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Diapycnal diffusivities in homogeneous stratified turbulence

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[1] Quantifying diapycnal mixing in stably stratified turbulence is fundamental to the understanding and modeling of geophysical flows. Data of diapycnal mixing from direct numerical simulations of homogeneous stratified turbulence and from grid turbulence experiments, are analyzed to investigate the scaling of the diapycnal diffusivity. In these homogeneous flows the instantaneous diapycnal diffusivity is given exactly by $K_d = \epsilon_\rho/(\partial\overline{\rho}/\partial z)^2$ where ϵ_ρ is the dissipation rate of density fluctuations, and $\partial\overline{\rho}/\partial z$ is the mean density gradient. The diffusivity K_d may be expressed in terms of the large scale properties of the turbulence as $K_d = \gamma L_E^2/T_L$, where L_E is the Ellison overturning length-scale, T_L is the turbulence decay time-scale, and γ is half the mechanical to scalar time-scale ratio. Our results show that L_E and T_L can explain most of the variations in K_d over a wide range of shear and stratification strengths while γ remains approximately constant. **Citation:** Stretch, D. D., and S. K. Venayagamoorthy (2010), Diapycnal diffusivities in homogeneous stratified turbulence, *Geophys. Res. Lett.*, *37*, L02602, doi:10.1029/2009GL041514.

1. Introduction

[2] Many geophysical flows such as in the oceans, atmosphere, lakes, or estuaries, are influenced by the presence of stable density stratification. Quantifying diapycnal mixing in these flows is important for estimating the effects on the overall mass/energy balance [see, e.g., *Gregg*, 1987]. Methods of inferring diapycnal fluxes from microstructure measurements have therefore been developed and are widely used in oceanography. These methods, principally due to *Osborn and Cox* [1972] and *Osborn* [1980], are based on the notion that for homogeneous stationary turbulence, the irreversible mixing at small scales is balanced by the average advective transport by large scales. The applicability of these methods has been widely questioned [e.g., *Davis*, 1994] because the basic assumptions are seldom fully valid in practice. However, even in cases where the simplifying assumptions are valid (e.g., in some lab experiments and/or numerical simulations) a unifying scaling framework for diapycnal mixing that relates the diffusivity to the large scale properties of the flow has not emerged. This is required for numerical models of geophysical flows where small scale mixing cannot be directly resolved and must be parameterized as a sub-grid scale process.

[3] The non-dimensional parameter $\epsilon/\nu N^2$ (where ϵ is the dissipation of turbulent kinetic energy, ν is the kinematic viscosity, and N is the buoyancy frequency) is widely used in oceanography to characterize turbulence "intensity" or "activity" and therefore mixing rates. *Barry et al.* [2001] and *Shih et al.* [2005] used it to parameterize the turbulent fluxes and diffusivities obtained from lab experiments and direct numerical simulations (DNS) respectively [see also *Ivey et al.*, 2008]. Their results indicate that there are three mixing regimes that may be identified in terms of $\epsilon/\nu N^2$: (1) a "diffusive regime" with $\epsilon/\nu N^2 \sim 1$ where the diffusivity tends towards molecular values; (2) an "intermediate regime" with $10 \lesssim \epsilon/\nu N^2 \lesssim 100$ where the diffusivity increases linearly with ϵ/N^2, consistent with the *Osborn* [1980] model; (3) an "energetic regime" with $\epsilon/\nu N^2 \gtrsim 100$ where the diffusivity increases more slowly with ϵ/N^2, e.g., diffusivity $\propto(\epsilon/\nu N^2)^{1/2}$.

[4] There are two issues raised by the diffusivity scaling presented by *Barry et al.* [2001] and *Shih et al.* [2005]. Firstly, their scaling, if generally applicable, seems to imply that the diapycnal diffusivity depends on the viscosity, even at very high Reynolds numbers, which is inconsistent with theoretical concepts and observations. *Pope* [1998] and *Donzis et al.* [2005] provide in depth discussions of this issue. Secondly, their scaling does not yield any clear "passive" scalar limit for the diffusivity; i.e., how do diascalar diffusivities in weakly stratified flows where $\epsilon/\nu N^2 \rightarrow \infty$ fit into the suggested scaling?

