Scalable turbocharger performance maps for dynamic state-based engine models

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Abstract
Adapting turbocharger performance maps to a form suitable for dynamic simulations is challenging for the following reasons: (1) the amount of available data is typically limited, (2) data are typically not provided for the entire operating range of the compressor and turbine and (3) the performance data are non-linear. To overcome these challenges, curve fits are typically generated using the performance data individually for each device. The process, however, can take uneconomical amounts of effort to implement for a range of compressors and turbines. This article introduces a method to implement non-dimensional performance maps thereby allowing a range of turbochargers to be modeled from the same performance data, reducing the effort required to implement models of different sizes. The non-dimensional maps seek to model the performance of compressor and turbine families in which the geometry of the rotor and housing are similar and allow the turbocharger to be scaled for simulation in much the same way used to design customized sizes of turbochargers. A method to match the non-dimensional compressor map to engine performance targets by selecting the compressor diameter is presented, as well as a method to match the turbine to the selected compressor.

Keywords
Turbocharger, compressor maps, turbine maps, dynamic modeling, scaling

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Introduction
Compressor and turbine performance is typically implemented in dynamic state-based engine models such as those developed in Kao and Moskwa,1 Malkhede et al.2 and Millet et al.3 and used for power system studies as in Katrasnik et al.4 and Ibrahim et al.5 as lookup tables or curve fits developed from performance maps; however, a significant amount of effort is required to convert the data to a form suitable for simulation. Manufacturers frequently only provide data on the maps around the region of high efficiency where the turbocharger is operating at high speeds and pressure ratios near the surge margin. The performance at low rotor speeds and low pressure ratios, a region frequently encountered during simulation at low engine load, must be extrapolated from the available data as illustrated by Moraal and Kolmanovsky6 and discussed in Gamma Technologies.7 Additionally, at compressor surge, the mass flow rate through the compressor drops suddenly and can even experience a flow reversal, creating stiff state equations. This region of the map is difficult to capture using a lookup table which is indexed by pressure ratio and speed parameter; the output is undefined for inputs above the surge line and it is difficult to capture the compressor surge well in the fits produced by the methods discussed in Moraal and Kolmanovsky.6

Collecting performance data specific to a particular compressor and turbine and generating curve fits which accurately capture the characteristics of the data often require a significant amount of time and effort. In order to model several engines ranging in displacement and power, the process must be repeated for each engine. A scalable approach to modeling the turbocharger would streamline this process, but has not been presented in the literature. The approach introduced in this article...
utilizes dimensional analysis techniques, often used in the design of compressors and turbines, to generate a suitable map once, and then adapt that map for use in simulation across a range of engine sizes. The process reduces the amount of data and effort required to model a series of engines.

The contributions of this article are threefold. First, a method is presented to normalize compressor performance data by the surge line in order to produce tabular data on rectilinear axes which capture the sudden reduction of mass flow at compressor surge and are readily implemented as a lookup table during simulation. Second, following well-established dimensional analysis of compressors and turbines, the published performance maps of a given device are converted into the non-dimensional terms which represent a family of devices for which design parameters associated with the rotor and housing remain constant. Third, a method is introduced to select the diameter of the compressor and turbine rotors (the remaining design parameter) in order to match the non-dimensional performance maps to an engine for which a state-based model is being constructed.

The remainder of this article is divided into three sections. The first briefly reviews methods to fit performance data and extrapolate into the low rotor speed, low pressure ratio regions of a compressor map. An approach to scaling a compressor map using the dimensional analysis similar to that presented in Yahya\(^8\) to produce maps for geometrically similar compressors is then illustrated. The compressor data are normalized by the surge line to produce a rectangular lookup table which (1) is readily implemented in simulation and (2) captures rapid change in mass flow near the surge line. A similar scaling approach is applied to normalize the turbine map. The second section introduces a method to select a compressor diameter to match the non-dimensional map to a given engine and to select a turbine diameter which will match the compressor torque requirements. Finally, a set of compressor and turbine maps are scaled to match several engine sizes in the results section in order to illustrate the utility of the method.

