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On downstream hydraulic geometry and optimal energy expenditure: case study of the Ashley and Taieri Rivers

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Abstract

The downstream distribution of channel geometry and of the rate of energy expenditure per unit channel area P_a are analyzed on extensive datasets from the Ashley and Taieri Rivers in New Zealand. We investigate whether the rivers conform to local optimality, by which P_a tends to be constant throughout the network. We look at energy expenditure from two perspectives. (1) In the context of downstream hydraulic geometry (DHG) relations, we derive equations for the general unconstrained optimal combination of the DHG exponents b and f under local optimality, and compare these with data-derived exponents from the Ashley and Taieri Rivers. (2) Treating P_a as a random variable, we examine the downstream scaling of moments of P_a. Results suggest that $E(P_a)$ is constant in the higher order stream network in both basins. The scaling of $\mathrm{Var}(P_a)$ with discharge illustrates that basin and channel heterogeneity play an important role in understanding the spatial distribution of energy expenditure and its variability in river systems.

Keywords: Downstream hydraulic geometry; River networks; Energy expenditure

1. Introduction

Regularity in the topological structure of river networks and in the distribution of their channel properties is an intriguing display of self-organization in nature. Efficiency and optimality in energy expenditure have been used to explain the regularity in hydraulic geometry (Leopold and Maddock, 1953; Leopold and Langbein, 1962; Langbein and Leopold, 1964), channel pattern (Bull, 1979; Chang, 1979), and river network structure (Howard, 1990; Rodriguez-Iturbe et al., 1992; Rigon et al., 1993). In this paper, extensive datasets from the Ashley and Taieri Rivers in New Zealand are used to further examine the issue of optimality in energy expenditure in the context of downstream hydraulic geometry (DHG) relations and the spatial distribution and scaling of energy expenditure.

DHG relations are functions of channel geometry (top width w, average depth d, average velocity v, and slope S) and streamflow Q of equal frequency of occurrence throughout the river network (Leopold and Maddock, 1953):

$$w = c_1 Q^b, \qquad d = c_2 Q^f, \qquad v = c_3 Q^m, \qquad S = c_4 Q^z \tag{1}$$

The exponents in these relations have been determined for many river systems, generally for the

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mean annual or bankfull flow conditions (Park, 1977; Griffiths, 1980; Knighton, 1984; Jowett, 1998). Their optimal values have been explored in the context of minimum entropy and equal adjustment (Leopold and Langbein, 1962; Langbein, 1964; Langbein and Leopold, 1964; Williams, 1978) and optimal energy expenditure (Rodriguez-Iturbe et al., 1992; Molnar and Ramirez, 1998a).

In their study of the structure of river networks, Rodriguez-Iturbe et al. (1992) postulated three principles that define the optimal topological structure: (1) minimum energy expenditure in a river link, (2) constant energy expenditure per unit channel bed area, and (3) minimum energy expenditure in the whole network. A combination of these principles led to the definition and modeling of optimal channel networks (OCNs) that exhibit remarkable similarities with natural river networks in their fractal aggregation structure, as well as other empirical geomorphological properties (Rodriguez-Iturbe et al., 1992; Rinaldo et al., 1992; Ijjasz-Vasquez et al., 1993; Rigon et al., 1993; Sun et al., 1994). Furthermore, the first two principles were used to derive optimal values for the DHG exponents $b = f = 0.5$ assuming constant velocity (Rodriguez-Iturbe et al., 1992), and $b = f = 0.5 - m/2$ assuming variable velocity (Molnar and Ramirez, 1998a). However, both of these OCN results were obtained for the special case $b = f$, which is not common in nature. In most rivers, the width–depth ratio increases in the downstream direction, $b > f$ (Knighton, 1984).

Despite the interest in the OCN concept, there have been relatively few comprehensive basin studies of the distribution and variability of energy expenditure in river networks, mostly due to insufficient data (Graf, 1983; Magilligan, 1992; Lecce, 1997; Molnar and Ramirez, 1998b; Knighton, 1999). This study is motivated by (1) the unique dataset with high spatial resolution of concurrent measurements of streamflow and channel geometry in the Ashley and Taieri Rivers, and (2) the finding that the DHG exponents of these rivers do not conform with OCN exponents, which suggests that the basins are sub-optimal (Ibbitt, 1997).

The underlying hypothesis in this study is that, in an average sense, alluvial streams adjust towards a condition of uniform energy expenditure. This condition is a dynamic measure of equilibrium, where the sediment transport and erosive capacity is in balance across the basin, and variability around this condition reflects the geomorphic and lithologic basin controls on channel adjustment. The local optimality principle of constant energy expenditure rate per unit channel area, P_a, is used to examine the spatial distribution of channel geometry in the Ashley and Taieri Rivers. We are not concerned with the minimization of energy expenditure in the river networks, i.e. with their optimal topological structure (Ibbitt et al., 1999), but rather we look at whether the observed channel properties of the river networks conform with local optimality, i.e. whether P_a approaches a constant value. In this case, DHG exponents may satisfy local optimality and at the same time be different from the traditional OCN exponents $b = f = 0.5$.