[5] *Venayagamoorthy and Stretch* [2006] (hereafter VS), using DNS of shear-free decaying homogeneous stably-stratified turbulence, showed that the diapycnal diffusivity K_d in these temporally developing flows is proportional to L_E^2/T_L where $L_E = (\overline{\rho'^2})^{\frac{1}{2}}/|\partial\overline{\rho}/\partial z|$ is the Ellison overturning length-scale and $T_L = k/\epsilon$ is the turbulence decay time-scale. This was found to hold over a wide range of stratification strengths, including the passive scalar limit. However the generality of this result could not be tested since the simulations did not include shear effects, nor did they include variations in Prandtl number. Furthermore, as with most DNS studies, they were limited to low Reynolds numbers that in the strongly stratified cases were within the "diffusive" regime ($\epsilon/\nu N^2 \sim 1$) mentioned above. The applicability of the suggested scaling to a wider class of flows therefore remains an unresolved issue.

[6] The objective of this paper is to address the above-mentioned issues by investigating the generality of the scaling suggested by VS using data from both DNS and laboratory experiments that cover a range of stratification strengths, shear rates, Reynolds numbers, and Prandtl (or Schmidt) numbers. In sections 2 and 3 we discuss the theoretical background to the scaling of the diascalar diffusivity. In section 4 we describe the DNS and experimental datasets used for the present analysis.

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The results are discussed in section 5 and conclusions in section 6.

2. Diascalar Flux in Homogeneous Turbulence

[7] *Winters and D'Asaro* [1996] used a geometric approach to derive an exact expression relating the instantaneous diascalar flux of an arbitrary scalar θ to its mean-square gradients averaged over iso-scalar surfaces, namely

$$\phi_d = \kappa \frac{\left\langle |\nabla\theta|^2 \right\rangle_{z*}}{|d\theta/dz*|} \quad (1)$$

where κ is the molecular diffusivity of θ, $z*$ is an iso-scalar co-ordinate and $\langle\cdot\rangle_{z_*}$ denotes an average over the iso-scalar surface corresponding to a given $z*$ value. Derivation of this result assumes only that the evolution of the scalar field θ is described by an advection–diffusion equation with a solenoidal velocity field. In their formulation, *Winters and D'Asaro* [1996] defined a reference state for the scalar field where all fluid particles are adiabatically re-arranged to form a profile $\theta(z*)$ that is monotonic in $z*$. When the scalar concerned is the density field in a stably stratified flow, this reference state corresponds to a state of minimum potential energy. VS argued that in the case of homogeneous turbulence with a uniform background mean scalar gradient $d\overline{\theta}/dz$ (where $\overline{\theta}(z) = \langle\theta\rangle_z$), the gradient of the reference state $\theta(z*)$ is equal to the gradient of the mean field $\overline{\theta}(z)$. In this case the diascalar flux given by (1), averaged over all iso-scalar surfaces, simplifies to

$$\phi_d = \frac{\epsilon_\theta}{|d\overline{\theta}/dz|} \quad (2)$$

with an associated diascalar diffusivity given by

$$K_d = \frac{\epsilon_\theta}{\left(d\overline{\theta}/dz\right)^2} \quad (3)$$

where $\epsilon_\theta = \kappa \langle|\nabla\theta'|^2\rangle$ is the volume averaged dissipation rate of the scalar variance. This result for the diascalar diffusivity is exact for the specified conditions. It is also the same as that inferred by *Osborn and Cox* [1972] but it is more generally applicable than suggested by the Osborn-Cox argument. Although it requires that the turbulent fields are statistically homogeneous, it does not require stationarity or a balance between large and small scale processes represented in the scalar transport budget.

[8] VS derived (3) using a Lagrangian analysis of fluid particle displacements which were decomposed into (reversible) isopycnal and (irreversible) diapycnal components. The growth rate of the mean square diapycnal displacements was shown to be proportional to K_d for high Reynolds and Peclet numbers where there is large separation between advective and diffusive scales.