**Scalable modeling approach**

**Review of fitting methods**

Steady-state compressor mass flow and isentropic efficiency data collected at selected rotor speeds and pressure ratios are typically available from the manufacturer as a table or map similar to that shown in Figure 1. Given the pressure ratio across the compressor and the speed parameter, the data can be used to determine the mass flow parameter, \( \dot{m}_{corr,C} = f(N_{corr,sp},(P_d/P_u)) \), and the isentropic compressor efficiency, \( \eta_c = f(N_{corr,sp},(P_d/P_u)) \). The speed parameter, \( N_{corr} \), and the corrected mass flow are used to account for temperature and pressure effects of the upstream air\(^9,10\)

\[
\dot{m}_{corr,C} = \frac{m_c \sqrt{T_u}}{P_u} \quad (1)
\]

\[
N_{corr} = \frac{N_{TC}}{\sqrt{T_u}} \quad (2)
\]

The tabular data are not well-suited for simulation since it (1) is limited to few data points representing non-linear performance characteristics of the compressor and turbine which would introduce significant error when used with linear interpolation and extrapolation techniques; (2) does not illustrate the performance at low rotor speed and low pressure ratios, a region frequently encountered in simulations during initialization or where the engine is operating across a range of speed and load and (3) includes a region above compressor surge where the outputs (\( \dot{m}_{corr,C}, \eta_c \)) are undefined for inputs (\( N_{corr,sp}, (P_d/P_u) \)). For these reasons, curve fits are often used in simulation models. Four methods of curve fitting compressor data are reviewed and illustrated by Moraal and Kolmanovsky\(^6\) in order to extrapolate...
performance to low pressure ratios and low rotor speeds. The four methods illustrated are as follows:

- The Jensen and Kristensen method\(^{11}\) which expresses the dimensionless head parameter and the compressor efficiency as functions of normalized flow rate and inlet Mach number. Coefficients for the expressions are determined using a least-squares fit to experimental data.
- The Mueller method\(^{12}\) which models the dimensionless head parameter as a quadratic function of normalized compressor flow rate.
- The zero-slope line method\(^{6}\) which describes the compressor flow parameter as a function of pressure ratio and the speed parameter. The fit is divided into a linear region and an exponential region by the zero-slope line which connects the maximum mass flow of each speed line.
- A neural network as used by Nelson et al.\(^{13}\) and determined to be ill-suited to compressor modeling due to the large number of required coefficients and the limited number of data points available to train the neural network.

Steady-state turbine mass flow and isentropic efficiency data collected at selected rotor speeds and pressure ratios are also typically available from the manufacturer as a table or map similar to that shown in Figure 2. Given the expansion ratio across the turbine and the speed parameter, the data can be used to determine the mass flow parameter, \(\dot{m}_{\text{corr}, T} = f(N_{\text{corr}}, (P_u/P_d))\), and the isentropic compressor efficiency, \(\eta_T = f(N_{\text{corr}}, (P_u/P_d))\). Methods to curve fit turbine mass flow and efficiency data are also presented by Moraal and Kolmanovsky.\(^{6}\)

All these methods require a significant amount of experimental data, as well as effort and judgment to process each map. In order to reduce the amount of time and effort required to implement a turbocharger model into simulations of various engine scales and applications, a scalable approach using a “generic” map is desirable.

**Proposed scaling method—compressor**

Turbocharger designers use maps in terms of non-dimensional terms to approximate the performance of new compressor sizes from similar compressors before producing a prototype.\(^{8}\) The dimensional analysis discussed by Canova et al.\(^{14}\) shows the characteristic curves for compressors and turbines are dependent on the working fluid, Reynolds number, and three design parameters: the rotor diameter and trim associated with the blade design, and the A/R ratio associated with the housing design. The analysis illustrates how the design parameters influence the characteristic curves describing compressor and turbine mass flow and efficiency, enabling the performance maps to be predicted for a given set of design parameters.

