Hillslope drainage development with time: a physical experiment

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Abstract

Rainfall simulator experiments were structured to develop erosion rill-channel networks for $9^\circ$ and $5^\circ$ slopes subject to constant rainfall. Quantitative measurements included measurements of rill-channel width, depth, and width-to-depth ratios aggregated over the slope, and measures of the scaling characteristics and space filling tendencies of the networks. Trends in fractal dimensions and width functions with time are presented and compared to previous qualitative descriptions of network evolution. Our results imply that the equilibrium scaling characteristics of rill-channel networks are similar to those of river networks. For a given slope, the fractal dimension increases with time toward an equilibrium value. This equilibrium value is hypothesized to be a function of the effective storm, the initial hillslope-scale slope, and the geologic properties of the substrate. Results also imply that the rate of increase of the fractal dimension of the developing erosion networks (i.e., the rate at which the erosion networks fill space) may increase with increasing hillslope-scale slope. In addition, the growing rill-channel networks possess width functions whose bifurcation characteristics, as described by the power contained in the high wave numbers of the Fourier series fit, remain constant throughout the evolution of the networks.

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1. Introduction

The field of experimental fluvial geomorphology can be defined as the study of specific geologic and morphologic features of rivers under closely monitored and controlled experimental conditions (Schumm et al., 1987). The advantage of the experimental approach is that it permits the study of the geomorphic evolution of a fluvial system rather than just differences between equilibrium and non-equilibrium states. The drawbacks, however, are that initial and boundary conditions, as well as natural interactions (e.g., network growth rates) may not be analogous to those found in nature (Rodríguez-Iturbe and Rinaldo, 1997). In this paper, results of a physical experiment, fluvial geomorphic in nature, are presented and analyzed to study the evolution of rill-channel networks on a hillslope. As conducted here, the characteristics of hillslope network growth (i.e., scaling and space filling properties, and width functions) and of the associated equilibrium states are studied from a geomorphic point of view. These characteristics are an accurate representation of the physical processes that occur on non-vegetated natural slopes.
Erosion rill channels on hillslopes are the dominant means of sediment and water transport (Nearing et al., 1997). Fundamental knowledge of the rates of growth of erosion rill channels is paramount in advancing the development of improved prediction models of erosion. Initiation mechanisms for river channels that occur at the watershed scale also exist at the hillslope scale for rill channels. In the Hortonian model (Horton, 1945), channel initiation occurs at the point where the shear stress exerted on the substrate by overland flow exceeds some critical value. Shallow land sliding is another mechanism that has been suggested for river-channel initiation at the watershed scale (Dietrich et al., 1986; Montgomery and Dietrich, 1988) and has been shown to occur in rainfall simulator experiments at the hillslope scale similar to the type presented here (Schumm et al., 1987). A principal hypothesis of this work is that the processes and mechanisms that govern the evolution of large-scale river networks also govern the dynamics of the evolution of drainage networks on a hillslope void of vegetation. As a result of the similarity in processes and mechanisms, we hypothesize that hillslope networks at equilibrium possess the same scaling relationships as river networks at equilibrium. These scaling relationships can then, in turn, be used as predictive tools for evolving hillslopes. Rill channels may also be initiated along overland flow planes by discontinuities in the flow caused by a leaf or a twig, a process that does not have a direct upwardly scaled comparison.

River networks possess properties that exist over a very wide range of spatial scales. These properties have led to the development of predictive models like those based on the relationships of downstream hydraulic geometry (e.g., Leopold and Maddock, 1953) and energy expenditure characteristics (e.g., Molnar and Ramírez, 1998a,b). Landform development has been shown to have fractal properties at scales as small as 100 m² (Ouchi, 2001) and as large as 10⁶ m² (Rodríguez-Iturbe and Rinaldo, 1997). The evolution of networks on tilled hillslopes has been shown to minimize the global rate of energy expenditure (Gómez et al., 2003). The literature suggests that the network of water flow paths on hillslopes is fractal in nature (Ogunlela et al., 1989; Wilson, 1991; Wilson and Storm, 1993) and that hillslope erosion rill channels share many of the same statistical properties of watershed drainage networks (Gómez et al., 2003; Raff et al., 2003). From an erosion prediction viewpoint, sediment detachment and transport occurs where the water is flowing. If the water flow paths are fractal in nature, as is suggested above, then the erosion rill-channel network that develops on a hillslope should also be fractal in nature, a hypothesis tested herein.