3. Scaling of the Diapycnal Diffusivity in Homogeneous Stratified Turbulence

[9] VS noted that the diapycnal diffusivity in homogeneous stably stratified flows can be expressed in terms of the large scale properties of the turbulence as

$$K_d = \frac{\epsilon_\rho}{\left(\partial\overline{\rho}/\partial z\right)^2} = \gamma \frac{{L_E}^2}{T_L}, \quad (4)$$

where $\gamma = T_L\epsilon_\rho/\overline{\rho'^2}$ is half the mechanical to scalar time-scale ratio. Physically, the length scale L_E describes the generation of density fluctuations by vertical displacement (or stirring) of fluid particles within the background mean gradient, while T_L is the time-scale over which fluid particles exchange their density with their surrounding fluid by small-scale mixing (refer VS for details). Note that although there is no explicit buoyancy parameter in (4), the effects of stable stratification on K_d are implicit in L_E and T_L.

[10] An important reason to express K_d in the form of (4) is that the time-scale ratio is widely used as a parameter in second moment closure models [e.g., *Pope*, 2000]. Moreover, VS found that their DNS results suggested that the time-scale ratio was approximately independent of stratification. The value they obtained was about 1.4 (i.e., $\gamma \simeq 0.7$) which is similar to values measured by *Sirivat and Warhaft* [1983] and *Yoon and Warhaft* [1990] in thermally stratified decaying grid turbulence. If the time-scale ratio remains independent of stratification for a wider class of flows, including shear flows, this has important simplifying implications for modeling. Since both time scales are linked to the turbulence cascade from large to small scales, it seems reasonable that they maintain a constant ratio, provided there is no fundamental change to these processes.

[11] Equation (4) may also be expressed in terms of length scales as

$$K_d = \gamma\, \epsilon^{1/3} L^{4/3} \quad (5)$$

where $L = L_E^{3/2}\, L_\epsilon^{-1/2}$ is a mixture of the displacement length-scale L_E and the dissipation length-scale $L_\epsilon = k^{3/2}/\epsilon$. Note that (5), with $\gamma \simeq$ constant, is qualitatively consistent with classical inertial range scaling for homogeneous turbulence [e.g., *Richardson*, 1926].

[12] In the remainder of this paper we focus on investigating the generality of the scaling given by (4) (or equivalently (5)) with $\gamma \simeq$ constant.

4. Data Sources

[13] Several previous DNS studies of homogeneous turbulence, both with and without shear and stable stratification, as well as laboratory grid turbulence studies, have provided data that can be used to test the ideas presented in section 3. We have selected several of these studies based on accessibility of the data and the values of key parameters, i.e., Reynolds numbers, Richardson numbers, and Prandtl (or Schmidt) numbers. DNS studies that have used artificially forcing to produce stationary turbulence were not considered since the type of forcing scheme can affect the mechanical to scalar time-scale ratio [see, e.g., *Donzis et al.*, 2005]. The selected data sources and typical parameter values are summarized in Table 1. Further details are as follows:

[14] 1. The DNS study of VS comprised freely decaying homogeneous stably stratified turbulence. Each simulation was initialized as isotropic turbulence with no scalar fluctuations. Initial transients in the development of the scalar fields (times less than one turnover time) were ignored. Note that since these are temporally developing flows, each simulation yields time histories for the instantaneous

Table 1. Summary of DNS and Laboratory Experimental Data Analyzed for This Study[a]

References	Re_λ	Ri_t	Pr, Sc
VS (2006)	40	0 – 100	0.5
Shih et al. [2005]	90	0 – 10	0.7
Rogers et al. [1986]	40 – 90	0	1 – 2
Sirivat and Warhaft [1983]	40	0	0.7
Yoon and Warhaft [1990]	30	0.04 – 1	0.7
Itsweire et al. [1986]	40	0.25 – 100	700
Mydlarski [2003]	80 – 731	0	0.7

[a]Typical values are shown for the micro-scale Reynolds number Re_λ, turbulent Richardson number $Ri_t = (NT_L)^2$ and Prandtl or Schmidt number Pr, Sc = ν/κ.

diapycnal diffusivity K_d, L_E and T_L, which can be used for testing (4).