We look at energy expenditure from two perspectives. (1) In the context of DHG relations, we develop equations for the general optimal combination of the exponents b and f, without any constraints ($m \neq 0$, $b \neq f$), for comparison with the data-derived exponents from the Ashley and Taieri Rivers (Section 3). (2) We analyze the scaling of moments of P_a with discharge in the river systems, and discuss the role of variability in P_a in the geomorphological context (Section 4). Our work extends analyses of Ibbitt (1997) on the Ashley and Taieri Rivers, and provides additional verification and testing of the concepts of optimality in energy expenditure in natural river systems.

2. Basin and data description

The Ashley and Taieri basins are located on the South Island of New Zealand. The Ashley basin covers an area of 121 km^2 with elevations from 500 to 1800 m. Mean annual precipitation ranges between 1200 and 2000 mm and mean annual discharge at the outlet of the basin is about 4 $m^3 s^{-1}$ (runoff rate 1040 mm yr^{-1}). The Taieri basin is about 350 km further south and covers an area of 158 km^2. The relief is gentler, with elevation ranging between 600 and 1200 m. Mean annual precipitation is estimated at 1400 mm and mean annual discharge at the outlet is about 4.9 $m^3 s^{-1}$ (runoff rate 980 mm yr^{-1}). Both basins are largely undisturbed and sparsely inhabited, with occasional seasonal sheep and cattle grazing. Vegetation is tussock grassland with some native

Table 1
Analysis of DHG on the Ashley and Taieri Rivers

	Taieri River ($n = 286$)	Ashley River ($n = 225$)
DHG exponents (Eq. (1))		
b (width)	0.533 ± 0.020 (0.80)[a]	0.444 ± 0.022 (0.75)
f (depth)	0.256 ± 0.022 (0.45)	0.231 ± 0.019 (0.53)
m (velocity)	0.211 ± 0.029 (0.23)	0.325 ± 0.024 (0.57)
z (slope)	-0.390 ± 0.026 (0.56)	-0.442 ± 0.040 (0.48)
DHG constants (Eq. (1))		
c_1	0.261	0.285
c_2	0.052	0.054
c_3	0.074	0.064
c_4	0.096	0.513
Ranges of data used		
Q (l s^{-1})	0.1–633 (47.9)[b]	1.65–1559 (140)
w (m)	0.05–16.6 (1.54)	0.18–9.1 (2.07)
d (m)	0.01–0.546 (0.106)	0.023–0.491 (0.144)
v (m s^{-1})	0.004–0.575 (0.141)	0.031–0.802 (0.265)
S (m m^{-1})	0.001–0.333 (0.074)	0.002–0.798 (0.157)

[a] The term in parenthesis is r^2 of the linear regression on log-transformed data. The $\pm$ symbol represents 90% confidence limits of the estimate of the scaling exponents.

[b] The term in parenthesis is the arithmetic mean from n observations.

birch forest in the Ashley basin and exotic pasture species in the Taieri basin. The fluvial systems consists of alluvial, mostly gravel bed streams, on the average twice as steep in the Ashley basin. The basins are described in detail elsewhere (Ibbitt, 1997; McKerchar et al., 1998; Ibbitt et al., 1998).

In order to ensure conditions with approximately equal discharge frequency throughout the river network, data were collected in a short period of steady flow and equal antecedent conditions. Streamflow and channel geometry were measured at fairly evenly distributed sites (not at permanent gaging stations) and upstream of every major confluence throughout both river networks (McKerchar et al., 1998; Ibbitt et al., 1998).

The Ashley River experiment took place over a 5-day period in February 1994. Mean discharge at the outlet of the basin during this period was 1.55 m^3 s^{-1} (discharge exceeded 70% of the time at the gage). Altogether measurements of streamflow, channel geometry, elevation and other channel properties were made at 336 sites in the basin. In this analysis, only 225 sites with concurrent measurements of