The so-called downstream hydraulic geometry relationships for river networks describe flow width, flow depth, and flow velocity as a function of a volumetric flow discharge of equal frequency of occurrence throughout the network (Leopold and Maddock, 1953). This is appropriate for perennial river systems and long timescales because the flows often chosen are close to the effective discharge, defined as the singular discharge responsible for channel form (e.g., Andrews, 1980). Effective discharge, \( Q_{\text{eff}} \), is the discharge that does the most work in terms of sediment transport over time and is usually determined by the maximum of the product of the probability distribution function of the flow discharge and the sediment transport flux. If \( Q \) is volumetric flow rate [L³/T], \( f(Q) \) is the probability density function of \( Q \), and \( Q_s(Q) \) is the sediment transport flux as a function of discharge (i.e., sediment rating curve) then

\[
Q_{\text{eff}} = \max_{Q} [Q_s(Q)f(Q)]. \tag{1}
\]

The distribution of water fluxes on hillslopes, however, is not as easy to describe as for perennial rivers. Generally, water only occurs on a hillslope during rainstorms, and these vary in frequency as well as in intensity and duration. The erosional drainage network present on a hillslope is capable of transporting the water and sediment loads from a specific storm of intensity, duration, and spatial heterogeneity such that no local aggradation or degradation occurs anywhere within the network. Thus, at any point in time, the effective storm, that is, the storm reflecting the current structure of the rill drainage network, is of a specific intensity, duration, and spatial heterogeneity. These characteristics may vary with the properties of the hillslope (e.g., substrate material, soil moisture distribution, hillslope-scale slope). Only a storm of greater intensity than the effective rainfall rate will cause sediment production from the hillslope. Therefore, considering the evolutionary response of hillslopes with respect to the effective storm may be a better approach to model erosion from single storm
events than the current method based on lumped statistics (e.g., soil types, mean rainfall intensity). The difference between a model based on the concept of an effective storm and a lumped erosional model is the inclusion of history. The response of the erosion rill-channel network, characterized by the rates at which the network elongates and fills space, is a function of previous storms to which said network has previously responded. Thus, the implementation of an approach to hillslope erosion predictions based on the effective storm concept requires descriptions of the development of the drainage networks with respect to many variables including hillslope-scale slope, rainfall intensity, and duration.

The influence of rill-channel slope on erosion potential is highly debated (e.g., Nearing et al., 1997). The debate is largely based on the dependence of flow velocity on slope because velocity is often related to sediment transport capacity. Various researchers have shown that, for a non-erodible bed, rill-channel velocities increased with increasing slopes (e.g., Foster et al., 1984; Rauws, 1988; Abrahams et al., 1996). Govers (1992), however, showed that for an erodible bed, rill-channel velocity was not influenced by bed slope.

An examination of hillslope-scale slope on the development of drainage networks is presented here. Our approach utilizes mathematical descriptors that have been developed for the analyses of river networks (e.g., fractal dimension of the network, width functions, etc.). We use these descriptors to characterize and quantify the development and evolution of the erosion network on hillslopes. We show that the evolution of the network on a hillslope, described as a function of these fluvial geomorphologic mathematical descriptors, is predictable with respect to hillslope-scale slope.
2. Methods

2.1. Experimental setup

This experiment is designed to determine how hillslope-scale slope affects erosion-rill channel network evolution described as a function of changes in the space filling properties and width functions of the network. We show that geomorphological theories developed to describe the structure and evolution of watershed-scale drainage networks are also valid in the framework of the dynamics of hillslope erosion networks.

Six rainfall simulations on an artificial hillslope of (projected) dimensions $3 \times 10$ m (Fig. 1) are presented. Rainfall applications are spatially and temporally homogeneous, ending when the erosion rill-channel network ceases to vary significantly, determined qualitatively, over a period of 30 min. Typical rainfall durations are 4–5 h. Eight commercial sprinkler nozzles generate the rainfall. A constant pressure input to the rainfall nozzle array was used for all experiments (207 kPa); this corresponds to an average rainfall intensity of 65 mm/h, held constant throughout experiments. A sample rainfall distribution is shown in Fig. 2. This rainfall intensity is sufficient to cause formation of erosion networks relatively quickly and not too intense as to never occur naturally.

