosen

Converter steady-state equations obtained via charge balance on each capacitor and voltage-second balance on each inductor. Use care in applying small ripple approximation.

Small ripple approximation sometimes applies:

$$i(t) = I$$

is a poor approximation when $\Delta i > 1$

Charge balance:

$$\langle v(t) \rangle = \frac{1}{T} \int_{0}^{T} v(t) \, dt = 0$$

$$\langle i(t) \rangle = \frac{1}{T} \int_{0}^{T} i(t) \, dt = 0$$

Capacitor charge balance:

$$\langle i(t) \rangle = \frac{1}{C} \int_{0}^{T} i(t) \, dt = 0$$

Inductor volt-second balance:

$$\langle v(t) \rangle = -\frac{1}{L} \int_{0}^{T} i(t) \, dt = 0$$

Chapter 5: Discontinuous conduction mode
Example: Analysis of DCM buck converter $M(D,K)$

Three Unknowns: $D_1$, $D_2$, $D_3$

Chapter 5: Discontinuous conduction mode

New Circuit for $D_3$ opens for reverse diode.
Inductor volt-second balance

\[ v_L(t) \]

\[ V_g - V \]

\[ D_1 T_s \]
\[ D_2 T_s \]
\[ D_3 T_s \]

\[ 0 \]

\[ T_s \]
\[ t \]

Key to understanding

\[ V_L = 0 \text{ when } i_L = 0 \]

Volt-second balance:

\[ \langle v_L(t) \rangle = D_1 (V_g - V) + D_2 (-V) + D_3(0) = 0 \]

Solve for \( V \):

\[ V = V_g \frac{D_1}{D_1 + D_2} \]

Usually \( D_1 \) set by designer

Note that \( D_2 \) is unknown
Capacitor charge balance

\[ i_{L}(t) = i_{C}(t) + \frac{V}{R} \]

Node equation:

\[ i_{L}(0) = i_{C}(0) = 0 \]

Hence:

\[ \langle i_{L} \rangle = \frac{V}{R} \]

\[ \langle i_{C} \rangle = \frac{V}{R} \]

Diode is big!\[ i_{DC} = \frac{V}{R} \]

\[ T_{i} \leq T_{1} \leq T_{2} \]

Chapter 5: Discontinuous conduction mode

Fundamentals of Power Electronics
Inductor current waveform

\[ i_L(t) \]

\[ \left\langle i_L \right\rangle = 1 \]

\[ V_{\text{dc}} = \frac{V}{R} \]

\[ I_{\text{dc}} = \frac{V}{R} \]

Equate dc component to dc load current:

\[ V = \frac{D_1 T_1}{2L} (D_1 + D_2) (V_k - V) \]

Over the cycle:

\[ I_{\text{dc}} = \frac{1}{T} \int_0^T i_L(t) \, dt \]

Triangle area formula:

\[ \left\langle i_L \right\rangle = \frac{1}{2} \int_0^{2T} i_L(t) \, dt \]

Peak current:

\[ i_L(D_1 T_1) = i_{\text{pk}} = \frac{V_{\text{dc}} - V}{L} \]

Average current:

\[ i_{\text{avg}} = \frac{1}{T} \int_0^T i_L(t) \, dt \]

Fundamentals of Power Electronics

Chapter 5: Discontinuous conduction mode
Solution for V

Two equations and two unknowns (V and D):

\[ V = V_e \frac{D_1}{D_1 + D_2} \]

1. \[ V = \frac{D_1 T}{2L} (D_1 + D_2) (V_e - V) \]

(from inductor volt-second balance)

2. \[ V = \frac{D_1 T}{2L} (D_1 + D_2) (V_e - V) \]

(from capacitor charge balance)

Eliminate \( D_2 \), solve for \( V \):

\[ V = \frac{2}{2} \left( \frac{V_e}{1 + \sqrt{1 + 4K/D_1}} \right) \]

For \( D_2 \) buck DCM:

\[ V_m = \text{discontinuous conduction mode} \]

\[ K = 2L / RT \]

\[ K < K_{\text{crit}} \]

Methodology
Buck converter $M(D,K)$

Graph showing $M(D,K)$ vs $D$ for different values of $K$:
- $K = 0.01$ (blue curve)
- $K = 0.1$ (black curve)
- $K = 0.5$ (red curve)
- $K \geq 1$ (red line)

CCM limit is marked.

Mathematical expression for $M$:

$$M = \frac{2}{1 + \sqrt{1 + 4K/D^2}}$$

- for $K > K_{crit}$
- for $K < K_{crit}$

"Like a Boost": $D_{CM}$ is DC Gain Higher $V_{out}$

Recall PSpice simulations.