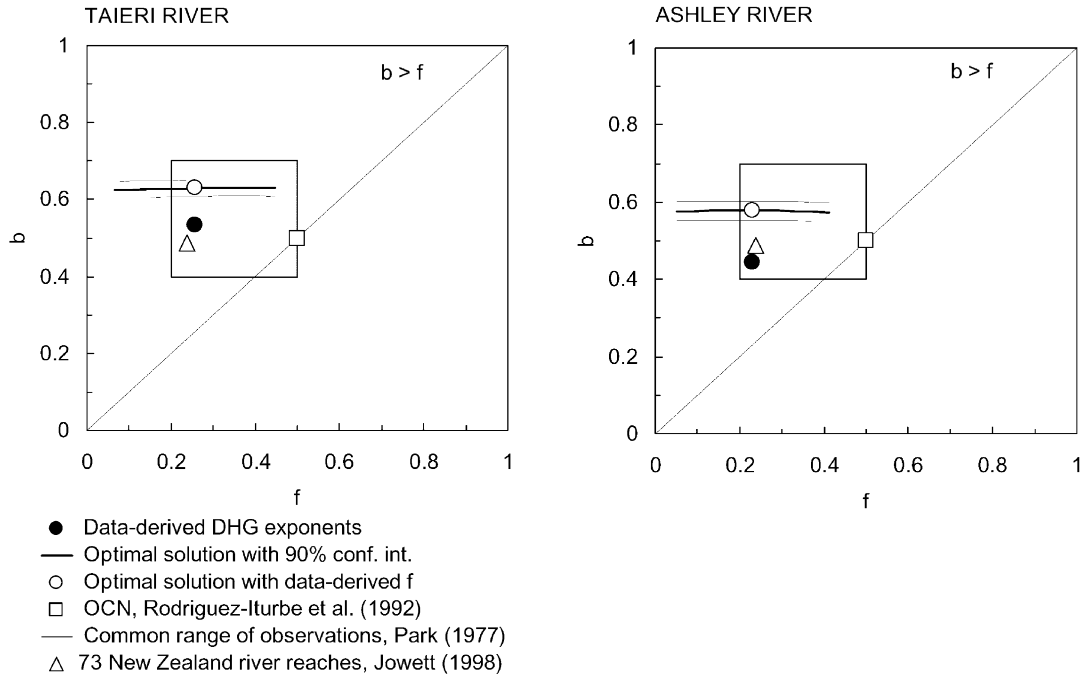

Fig. 1. DHG exponents for width b and depth f. The data-derived exponents are from Table 1. The optimal solution with 90% confidence limits for b is computed using the procedure described in this paper. The common range of observations is taken from river data in temperate and humid regions (Park, 1977). The average for 73 New Zealand river reaches is computed from data of Jowett (1998).

streamflow, flow-related cross-sectional geometry and channel bed slope were used. Channel bed slope is computed from the elevation difference and channel length to the nearest upstream site, or next upstream 20 m map contour for external sites. The average length of channel links between sites was 427 m.

The Taieri River experiment took place over a 4-day period of low summer flows in February 1995. Mean discharge at the outlet of the basin during this period was 0.7 $m^3 s^{-1}$ (discharge exceeded 99% of the time at the gage). Streamflow and channel measurements were made at 300 sites, out of which 286 had concurrent measurements of streamflow, flow-related cross-sectional geometry and channel bed slope. Channel bed slope was computed as in the Ashley River case, and the average length of channel links between sites was 726 m.

Details of the data collection experiments are described in McKerchar et al. (1998) and Ibbitt et al. (1998). Data and maps of the stream networks and measurement locations can be downloaded from the web page of the National Institute of Water and Atmospheric Research (NIWA).

In the context of the DHG relationships, it is assumed that the short measurement window during a period of steady flow provides discharges with approximately equal frequency of occurrence at every site. Secondly, it is assumed that the level of streamflow, which is much lower than bankfull or mean annual flow conditions, is representative of channel geometry. These assumptions will be addressed later. The DHG relations were evaluated using least squares linear regression fits to the log-transformed data in the form of Eq. (1) for both rivers. The results are reported in Table 1. The data-derived DHG exponents $b = 0.44, f = 0.23$ (Ashley) and $b = 0.53, f = 0.26$ (Taieri) are very similar to the average exponents computed from 73 river reaches across New Zealand for mean annual discharge conditions, $b = 0.49, f = 0.24$ (Jowett, 1998), and are within the ranges observed on rivers in humid and temperate regions of the world (Park, 1977), but they do not conform well with the OCN values, because $f < 0.5$ (Fig. 1). There is also a considerable downstream increase in the average velocity of flow: $m = 0.33$ (Ashley) and $m = 0.21$ (Taieri), contrary to the original OCN assumption. The slope scaling exponent z is smaller (in absolute value) than the OCN predicted value $z = -0.5$ for both basins: $z = -0.44$ (Ashley) and $z = -0.39$ (Taieri).

3. Optimal energy expenditure and downstream hydraulic geometry

The application of the local principle of optimality of Rodriguez-Iturbe et al. (1992) allows us to derive equations for the unconstrained optimal combination of the DHG exponents b and f, and to compare these with the data-derived exponents from the Ashley and Taieri Rivers. In this context, the DHG relations are treated as independent functions of average channel geometry and streamflow, see Troutman (1996). They represent a combination of channel topography and geometry that implies a certain distribution of the average rate of energy expenditure per unit channel area in the river network in the form of a 'downstream energy expenditure' (DEE) function. It is this function, which is, studied here.