The substrate material used for the experiment was obtained in Ft. Collins, CO, a semi-arid region where a storm of this intensity would cause rill-channel development under most circumstances. The substrate is composed of a sand mixture, whose grain size distribution is presented in Fig. 3. The hillslope is initially smooth and compacted as uniformly as possible utilizing a roller compactor. Two slope angles, $9^\circ$ and $5^\circ$, are considered. In between experimental runs, the substrate is roto-tilled, leveled, and compacted. A small amount of additional sediment is mixed in with the roto-tilled sediment surface between runs to replace the sediment lost from the previous experiment and to eliminate any effects of preferential sediment transport and armoring from one experiment to the next.

2.2. Measurements

Photographs are taken at 1-h intervals throughout the experiment. At the completion of the experiment, the erosion rill-channel network is mapped manually. Each erosion rill channel network is ordered using the

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![Fig. 3. Grain size distribution. Continuous line is distribution before beginning of an experiment. Dashed line is a sample grain size distribution of the sediment removed from the slope during the simulation. The difference in distributions shows preferential transport of finer sediments.](image-url)
Strahler stream ordering system (Strahler, 1957). Width and depth measurements are made along each rill-channel, same as a link, at intervals of 300 mm. The number of measurements made per rill channel, a function of the rill-channel length, has a mean of 4 and a standard deviation of 2.5. Widths and depths are aggregated to obtain average width and depth of erosion rill channels for the 9° and 5° experiments, respectively.

Scaling relationships are calculated to determine if the erosion rill-channel networks resemble the fractal characteristics of larger river networks. These scaling relationships include Horton’s law of stream lengths:

$$L_{w0} = L_1 R_L^{w0-1}$$

where $R_L$ is the length ratio, $L_{w0}$ is the (arithmetic) average of the length of streams of order $w_0$, and Horton’s law of stream numbers:

$$N_{w0} = R_B^{Q_{w0}}$$

where $R_B$ is the bifurcation ratio, $N_{w0}$ is the number of streams of order $w_0$, and $Q$ is the network order. In river networks, $R_B$ and $R_L$ typically average near 4 and 2, respectively, but range between 3 and 5 for $R_B$ and 1.5 and 3.5 for $R_L$.

The fractal dimension of the erosion rill-channel network is estimated from photographs taken at regular 1-h intervals using a functional box-counting method as described by Lovejoy et al. (1987). Tarboton et al. (1988) applied this method to a stream network by beginning with a set of points embedded in a $d$-dimensional space. Here, the method is applied to a rill-channel erosion network. The $d$-dimensional space (i.e., here the photograph where $d = 2$) is covered by a mesh of $d$-dimensional boxes of area $r^d$. By varying the length scale of the boxes, $r$, that is, by varying the size of the squares, a relationship develops between $r$ and the number of cubes of size $r$ within the mesh that contain elements of an erosion rill channel, $N(r)$, of the form:

$$N(r) \propto r^{-D_f}$$

where $D_f$ is the fractal dimension of the network defined by Hentschel and Procaccia (1983) as:

$$D_f = -\lim_{r \to 0^+} \frac{\log N(r)}{\log r}$$

for a set of samples approaching infinity in number. A projection of $N(r)$ vs. $r$ on a log–log scale is a straight line of slope $-D_f$. We use box sizes of $r = 4.0, 8.5, 13.0, 17.0$, and 30.0 mm; smaller box sizes are very difficult to work with and larger sizes do not provide a meaningful number of samples. This range of box sizes only covers one order of magnitude, a limitation of the approach applied in the manner presented here. As $D_f \rightarrow 2$, the network is said to become more space filling. A linear ANOVA model is used to study the dependence of the fractal dimension, $D_f$, on time and slope with the form:

$$D_f = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

where $x_1$ represents simulation time and $x_2$ represents initial hillslope-scale slope.

Antecedent sediment moisture content (concentration of water by weight) is inversely proportional to the time period since the previous rainfall experiment (Fig. 4). Drainage densities for the erosion rill-channel network are calculated in pixel units from the photographs taken at the finest mesh used throughout this analysis (4 × 4 mm).