[15] 2. The DNS study by *Shih et al.* [2005] concerned initially isotropic turbulent fields that were subjected to both a uniform mean shear rate ($S = \partial\overline{u}/\partial z$) and a uniform stable stratification (buoyancy frequency $N = \sqrt{(-g/\rho_0)\partial\overline{\rho}/\partial z}$). A measure of the buoyancy effects is the gradient Richardson number $Ri_g = N^2/S^2$. Values used in the simulations were $0.05 \leq Ri_g \leq 0.6$. The flows are temporally developing: for small Ri_g the turbulent energy grows in time while for large Ri_g the energy decays. A stationary state occurred when $Ri_g \simeq 0.17$ for the data series used here. Initial transients in the development of the scalar fields were omitted from our analysis.

[16] 3. The DNS study of *Rogers et al.* [1986] provides a reference case for passive scalars in homogeneous shear flows, i.e., $Ri_g = 0$. Available data comprised statistics at discrete non-dimensional times $St = 2, 4, 6, \ldots 12$.

[17] 4. The experiments of *Sirivat and Warhaft* [1983] and *Yoon and Warhaft* [1990] investigated the mixing of temperature in spatially decaying grid turbulence. The data used here was extracted from tables in the publications and were measured at fixed downstream positions $x/M = 100$ and $x/M = 76$ respectively, where M is the grid mesh size.

[18] 5. The experiments of *Itsweire et al.* [1986] investigated decaying stably stratified grid turbulence in a water channel using salinity as the active scalar. The Schmidt number of salt in water is about 700. Note that the small-scale salinity fluctuations could not be resolved by conductivity probes due to the high Schmidt number. The dissipation rate of the scalar variance was therefore inferred indirectly from the variance transport equations.

[19] 6. The experiments of *Mydlarski* [2003] [see also *Mydlarski and Warhaft*, 1998] investigated temperature mixing in decaying grid turbulence without significant buoyancy effects. An "active" grid was used to generate intense turbulence at high microscale Reynolds numbers.

5. Results and Discussion

[20] Consolidated results from DNS and grid turbulence experiments are shown plotted in Figure 1 in the form of the non-dimensional diascalar diffusivity K_d/κ (or Cox number) versus the parameter $Pe_t = L_E^2/T_L\kappa$. The Cox number is the ratio of turbulent to molecular diffusivity and is thus a measure of the intensity of turbulent mixing, while Pe_t may be interpreted as a turbulent Peclet number based on the vertical overturning scale L_E and velocity scale L_E/T_L. Equation (4) with $\gamma = 0.7$ and with 5th and 95th percentile values (0.5 and 1.0 respectively) are also shown on the plot for comparison. More detailed insight into values of the parameter γ can be obtained from Figures 2 and 3 where it is shown plotted versus the Cox number and local Richardson number $Ri_t = (NT_L)^2$ respectively. The Richardson number indicates the ratio of buoyancy to inertial forces.

[21] From Figure 1 it is evident that the diapycnal diffusivity remains proportional to L_E^2/T_L for all cases and all times, and over at least five orders of magnitude in the