3.1. Energy expenditure in a river network

Rivers are non-conservative systems in which energy is dissipated by transporting water and sediment, and in geomorphic adjustment of the channel. The problem of estimating the amount of energy expenditure associated with overcoming surface and system resistance in the river network, transporting sediment and flood debris, and eroding the channel bed is extremely complicated (if not impossible) to solve in the natural environment. A simplification on a river network scale is to consider energy expenditure to be primarily associated with friction of the fluid at the channel boundary. A river network can be schematized to consist of a series of connected rectangular channel links of different lengths, widths, and flow depths. The term in the power equation (Rouse, 1959) that represents the rate at which mechanical energy is dissipated in a given link, P_i, can then be approximated as (Rodriguez-Iturbe et al., 1992; Molnar and Ramirez, 1998a):

$$P_i = LP_{\mathrm{w}} \int_0^l \tau_x \frac{\partial v_x}{\partial z} \mathrm{d}z = LP_{\mathrm{w}} \tau_0 v \quad (2)$$

where L is the channel link length, P_{w} is its wetted perimeter, τ_x is the tangential shear stress, and v_x is the flow velocity in the downstream direction. The most important assumptions made in obtaining Eq. (2) were: (1) one-dimensional steady flow in a channel link; (2) equality of tangential shear stresses and velocity gradients at the bed and banks in the downstream direction; and (3) energy dissipation occurring predominantly in a boundary layer with depth l at which the velocity v_x equals the depth-averaged velocity v, and in which shear stress is constant and equal to the average boundary shear τ_0 (Molnar and Ramirez, 1998a).

The rate at which energy is dissipated per unit channel area P_a in a link ($P_a = P_i/LP_w$) is then equal to:

$$P_a = \tau_0 v \tag{3}$$

and it is sometimes referred to as unit stream power. Under uniform flow, average boundary shear stress can be approximated as:

$$\tau_0 = \gamma R_h S \tag{4}$$

where γ is the specific weight of water (for sediment laden flows, the specific weight of the fluid-sediment mixture can be used), R_h is the link hydraulic radius, and S is the link channel bed slope. Then, P_a becomes the well-known formula:

$$P_a = \gamma S \frac{Q}{w + 2d} \tag{5}$$

where P_a, w, d, Q, and S correspond to a particular channel link.

Combining the DHG relations in Eq. (1) with the rate of energy expenditure in Eq. (5), leads to the following expression for P_a:

$$P_a = \gamma \frac{c_4 Q^{z+1}}{c_1 Q^b + 2c_2 Q^f} \tag{6}$$

where the constants c_1, c_2, c_4 depend on streamflow frequency. Eq. (6) gives P_a as a function of the spatial distribution of discharge with equal frequency of occurrence throughout a river network, a DEE relation of the form $P_a \propto Q^\beta$.

3.2. Local optimality

The principle of local optimality states that in an optimal river network, average channel properties are such that P_a is constant throughout the network. Here, a general methodology is adopted (following Molnar and Ramirez, 1998a) to determine the unconstrained optimal combinations of the DHG exponents that will satisfy local optimality.

The downstream distribution of P_a is a function of Q and DHG parameters, $P_a = P_a(Q; b, f, \ldots)$, in the form of Eq. (6). For P_a to be constant,

$$\frac{d(P_a(Q; b, f, \ldots))}{dQ} = 0 \tag{7}$$

The analytical solution to Eq. (7) gives the following relationship between the optimal width and depth exponents for a given topography of a basin represented by the slope scaling exponent z and flow-related channel size represented by c_1 and c_2:

$$b = f + \frac{\ln\left(\frac{2c_2}{c_1}\right) + \ln\left(\frac{f - z - 1}{z + 1 - b}\right)}{\ln Q} \tag{8}$$

The constants c_1 and c_2 from Eq. (1) are required to best fit channel geometry data, in that they minimize the mean square errors s^2 in the DHG relations:

$$s_W^2 = \frac{1}{n-2} \sum_{i=1}^{n} (\ln w_i - \ln c_1 - b \ln Q_i)^2 = \min \tag{9}$$

$$s_D^2 = \frac{1}{n-2} \sum_{i=1}^{n} (\ln d_i - \ln c_2 - f \ln Q_i)^2 = \min \tag{10}$$

where n is the total number of sites analyzed. An iterative solution is required to simultaneously satisfy Eqs. (8)–(10) and compute the optimal combinations of b and f. Note that the analytical solution does not contain the unique case, where $b = f$. It can readily be shown that in this case $b = f = 1 + z$.

However, the above analytical solution to the unconstrained optimal combination of b and f under local optimality remains a function of discharge in Eq. (8). It is desirable to find an optimal solution for the downstream range in discharges. In order to determine the best combination of b and f throughout a river network, Molnar and Ramirez (1998a) defined an optimality function, which is modified here as:

$$h(b, f, \ldots) = \int_{Q_{\min}}^{Q_{\max}} \left| \frac{d(P_a(Q; b, f, \ldots))}{dQ} \right| Q \, dQ \tag{11}$$

where $Q_{\min}$ and $Q_{\max}$ are the minimum and maximum streamflow in a river network (from the spatial