Every pair of points within a drainage network tree has a unique path connection. Therefore, the distance a drop of water travels from input to basin outlet is uniquely determined by the initial position of the droplet in the basin and by the basin topography and network topology. Hydrologic response of a drainage system subject to rainfall input is particularly dominated by the arrangement of the concentrated flow paths (e.g., Rodriguez-Iturbe and Rinaldo, 1997). The spatial organization of these paths relative to the network is often characterized by the width function (Shreve, 1969), which gives the number of links in the network as a function of a flow distance $x$ from the outlet, $N_L(x)$. Width functions can be readily modeled by Fourier series of the form:

$$N_L(x) \equiv \frac{A_0}{2} + \sum_{n=1}^{m} a_n \cos \left( \frac{n \pi x}{p} \right) + b_n \sin \left( \frac{n \pi x}{p} \right)$$

where $p$ is the period, $x$ is distance from outlet, $m$ is the number of harmonics or frequencies, and $A_0$, $a_n$, and $b_n$ are constants fit to the data using a nonlinear optimization technique that reduces the total sum of squared errors. The low frequencies of the width
function describe the shape of the watershed drained by channels; and the high frequencies describe the bifurcation characteristics of the network, the rate of change of numbers of channels by distance from outlet (Rodríguez-Iturbe and Rinaldo, 1997). Width functions were obtained also from the photographs. Fourier fits for \( m = 3 \) have mean \( r^2 \) values of 0.98 and standard deviation of 0.01.

The power, \( S_p(f) \), contained in a single frequency is proportional to the sum of the squares of the corresponding Fourier coefficients; that is:

\[
S_p\left(\frac{n\pi}{p}\right) \propto (a_n^2 + b_n^2)
\]

(8)

where the frequency is \( f = n\pi/p \). An ANOVA model is used to study the dependence of the scale \( A_0 \) and signal power contained in the wave number \( (n_1, n_2, \text{ and } n_3) \) on time, antecedent sediment moisture content (specified as a time interval since last rainfall), and slope:

\[
(A_0, S_p\left(\frac{n\pi}{p}\right)) = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_3x_3
\]

(9)

where \( x_1 \) represents time, \( x_2 \) represents the number of days since the previous rainfall, and \( x_3 \) represents experimental slope.

3. Results and discussion

3.1. Rill-channel geometry and geomorphology

As mentioned previously, simulations are continued until the network ceases to vary significantly for a period of 30 min; that is, until the sediment outflow reaches a quasi-steady state or ceases completely. This requires limitation of the sediment supply for the water flowing in the erosion rill channels. As can be seen in Fig. 3 a degree of preferential transport of the finer sediments occurred in the experiments. Thus, the erosion rill channels become stable over time and that eventually limits the supply of sediment to that coming from non-channelized areas only (which may or may not be produced). As the erosion network evolves toward an equilibrium state, the shear stress exerted on the non-channelized substrate surface is not large enough to detach any sediment; and the channel stability prevents further channelization.

Mean values for Horton’s bifurcation and length ratios measured at the end of rainfall simulations are 3.8 and 2.1 and have variances of 0.55 and 1.1, respectively. The measured values are within the expected ranges indicated above for stream networks.
Average widths and depths for the $9^\circ$ slopes are greater than those for the $5^\circ$ slopes (Table 1; Figs. 5 and 6). The average width-to-depth ratio for $9^\circ$ slopes is less than those for $5^\circ$ slopes (Table 1; Fig. 7). The relationship between width and depth with slope indicates a higher degree of incision on the steeper slope. The transport capacity of the larger $9^\circ$ slopes is greater than that of the $5^\circ$ slopes because larger incisions are observed and implied by the increased depth observed in the rill channels at $9^\circ$ slopes (Fig. 6). Hillslope-scale slope and local slope are expected to play a significant role in erosion network evolution under optimality principles. Optimality in energy expenditure requires that the total energy expenditure in a drainage network approach a minimum and that the distribution of energy expenditure per unit flow area approach a constant during evolution (e.g., Rodriguez-Iturbe et al., 1992; Molnar and Ramirez, 1998a). The drainage network that develops within a given drainage area subject to the same energy input as well as uniform surface and subsurface substrates should have the same relationship between the local slope at a point in the network, $S$, and the area drained by that point, $A$; that is, the same slope–area relationship (Rodriguez-Iturbe and Rinaldo, 1997; Knighton, 1998). Observed slope–area relationships are power laws of the form $S \propto A^z$. For $S \propto A^z$ to remain constant from the $5^\circ$ slope to the $9^\circ$ slope, more incision would be required, meaning flattening of rill-channel scale slope, on the steeper slope as observed in the experiments; no direct measurements of slope–area relationships, however, were made in this experiment.