The method proposed here seeks to use a single set of available performance data to generate a scalable map for state-based models. Although the non-dimensional data represent a device with two fixed design parameters, namely, the trim and A/R ratio, the method allows a first-cut model to be developed for various engine sizes from a single set of device-specific data.

Starting with manufacturer data for a single compressor design, if the diameter of the compressor wheel, \(D_c\), is known, the mass flow parameter and efficiency data can easily be converted into non-dimensional compressor mass flow coefficient, \(\vartheta_C\), and Mach number based on the rotor tip speed, \(c_0\).

\[
\vartheta_C = \dot{m}_{\text{corr}, T} \sqrt{\frac{R}{D_c^2}}
\]

\[
c_0 = \frac{2\pi}{60} \frac{N_{\text{corr}} D_c}{\sqrt{\gamma R}}
\]

Maps implemented in terms of the compressor mass flow coefficient and the rotor tip Mach number represent a family of similar compressors with the same trim.
and A/R ratio, rather than a single design. Manufacturers typically present data in terms of speed parameter and mass flow parameter rather than these non-dimensional parameters because the map is presented for a particular selection of rotor ($D_c = constant$) and it is assumed the compressor will always handle air near standard temperature and pressure ($R = R_{air}$, $\gamma = \gamma_{air|STP}$).

Figure 3 shows a compressor map in terms of the non-dimensional parameters developed from data presented in Figure 1. The data were first fit using the zero-slope line method and extrapolated into the low rotor speed, low pressure ratio region. The map in Figure 4 is generated from the non-dimensional map by selecting a new diameter, $D_c = 100$ mm (smaller than the original rotor). By selecting a diameter, $D_c = 200$ mm (larger than the original diameter), the map in Figure 5 is produced.

From equations (3) and (4), it is easy to see that the mass flow rate is proportional to the square of the rotor diameter, and the rotor speed is inversely proportional to the rotor diameter. By selecting a smaller diameter, the rotational speed of the rotor is increased and the mass flow parameter is reduced as illustrated in Figure 4. By selecting a larger diameter, the rotor speeds are reduced and the mass flow rate is increased as shown in Figure 5.

**Compressor performance as lookup tables**

Lookup tables generally are not used for turbocharger data because linear interpolation methods result in significant error, particularly in regions with low resolution data or significant non-linearity. Lookup tables, however, are readily implemented in simulations and are computationally efficient. By first fitting the data using one of the methods discussed previously, the curve fit can be evaluated to create a lookup table at a higher resolution (using smaller steps in pressure ratio and rotor speed) than the data provided by the manufacturer. This method reduces the error associated with interpolation within the table data.

At the surge line, the compressor stalls and mass flow stops or may reverse flow. The region of the map above the surge line is typically not communicated by the manufacturer, which makes it difficult to implement the data as lookup tables indexed by pressure ratio and rotor speed (either speed parameter or rotor tip Mach number). The challenge is twofold: First, in the surge region, a lookup table would have combinations of inputs ($P_d/P_u$, $c_{0,c}$) where the outputs ($\theta_c$, $\eta_c$) are undefined. This region cannot be avoided during dynamic simulation due to events such as a sudden change in speed and/or load and results in undefined behavior, where there is no counteracting “force” to push the simulation back onto the map. Second, for a lookup table of practical size (or a curve fit of practical order), it is difficult to represent the sudden change in output parameters near the surge line. Therefore, the simulation can easily jump over this boundary and get stuck in the undefined area above the surge line.
The pressure ratio at compressor surge is found for a given rotor speed \((P_d/P_u)_{surge} = f(N_{corr})\), or rotor tip Mach number \((P_d/P_u)_{surge} = f(c_{0,c})\). The pressure ratios corresponding to efficiency and mass flow data at that rotor speed are then normalized by the pressure ratio at surge as

\[
P_{idx} = \frac{(P_d/P_u) - 1}{(P_d/P_u)_{surge} - 1}
\]