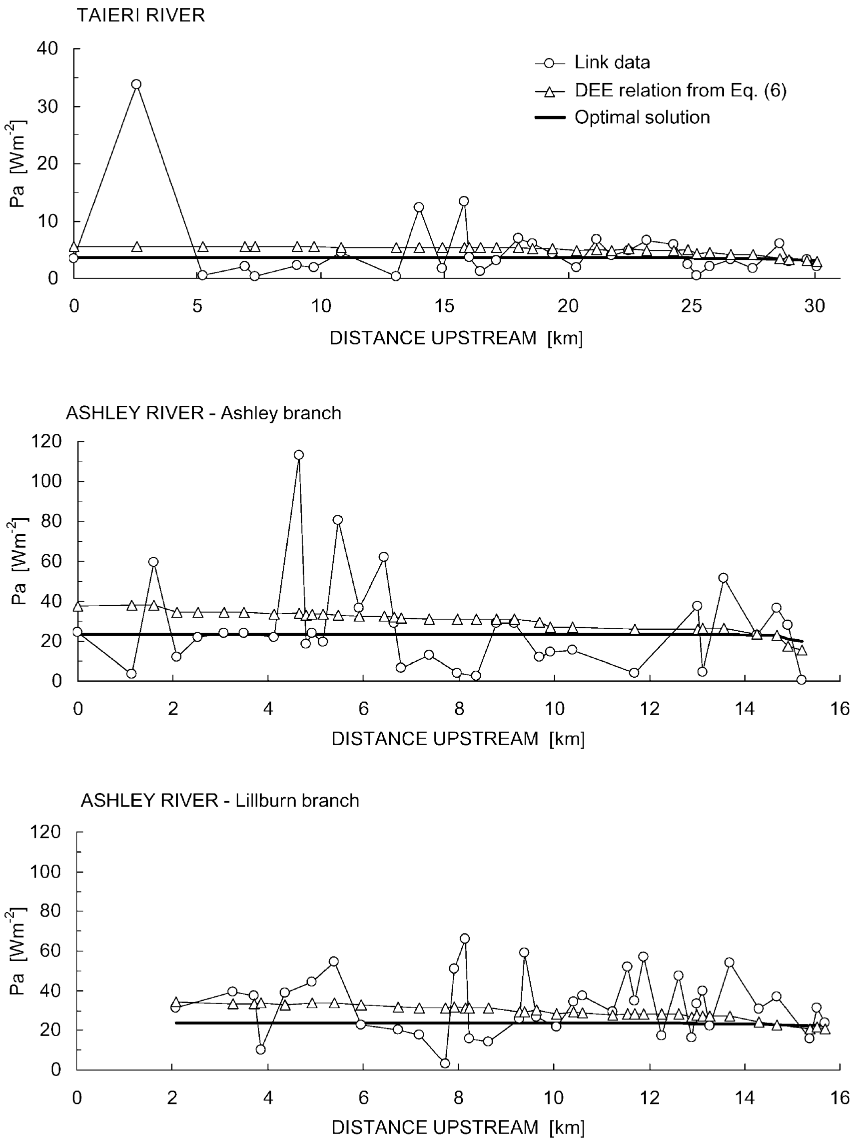

Fig. 2. Downstream distribution of energy expenditure per unit channel area P_a on the main stream segment of the Taieri River and two main branches of the Ashley River. Distance is upstream from the basin outlets. Energy expenditure computed by Eq. (6) using hydraulic geometry relationships is shown with triangles. The optimal solution is one with the data-derived depth exponent *f* from Table 1.

distribution of measurements), and

$$\frac{d(P_a(Q;b,f,\ldots))}{dQ} = \gamma c_4 \frac{c_1(z+1-b)Q^{b+z} + 2c_2(z+1-f)Q^{f+z}}{(c_1Q^b + 2c_2Q^f)^2} \quad (12)$$

The best combination of river network parameters $(b^*,f^*,\ldots)$ is the one that minimizes the optimality function h:

$$h(b^*,f^*,\ldots) = \min_{\forall(b,f,\ldots)} h(b,f,\ldots) \quad (13)$$

For all sub-optimal cases, P_a in the form of Eq. (6) consistently increases or decreases in the downstream direction. Note also that the optimality function h in Eq. (11) weights the derivative dP_a/dQ with discharge magnitude, thereby giving more importance to the downstream sections of the river network.

3.3. Application to Ashley and Taieri River data

In practical application, for a given depth exponent f, we first computed the parameter c_2 from Eq. (10) using flow depth data. Then, the exponent $b = b^*$ was found by minimizing h from Eq. (11), keeping in mind that at every stage in the minimization, the parameter c_1 (itself dependent on b) had to simultaneously satisfy Eq. (9) using flow width data. This iterative procedure was repeated for an applicable range of f values (such that $r^2 > 0.2$ for width DHG), and optimal combinations of b and f were obtained. These optimal combinations are depicted with 90% confidence intervals for the estimate of b in Fig. 1.

The traditional OCN exponents derived by minimizing energy expenditure in a network with constant velocity plot in the center of the figure $b = f = 0.5$ (Rodriguez-Iturbe et al., 1992) and imply that the OCN slope scaling exponent $z = -0.5$. For both analyzed rivers, the data-derived exponents are closer to the unconstrained optimal combinations than to the OCN exponents. However, they still depart from local optimality, having a lower value of b than optimal.