Corresponding arguments with downstream hydraulic geometry relationships describing flow depth and width are more difficult with the data collected during this experiment as cross-section shapes are quite variable and flow depths are very small compared to the erosion rill-channel depth. In addition, the depth and width of the erosion rill channels themselves may not be good representations of the depth and width of the flow.

### 3.2. Network growth

How drainage networks develop through time has mostly been studied qualitatively through descriptions...
such as those of Horton or Glock for lack of quantitative measures (Glock, 1931; Schumm et al., 1987). Recently, many quantitative measures of the characteristics of river networks have been developed, as discussed in the introduction; however, these have generally been applied to equilibrium systems, not those under active development. Niemann et al. (2001) studied the growth of numerically simulated networks using a theoretical model and concluded that only when elevation, a substitute for erosion potential, is considered during network growth do the end states have forms quantitatively similar to actual river networks. Schorghofer et al. (in press) studied channel network and postulated the mathematical development of an elongation mechanism they call “Wentworth instability”. Here, quantitative tools developed for river systems are applied to describe the growth of evolving hillslope erosion rill-channel networks.

Measures of the fractal dimension, $D_f$, determined by fitting a line to the log–log plot of $N(r) \propto r$, show a strong dependence on time and a weak dependence on slope. The linear fits to the log–log plots, not shown, have a mean $r^2$ of 0.98 and a standard deviation of $r^2$ of 0.012. The ANOVA model of Eq. (6) accounted for more than 50% of the variability in the fractal dimension values, and the $p$-values for $b_1$ and $b_2$ are $6.97 \times 10^{-4}$ and 0.02 and have means of $4.40 \times 10^{-3}$ and $1.86 \times 10^{-3}$, respectively. The plot of $D_f$ with time for all six simulations is shown in Fig. 8. However, the experiment represented by the gray continuous line in Fig. 8 (referred to heretofore as Ex. 4) appears to be an anomaly and, therefore, the ANOVA model of Eq. (6) was repeated excluding that experiment. In the absence of the data from Ex. 4 the $p$-values for $b_1$ and $b_2$ were determined to be 0.002 and 0.083, respectively. At the $p < 0.05$ significance level $b_2$ is not significant although at the $p < 0.1$ level it is significant. There is some question therefore as to the significance of slope on the rate of growth of $D_f$ although there appears to be some influence. Fig. 8 shows that $D_f$ (i.e., the degree of space filling) grows, as does the rill-channel network, and that it approaches the fractal dimension characteristic of the equilibrium network asymptotically. The variability observed in the plot results from the methods utilized, specifically the reliance on manual interpretation of photographic information.

For river networks, Tarboton et al. (1988) noted that two asymptotic trends occur on a log–log plot of $N(r)$ vs. $r$ distinguished by the magnitude of $r$; for small-scale boxes relative to the scale of the map, $D_f \rightarrow 1$, whereas for large-scale boxes, $D_f \rightarrow 2$. For

![Fig. 8. Fractal dimension, $D_f$, as a function of time. The secondary index 1–3 for each slope identifies the experimental runs.](image-url)
small box sizes, the fractal dimension reflects the characteristics of the individual channels within the network and \( D_t \to 1 \), whereas for large box sizes, it actually represents the fractal dimension of the network and \( D_t \to 2 \). The measurements made here were taken over a small range of box sizes (one order of magnitude), which could play a significant role in defining the observed fractal dimension \( D_t \). The scale of the boxes used relative to the picture size, however, is in the appropriate range to allow determination of \( D_t=2 \) if the networks are completely space filling.