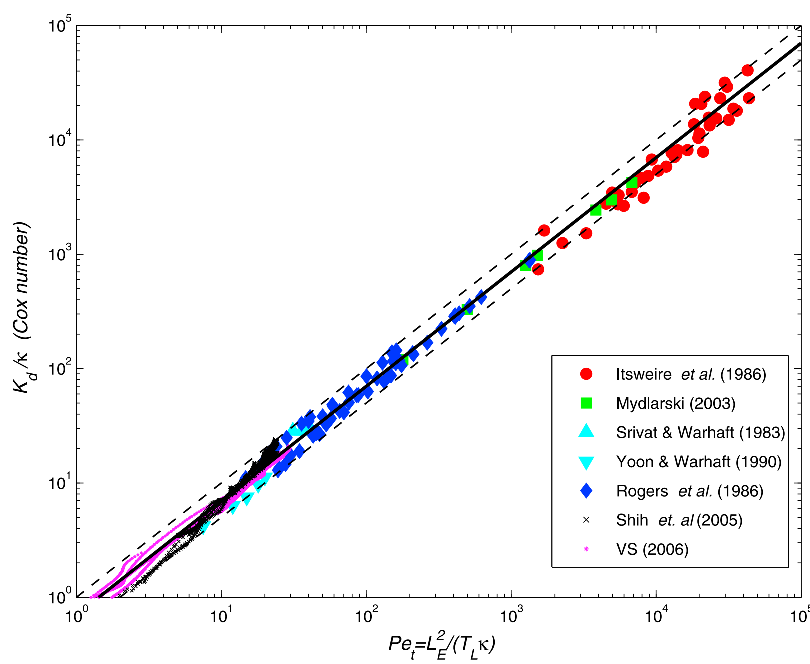

Figure 1. The non-dimensional diapycnal diffusivity K_d/κ (or Cox number) plotted as a function of $L_E^2/T_L\kappa$ for DNS and experimental data. The solid line is $\gamma = 0.7$, and the dashed lines are $\gamma = 0.5$, 1.0.

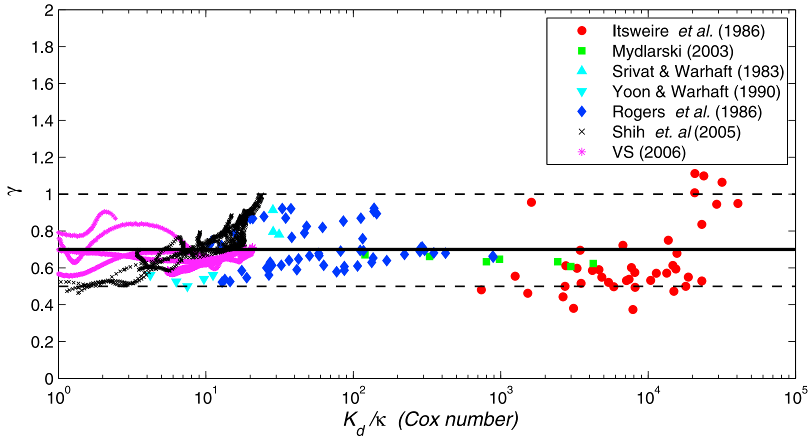

Figure 2. The coefficient γ plotted as a function of the Cox number K_d/κ for DNS and experimental data. The solid line is γ = 0.7, and the dashed lines are γ = 0.5, 1.0.

Cox number. Figures 2 and 3 further show that the coefficient γ does not vary systematically with Cox number or with Ri_t.

[22] The stratified DNS of VS and *Shih et al.* [2005] show only weak mixing with Cox numbers generally less than 30 and reducing as buoyancy effects increase. The passive scalar simulations of *Rogers et al.* [1986] with Pr = 2 attained Cox numbers of up to 1000 but Reynolds numbers were a factor of two lower than the Pr = 1 simulations (Table 1). Diffusivities in these DNS are all consistent with (4) and suggest that γ is insensitive to both shear and stratification.

[23] The data of *Itsweire et al.* [1986] have the highest Cox numbers ranging from about 1000 to 30000. The Reynolds numbers for these experiments were low (Table 1) and the high Cox numbers are a consequence of the high Schmidt number of salt in water (Sc $\simeq$ 700). The scatter in this data is probably due to the indirect method of measuring the scalar dissipation rate. Nevertheless the data are consistent with the suggested scaling. Note that as the turbulence decays downstream in these experiments, local Richardson numbers increase while the Cox numbers decrease.

[24] The high Reynolds number passive scalar grid turbulence experiments of *Mydlarski* [2003] attained Cox numbers ranging from 120 to 4250. Diffusivities from these experiments are again consistent with (4) with $\gamma \simeq 0.7$.

[25] In summary the results shown in Figures 1, 2, and 3 support the scaling suggested in section 3 and the data are well described by (4) with $\gamma \simeq 0.7$ for all stratifications (including passive cases) and both with/without shear. The scaling also seems to be valid for Prandtl (Schmidt) numbers $0.7 \leq Pr \leq 700$ although simulations or experiments at higher Reynolds numbers (and with shear and stable stratification) are needed to fully address this issue.