An important observation from Fig. 1 is that optimal b is essentially independent of f, and it is a function of the slope scaling exponent z for both basins. If the effect of channel depth on energy expenditure is neglected, it can readily be shown from Eq. (6) that the condition of local optimality with constant P_a is satisfied when

$$z = b - 1 \tag{14}$$

This well-known relation demonstrates the strong connection between basin topography and channel width under local optimality (the linear correlation coefficients between channel bed slopes and widths were $r = -0.52$ for the Ashley River and $r = -0.41$ for the Taieri River). If one takes P_a to represent the active erosive force on the channel boundary (many sediment transport equations are based on this premise), constant P_a would represent a network, where the basin sediment transport and erosive capacity of the stream would be evenly distributed. One can then argue that in a river network, where P_a is not constant, river sections with high average P_a will have a tendency to degrade relative to the remainder of the network, thereby constantly adjusting channel gradient (Langbein and Leopold, 1964; Bull, 1979).

Based on the above results, what can be said about the downstream distribution of energy expenditure P_a in the Ashley and Taieri Rivers? Fig. 2 shows computed P_a in the main stream segments of the Taieri River and two main branches of the Ashley River compared with the DEE relation from Eq. (6) and the optimal case in which the depth exponent is equal to the data-derived value f from Table 1. There is considerable scatter, but no apparent trend in data-derived P_a. We do not observe headwater maxima in P_a as in Lecce (1997) and Knighton (1999). The DEE relationship from Eq. (6) predicts a small downstream increase in P_a ($P_a \propto Q^\beta$, $\beta > 0$), while the chosen optimal solution is almost constant throughout all streams. Generally, the mainstream segment of the Taieri River seems to conform better to local optimality than the Ashley River. The outlier with high P_a 2.6 km upstream from the outlet on the Taieri River was in a geologically constrained reach with small channel width and unusually high gradient.

The downstream distribution of energy expenditure in the context of DHG relations represents an average state around which variability exists. In Section 4, we look at the variability in P_a.

4. Variability and scaling of energy expenditure

Studies of scaling and variability in channel properties of river networks have largely concentrated on analyses of slopes extracted from digital elevation models (Tarboton et al., 1989; Ijjasz-Vasquez and Bras, 1995), but have also included analyses of energy dissipation and stream power (Rodriguez-Iturbe et al., 1992; Lecce, 1997). In rivers, the observed geomorphological variability in channel geometry and thereby P_a is due to external driving forces, internal channel conditions, and the response of the river network at different spatial and temporal scales (Petts and Foster, 1985). Here, we treat energy expenditure as a random variable (rather than as a function of discharge in the DHG context) and we analyze the distributions and moment scaling of P_a.

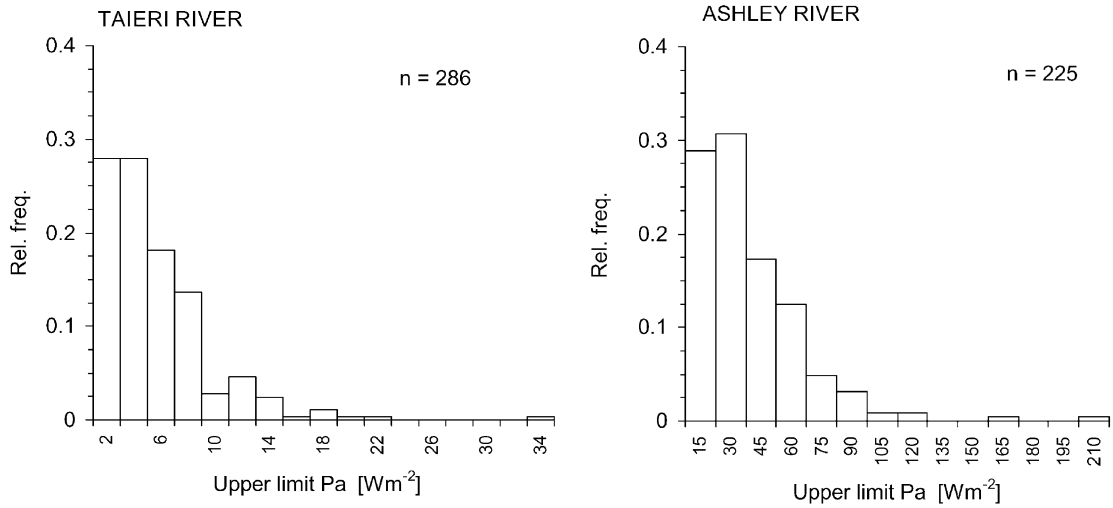

Fig. 3. Histograms of the energy expenditure rate P_a. Mean P_a for the Ashley River is 31.1 W m^{-2}, the coefficient of variation is $C_v = 0.83$ and the coefficient of skewness is $C_s = 2.31$. For the Taieri River, mean P_a is 4.4 W m^{-2}, the coefficient of variation is $C_v = 0.89$ and the coefficient of skewness is $C_s = 2.51$.