At any given point in time, \( D_t \) represents how much space comprises the erosion rill-channel network. Hillslope drainage networks become more space filling with time, as has been described qualitatively throughout the literature (e.g., Schumm et al., 1987) and as has been shown quantitatively here (Fig. 8). These experiments indicate, however, that a unique value of this fractal dimension exists toward which the networks evolve and that the fractal dimension stops increasing once this asymptotic value is reached, presumably at equilibrium. In this study, the asymptotic value appears to be approximately 1.2, regardless of initial condition and hillslope-scale slope. The dependence of the equilibrium value of \( D_t \) on the time it took to reach equilibrium and initial scale slope was tested, with and without the data from Ex. 4, using an ANOVA model as described by Eq. (6) and neither \( \beta_1 \) nor \( \beta_2 \) were determined to be significant (\( p \gg 0.05 \)). There is no statistical evidence, therefore, that the equilibrium value of \( D_t \) is dependent on the time it took to reach equilibrium or the initial hillslope-scale slope. From Eqs. (4) and (5), the statistically significant \( \beta_1 > 0 \) implies that the erosion network becomes more space filling with time. In addition, a significant \( \beta_2 > 0 \) would be an indication that the erosion rill-channel networks become more space filling more quickly on steeper hillslope-scale slopes when subject to the same rainfall intensity for the same period of time.

Width functions generally have a form such that \( N_t(x) \) approaches a minimum as \( x \to 0 \) and as \( x \to D_D \) (the distance to the drainage divide). \( N_t(x) \) approaches a maximum near \( D_D/2 \) and is subject to oscillations occurring at different length scales. Tests to determine whether the characteristics of the width function expressed in terms of the scale factor, \( A_0 \), and the signal power, \( S_p \), depend on time (i.e., the stage of network development), antecedent initial moisture content, or hillslope-scale slope show that only time is a significant variable (Table 2). Further, the tests show that time is only significant in determination of the scale coefficient, \( A_0 \), and of the signal power contained within the \( n_1 \) frequency. The scale factor increases with time and the power contained within the most dominant “low” frequency also increases with time. The signal power contained within the higher frequencies does not significantly change as a function of time but is an important variable in the ANOVA model.

Initial conditions, determined by base-level position, played a large role throughout formation of the networks during the landscape simulator experiments of Schumm and Khan (1971), Schumm et al. (1972), and Schumm (1977). In this research, the influence of initial conditions, excluding hillslope-scale slope, cannot be determined to persist throughout development of the networks, although initial conditions play an important role in early formation of the networks. Fig. 9 shows the drainage density, in pixel units as a function of the number of days that the hillslope was allowed to dry between experimental runs without rewetting. The number of days that the hillslope was allowed to dry between experiments without rewetting is a surrogate measure for initial moisture conditions. Because the time required for water to begin flowing on the substrate surface is nonlinearly related to the initial soil moisture, the data is fit with exponential functions. The results shown in Fig. 9 indicate that the drainage density after 1 h of simulation is highly dependent on initial moisture condition (\( r^2 = 0.79 \)),

<table>
<thead>
<tr>
<th>Scale factor</th>
<th>Signal power</th>
</tr>
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<tbody>
<tr>
<td>( A_0 )</td>
<td>( S_p(n_1) )</td>
</tr>
<tr>
<td>Hour Coefficient ( 0.071 )</td>
<td>( 0.008 )</td>
</tr>
<tr>
<td>Significance ( &lt; 0.05 )</td>
<td>( &lt; 0.05 )</td>
</tr>
<tr>
<td>Antecedent moisture Coefficient ( -0.00 )</td>
<td>( -0.001 )</td>
</tr>
<tr>
<td>Significance ( 0.22 )</td>
<td>( 0.34 )</td>
</tr>
<tr>
<td>Slope Coefficient ( -0.007 )</td>
<td>( 0.002 )</td>
</tr>
<tr>
<td>Significance ( 0.55 )</td>
<td>( 0.65 )</td>
</tr>
</tbody>
</table>
and that this dependence decreases with time as expected. For drier initial conditions, more water is infiltrated at the beginning of each simulation than for wetter initial conditions and, thus, the evolution of the network is delayed. Once evolution of the networks begins, the observed rate of growth during the first hour is dominated by the initial moisture condition of the sediment substrate. Therefore, even for a storm of intensity greater than that of the effective storm, the duration of the storm must be greater than the time necessary to begin evolution of the network as defined by the initial moisture characteristics of the sediment. The high $r^2$ value for the first hour shows the dominance of initial conditions on early development of the networks. This result also indicates that early development of the erosion network is nonlinearly related to the rainfall history of the site.