6. Conclusions

[26] In homogeneous (but not necessarily stationary) turbulent flows the instantaneous diapycnal diffusivity is given exactly by $K_d = \epsilon_\rho/(\partial\overline{\rho}/\partial z)^2$ and may be formulated in terms of the large scale properties of the turbulence as $K_d = \gamma L_E^2/T_L$. Our analysis of DNS and grid turbulence data shows that L_E and T_L can explain all the variations in K_d (over several orders of magnitude in the Cox number K_d/κ) for a broad range of shear and stratification strengths (including shear-free and neutrally stratified cases) while γ remains approximately constant.

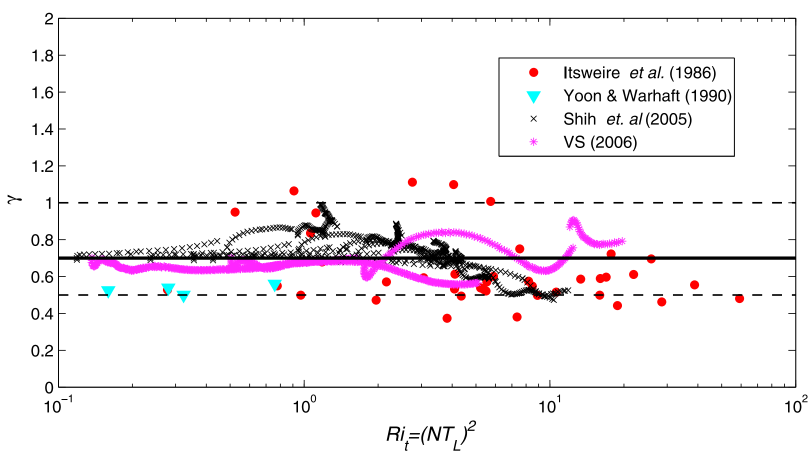

Figure 3. The coefficient γ plotted as a function of the Richardson number $Ri_t = (NT_L)^2$ for DNS and experimental data. The solid line is γ = 0.7, and the dashed lines are γ = 0.5, 1.0.

[27] This result suggests a unified scaling framework for diascalar fluxes in homogeneous turbulence that seems to have considerable generality and may therefore be useful for turbulence models. It is worth noting that advective vertical fluxes $\overline{\rho' w'}$ for the flows discussed in this paper show very different characteristics from the diapycnal fluxes. For example in developing stably stratified flows, the advective fluxes can oscillate and change sign due to (reversible) internal wave motions that do not contribute significantly to irreversible diascalar mixing.

[28] An example of how these results may be applied to turbulence modeling is given by *Venayagamoorthy and Stretch* [2010], who used them to derive a new formulation of the turbulent Prandtl number based on irreversible scalar and momentum fluxes.

[29] Our results also clarify the issue of Reynolds number effects that was discussed in section 1 regarding previously suggested scalings, especially for the energetic regime where $\epsilon/\nu N^2 \gtrsim 100$. The new scaling we have presented in this paper does not show any distinct regime(s) of applicability, but remains valid over the whole range $1 < \epsilon/\nu N^2 \leq \infty$ represented in the data discussed here. Furthermore the results do not show significant Prandtl (or Schmidt) number effects on the diapycnal diffusivity. However, this issue requires further investigation due to the limited range of Reynolds numbers in both the DNS and laboratory experiments, particularly for stably stratified flows. The DNS studies used here have relatively low resolution compared to current state of the art so it is feasible to extend the simulations to higher Reynolds numbers to further check the validity of the suggested scaling for K_d. Another important extension of this study is to explore the applicability of these scaling results to inhomogeneous turbulent flows and to field-scale flows in the atmosphere and oceans. Natural geophysical flows are more complex than the idealized flows considered here, e.g., they are typically intermittent in space and time and have both high Reynolds and Richardson numbers, a regime which is not represented in the data considered here. These flows can develop a quasi two-dimensional layered structure in the strongly stable limit [see, e.g., *Riley and Lelong*, 2000]. Whether these changes in structure influence the scaling results obtained in our study requires further investigation. The detailed measurements now becoming available from field studies [e.g., *Zaron and Moum*, 2009] are making it feasible to test the K_d scaling in natural geophysical flows.

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