Histograms of the energy expenditure rate P_a for both rivers are shown in Fig. 3. It is notable that, although the average energy expenditure rate is seven times larger at the Ashley River than at the Taieri River, because of higher streamflow conditions, the variability and skewness of the distributions are almost identical. In fact, variability in P_a using data from Jowett (1998) for 73 river reaches across New Zealand also gave $C_v = 0.84$ (and skewness $C_s = 1.94$), which compares very well with Ashley and Taieri data (see Fig. 3). The large positive skewness is caused by a few sites that are expending three to five times more energy per unit channel area than the mean. Generally, these are links with very steep slopes that contain coarse bed material (more than 70% boulders in the Ashley River). In terms of

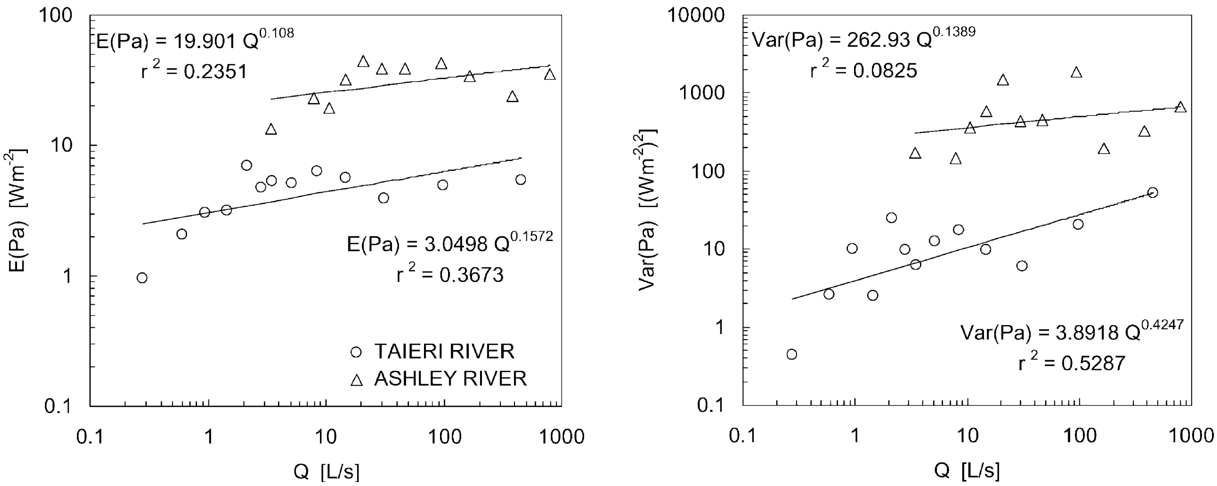

Fig. 4. Downstream scaling of moments of the energy expenditure rate $E(P_a)$ and $\mathrm{Var}(P_a)$ with discharge. Data are grouped into bins with size $n = 22$ for the Taieri River and $n = 20$ for the Ashley River.

channel adjustment, these are geologically constrained channel reaches.

Consider now the scaling of the first two conditional moments of P_a with discharge throughout the river networks:

$$E(P_a|Q) \propto Q^{\gamma}, \qquad \mathrm{Var}(P_a|Q) \propto Q^{\delta} \tag{15}$$

Under local optimality in energy expenditure, where P_a is a random variable, its expected value is constant throughout the river network, $\gamma = 0$. The comparison of the optimal and data-derived exponent b in the DHG-based analysis in Fig. 1 suggests that for the Ashley and Taieri Rivers $\gamma > 0$. Scaling of the conditional mean in the form of Eq. (15) is shown in Fig. 4, where moments were computed for data grouped into bins with size $n = 22$ (Taieri) and $n = 20$ (Ashley). Both rivers show a slight downstream increase in mean energy expenditure in accordance with DHG results ($\gamma = 0.11$ for the Ashley and $\gamma = 0.16$ for the Taieri). It is especially noteworthy that the headwater segments of the river networks are largely responsible for the downstream increase in mean energy expenditure. For instance, when considering the higher order network separately, i.e. only streams with order $\omega \geq 3$, the exponents γ for both rivers were found to be statistically insignificantly different from 0.

Variability in P_a is predominantly caused by the variability in channel bed slopes. Variability in channel bed slopes explains 59% of the variability in P_a in the Ashley River and 85% in the Taieri River. The variance of channel bed slopes scales as:

$$\mathrm{Var}(S|Q) \propto Q^{\theta} \tag{16}$$

and generally decreases in the downstream direction: $\theta = -0.78$ ($r^2 = 0.82$) for the Ashley and $\theta = -0.55$ ($r^2 = 0.75$) for the Taieri. Analyses of θ are often used to express the simple or multi-scaling properties in slope scaling (Gupta and Waymire, 1989; Tarboton et al., 1989).