3.3. Generalization

In the real world, hillslopes are subject to storms of spatially heterogeneous intensity and duration, and the relationship between slopes and erosion potential becomes more complex. Let us consider this relationship with respect to an effective storm for the development of hillslope erosion networks, which is defined conceptually in a similar manner as effective discharge is for the development of a river. An effective storm is defined herein as a rainfall of constant intensity and duration such that the existing network of hillslope erosion rill-channels is neither erosional nor depositional. For a storm of smaller intensity than the effective storm, little opportunity exists for adjustment of the erosion network, and little rill-channel sediment production will occur; any adjustment, however, would be depositional. For a storm of greater intensity than the effective storm, the rill-channel network will be in active erosional development adjusting to the new intensity and duration at a rate determined partially by the slope of the rill-channel network in adjustment and the distribution of energy within the system. We, therefore, hypothesize that for systems in which the development of rill-channels is the dominant contribution of sediment yield from the hillslope, slope may play a role only during storms greater than the effective storm.

Schumm et al. (1987) suggested that, in a system with a fixed base level, networks progress through time from a very chaotic dendritic structure to a more organized network with fewer segments. While what Schumm et al. meant by chaotic is difficult to say, here the interpretation is made that little structure is associated with the drainage network. One description of network evolution holds that networks evolve through a process of “slow headward growth with full elaboration of the entire texture by streams of all orders” (Rodríguez-Iturbe and Rinaldo, 1997), a general view based on combinations of theories developed by Ruhe (1952), Glock (1931), and Morisawa (1964). This network development is described here through the characteristics of width functions as encoded in Fourier series approximations. The power

![Fig. 9. Influence of antecedent moisture on initial drainage density measurements.](image-url)
contained within the lowest frequency significantly grows with time. The powers in the higher frequencies do not significantly change with time; however, they are important in describing the width functions. These relative frequencies have been associated respectively with the shape of the region in which the network develops and the bifurcation structure of the network itself. Our results indicate that if the “chaotic” network of Schumm et al. is understood as lacking inherent order, then our simulated networks are not chaotic for any time after 1 h of rainfall simulation because the bifurcation structure does not significantly vary throughout the evolution of the networks. The increase in power associated with the low frequency can only be associated with filling the area more fully.

4. Conclusions

Developing networks of erosion rill-channels on a slope void of vegetation and subject to constant rainfall exhibit time-varying fractal characteristics that describe the space filling properties of the networks and may be a function of hillslope-scale slope. The networks have been shown to become progressively more space filling with time as described by temporally increasing fractal dimensions, $D_f$. The fractal dimension, however, does not increase indefinitely but approaches an upper limit that, when reached, remains constant. Steeper slopes have been shown to enhance the space filling characteristic of the erosion rill-channel network; that is, to produce larger fractal dimensions more quickly. Through an analysis of the width functions of actively developing erosion rill-channel networks, we have shown that the network appears to fill the drainage area more completely with time. The bifurcation characteristics of the width functions as described by the power contained in the high wave numbers of the Fourier series appear to remain constant throughout network evolution. An inherent order is, therefore, contained within these erosion rill-channel networks throughout development.

Slope plays a significant role in individual erosion rill-channel geometry, 9° slopes leading to wider and deeper erosion rill-channels but having a smaller width to depth ratio than 5° slopes. The result that rill-channels on a 9° slope incise more than on a 5° slope is consistent with optimality theories based on the distribution of energy expenditure within a channel network.

Further research is warranted to find the rates of growth of hillslope erosion networks under different rainfall environments, substrate properties, in the presence of vegetation, as well as to establish the role that hillslope-scale slope plays in the rate of network development. Erosion prediction models would benefit greatly from the inclusion of information regarding rates of growth of the erosion rill-channel network as a whole in response to individual storm events and including the history of previous rainfall events.

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