By normalizing the pressure ratio by the surge line, a rectangular lookup table is produced, indexed by a surge-normalized pressure ratio index, \(P_{idx}\), instead of the pressure ratio, \((P_d/P_u)\). This process effectively maps the region below the surge line onto rectilinear axes as illustrated in Figure 6. Using this index, there is no longer a region above compressor surge in the data where the outputs are undefined. The new pressure index ranges from \(P_{idx} = 0\) where there is no boost across the compressor, \((P_d/P_u) = 1\), to \(P_{idx} = 1\) at compressor surge, \((P_d/P_u) = (P_d/P_u)_{surge}\), where the flow quickly rolls off to 0. The high-resolution lookup table created by this process can be used for a range of engine sizes.

**Proposed scaling method—turbine**

An approach similar to that used for the compressor data can be applied to the turbine data. If the diameter of the turbine wheel, \(D_T\), is known, the data can easily be converted into the non-dimensional parameters. The turbine mass flow coefficient, \(\phi_T\), and Mach number based on the rotor tip speed, \(c_{0,T}\), are calculated as

\[
\phi_T = \frac{m_{cor,T}}{D_T} \sqrt{\frac{R}{T_u}}
\]

(6)

Again, the manufacturers typically present the data as in Figure 2 in terms of speed parameter, \(N_{corr} = (NTC/\sqrt{T_u})\), and mass flow parameter, \(m_{cor,T} = ((\dot{m}_T/\sqrt{T_u})/P_u)\), rather than these non-dimensional parameters, for similar reasons. Figure 7 shows the turbine efficiency and non-dimensional mass flow coefficient presented in terms of the expansion ratio, \((P_u/P_d)\), and rotor tip Mach number. The data were first fit using techniques in Gamma Technologies\(^7\) and extrapolated into the low rotor speed, low pressure ratio region, and then normalized using equations (6) and (7).

**Turbocharger selection using non-dimensional maps**

Implementing performance maps in terms of the non-dimensional parameters allows a single set of maps to approximate the turbocharger performance for a range of engine sizes by adjusting the compressor and turbine diameters to match the target brake power and air–fuel (A/F) ratio of each engine. A single set of non-dimensional maps can be utilized in the series of engine models instead of repeating the process of finding the device-specific compressor and turbine maps, fitting the data to extrapolate into the region of low pressure and low rotor speed and converting the maps into a tabular data format at a higher resolution than provided by the manufacturer. The non-dimensional maps are limited to the assumption that the trim and A/R ratio in the family is fixed. A process to match the non-dimensional compressor and turbine maps to a given engine is illustrated below.

**Selecting compressor diameter**

The fuel mass flow rate to meet a target engine power, \(W\), can be calculated as \(\dot{m}_F = W/\eta_{Th} E_{LHV}\) where \(\eta_{Th}\) is...
the approximate thermal efficiency of the engine and $E_{LHV}$ is the lower heating value of the fuel. The air mass flow rate required to operate at a target A/F ratio, $R_{A/F}$, can then be approximated as

$$m_{air} = \frac{\dot{W}R_{A/F}}{\eta_VE_{LHV}}$$  \hspace{1cm} (8)

The intake manifold pressure, $P_m$, required to meet this air flow rate in a four-stroke engine can then be approximated by rearranging the speed-density equations (1) and (2) and assuming an ideal gas as

$$P_m = \frac{120m_{air}RT_m}{\eta_VN_{V_d}}$$  \hspace{1cm} (9)

where $T_m$ is the intake manifold temperature, $\eta_V$ is the volumetric efficiency of the engine, $N$ is the engine speed and $V_d$ is the displacement of the engine.

Assuming a small pressure loss across the air filter, $P_{loss1}$, and across the aftercooler, $P_{loss2}$, the pressure ratio across the compressor can be determined as

$$\left(\frac{P_d}{P_a}\right)_{Req} = \frac{P_m + P_{loss2}}{P_{in} - P_{loss1}}$$  \hspace{1cm} (10)

where $P_{in}$ is the atmospheric pressure.

Referring to the non-dimensional map, the required pressure ratio can be found in the region of high efficiency and the required compressor mass flow coefficient, $\phi_{C,Req}$, can be read as illustrated in Figure 8.

The diameter of the compressor wheel can be calculated by combining equations (1) and (3), substituting $T_{atm}$ for $T_u$, ($P_{in} - P_{loss1}$) for $P_u$ and $\dot{m}_{air}/N_{Parallel}$ for $\dot{m}_c$, where $N_{Parallel}$ has been introduced to divide the air flow among parallel turbochargers

$$D_c = \sqrt{\frac{\dot{m}_{air}\sqrt{RT_{atm}}}{N_{Parallel}\phi_{C,Req}(P_{atm} - P_{loss1})}}$$  \hspace{1cm} (11)

This diameter will match the normalized compressor map to the given engine. The torque required to drive the compressor, $\tau_C$, will be required to match the turbine and can be calculated from isentropic relations as

$$\tau_C = \frac{\eta_Tm_c\dot{C}_p T_u}{N_{Parallel}\eta_{TC}} \left(\left(\frac{P_d}{P_u}\right)^{\frac{\gamma}{\gamma - 1}} - 1\right)$$  \hspace{1cm} (12)

Selecting turbine diameter

The turbine diameter must be approximated to match the torque produced by the turbine to that required by the compressor during operation, $\tau_T = \tau_C$. The mass flow rate through the turbine can be calculated from the compressor mass flow and the A/F ratio as $m_T = m_c(1 + (1/R_{A/F}))$. The isentropic relation for turbine torque is

$$\tau_T = \frac{\eta_Tm_T\dot{C}_p T_u}{N_{Parallel}\eta_{TC}} \left(1 - \left(\frac{P_d}{P_u}\right)^{\frac{\gamma}{\gamma - 1}}\right)$$  \hspace{1cm} (13)

The lookup tables provide a relationship between the mass flow coefficient, efficiency, rotor tip Mach number and expansion ratio

$$\phi_T = f\left(\frac{P_u}{P_d}, C_{0,T}\right)$$  \hspace{1cm} (14)

$$\eta_T = f\left(\frac{P_u}{P_d}, C_{0,T}\right)$$  \hspace{1cm} (15)

The definition of the rotor tip Mach number and the mass flow coefficient provides two more equations with relation to the diameter

$$\phi_T = \frac{m_T\sqrt{RT_u}}{P_dD_T^2}$$  \hspace{1cm} (16)

$$\eta_{0,T} = \frac{\omega_{TC}D_T}{\sqrt{\gamma RT_u}}$$  \hspace{1cm} (17)

Assuming the temperature of the exhaust gas upstream of the turbine, $T_u$, is known and the turbine outlet pressure, $P_d$, is near ambient, equations (13)–(17) provide a set of five equations and five unknowns ($\eta_T$, $P_u$, $\phi_T$, $C_{0,T}$ and $D_T$) which are solved for the turbine rotor diameter.

Results—matching non-dimensional maps to four diesel engines

To illustrate the utility of the non-dimensional approach, compressor and turbine wheel diameters are selected to match the non-dimensional maps presented in Figures 6 and 7 to several engines of various sizes. Table 1 summarizes the power and displacement of these engines at 20%, 40%, 60%, 80% and 100% load. The compressor performance was monitored during simulation and is plotted on the normalized...
compressor maps in Figures 9 and 10. At each load, each engine operates at nearly the same point on the map. The turbine performance during simulation is also presented in Figure 11. Again at each load, each engine operates in nearly the same location on the normalized turbine map to balance the compressor torque. The clusters of simulated points correspond to increasing loads from left to right in each figure.

The results illustrate that the non-dimensional maps can be utilized to model the turbocharger compressor and turbine across a range of engine models. Using the methods illustrated in this article, the work flow and effort required to model engine dynamics in control studies where turbocharging plays an important role can be reduced by eliminating the need to fit compressor and turbine data for each engine individually, but rather match the “generic” family maps to the engines through the selection of wheel diameters. In this way, a single map can be utilized across a range of studies involving different engines.

**Conclusion**

Normalizing the pressure ratio in compressor data by the pressure ratio at compressor surge produces rectilinear axes which are readily implemented as a lookup table in simulation models. The rectangular surge-normalized map captures the sudden decrease in flow at compressor surge and avoids modeling the surge region using empirical fits generated from data in the normal operating region of the compressor and extrapolated beyond surge.

The non-dimensional maps represent devices where the turbocharger design parameters, namely, the A/R ratio and trim, are constant within a family. Implementing turbocharger performance data in state-based engine models using the non-dimensional mass flow coefficient and rotor tip Mach number allows the turbine and compressor to be scaled to match the displacement and power of the engine. This eliminates the need for data specific to each turbocharger and reduces the time and effort required to model a range of engine sizes. For design applications, additional maps representing geometric families of devices could be developed in order to evaluate the influence of trim and A/R ratio on the engine dynamics.

The method presented to scale the compressor and turbine is a useful tool when implementing engine
models across a range of engine sizes. The method was used to match non-dimensional compressor and turbine performance data generated from a single device for a family with a specific A/R ratio and trim to four diesel engines ranging from 500 kW to 2.5 MW. The results show that the compressor and turbine operate similarly across the range of engine sizes when the maps for the family are scaled to match the engine power and displacement.

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**References**

**Appendix 1**

**Notation**
- \( c_{0,C} \): compressor rotor tip Mach number based on inlet conditions
- \( c_{0,T} \): turbine rotor tip Mach number based on inlet conditions
- \( C_p \): specific heat at constant pressure, J/(kg K)
- \( D_T \): turbine wheel diameter, m
- \( D_C \): compressor wheel diameter, m
- \( E_{LHV} \): lower heating value of fuel, J/kg
- \( \dot{m}_{air} \): engine intake air flow rate, kg/s
- \( \dot{m}_{corr,C} \): compressor mass flow parameter, kg K\(^{0.5}/\) (s Pa)
- \( \dot{m}_{corr,T} \): turbine mass flow parameter, kg K\(^{0.5}/\) (s Pa)
- \( \dot{m}_C \): compressor mass flow rate, kg/s
- \( \dot{m}_T \): turbine mass flow rate, kg/s
- \( N \): engine speed, r/min
- \( N_{corr} \): rotor speed parameter, r/min K\(^{-0.5}\)
- \( N_{parallel} \): number of turbochargers in parallel
- \( N_{TC} \): turbocharger rotor speed, r/min
- \( P_{atm} \): atmospheric pressure, Pa
- \( P_d \): downstream pressure, Pa
- \( P_{dx} \): surge-normalized pressure ratio index
- \( P_{loss1} \): pressure loss across air filter, Pa
- \( P_{loss2} \): pressure loss across aftercooler, Pa
- \( P_m \): intake manifold pressure, Pa
- \( P_u \): upstream pressure, Pa
- \( R \): specific gas constant, J/(kg K)
- \( R_{a-f} \): air–fuel ratio
- \( T_{atm} \): atmospheric temperature, K
- \( T_u \): upstream temperature, K
- \( W \): engine displacement, m\(^3\)
- \( \gamma \): specific heat ratio
- \( \eta_c \): compressor isentropic efficiency
- \( \eta_T \): turbine isentropic efficiency
- \( \eta_{rh} \): engine thermal efficiency
- \( \eta_v \): engine volumetric efficiency
- \( \tau_c \): torque required to drive compressor, N m
- \( \tau_T \): torque produced by turbine, N m
- \( \Theta_C \): compressor mass flow coefficient
- \( \Theta_T \): turbine mass flow coefficient
- \( \omega_{TC} \): turbocharger rotor speed, rad/s