Rodriguez-Iturbe et al. (1992) show that in a first-order analysis, using drainage area as a surrogate for discharge, and assuming OCN conditions $b = f = 0.5$, $z = -0.5$, the scaling of the variance of energy expenditure P_a in Eq. (15) under local optimality is approximately a function of the scaling of the variance of slope: $\delta = 1 + \theta$ (which gives $\delta = 0.22$ for the Ashley and $\delta = 0.45$ for the Taieri). Scaling of the conditional second moment in Eq. (15) for the Ashley and Taieri Rivers in Fig. 4 shows that the data-derived values are $\delta = 0.14$ for the Ashley and $\delta = 0.42$ for the Taieri (with large scatter), which is close to the first-order OCN approximation.

Under local optimality ($\gamma = 0$) energy expenditure has a multi-scaling character, in which the coefficient of variation is not constant, but increases in the downstream direction as $C_v \propto Q^{\delta/2}$ with $\delta > 0$. It has been argued that the downstream increase in variance, observed here specially in the Taieri River case, is similar to that of a diffusion process, in which the variance is proportional to the length of the path or average travel time for the flow to reach a particular link (Rodriguez-Iturbe et al., 1992). Trends in the variance of P_a are also an indication of watershed heterogeneity in general. For instance, spatial variability in P_a has been connected with geomorphic and lithologic controls on the Galena and Blue rivers in Wisconsin (Magilligan, 1992; Lecce, 1997).

5. Discussion and conclusions

In this paper, we used data from the Ashley and Taieri Rivers to further explore the concept of optimality in energy expenditure in undisturbed river basins. We analyzed the principle of local optimality—where the rate of energy expenditure per unit channel area is constant—and its relationship to channel geometry and basin topography.

In the first part of the paper, we adapted a methodology to find the best optimal unconstrained combination of DHG exponents for width b and depth f in both rivers (following Molnar and Ramirez, 1998a) considering P_a as a function of spatially variable discharge: a DEE relation. We then compared the optimal exponents to the data-derived and traditional OCN exponents. We are in agreement with Ibbitt (1997) that the data-derived exponents for both rivers are not identical with the OCN exponents. However, our results illustrate that the OCN exponents are just a specific case of DHG exponent combinations under local optimality, and an unlikely combination to occur in nature. Especially, in the Taieri River, Fig. 1 shows that the optimal combinations of b and f are in fact not far removed from the data-derived exponents.

This is also well illustrated on the mainstream segment of the river in Fig. 2. Indeed, the Taieri basin is reported to be more likely in a state of dynamic equilibrium, it is in one of the oldest landscapes in New Zealand (Ibbitt et al., 1998), while the Ashley basin is nested in an active region near the Alpine Fault (Ibbitt, 1997).

In the second part of the paper, we treat energy expenditure P_a as a random variable, the mean of which, under local optimality, should be constant throughout the river network. The observed variability in P_a for both river networks reflects the natural variability in watershed topography, channel geometry, and watershed heterogeneity in general. The distributions of P_a, depicted in Fig. 3, are highly (positively) skewed in both basins, the outliers being mostly geologically controlled channel links. The coefficient of variability in P_a was similar for both basins and for New Zealand gravel bed rivers in general. The scaling of first two moments of energy expenditure with discharge in Fig. 4 illustrates that the headwater basins with low mean P_a, are responsible for the downstream increase in $E(P_a)$, i.e. $\gamma > 0$ in Eq. (15). Analyses of the higher order network, which is presumed to be the more stable network, show much better conformity with local optimality. Excluding low order streams gave $\gamma \approx 0$ for both river networks.

Our analyses of the variability in P_a point to the role of basin heterogeneity in channel adjustment. Ibbitt (1997) sees predominantly the controlling factors of geology and heterogeneity in watershed properties as the main reasons for discrepancy between the data-derived DHG exponents and traditional OCN exponents. Just as the downstream distribution of mean P_a may reflect the general control of local optimality on the river network, the spatial variability in P_a may reflect the local lithologic and other basin controls on channel adjustment. In the context of sediment transport and erosion, it is this variability in P_a that will influence the sediment dynamics in the basin (Magilligan, 1992).

An important question that remains is the geomorphic representativeness of the channel observations at the Ashley and Taieri Rivers. There are two main concerns: (1) constant discharge frequency and (2) low flow condition.

It is clear that, in order for streamflow to be determining (and adjusting) channel shape, it has to be of sufficient magnitude. It also, by DHG definition, has to be of equal frequency throughout the network. We believe that the frequency requirement is met adequately with the short time in which measurements were made in both basins (Ibbitt, 1997; Ibbitt et al., 1998; McKerchar et al., 1998). The magnitude of streamflow is of greater concern, since the flow condition during the experiments was well below average for the period (February) of measurement. Especially, in the Taieri River case, average $P_a = 4.4\ \mathrm{W\ m^{-2}}$ is certainly low stream power in the geomorphological context. In the Ashley River average $P_a = 31.1\ \mathrm{W\ m^{-2}}$, which is closer to the average $P_a = 20.1\ \mathrm{W\ m^{-2}}$ for the mean annual discharge conditions for 73 river reaches across New Zealand. But, it has been proposed that a threshold for major geomorphic adjustments in alluvial channels in humid and sub-humid environments may be on the order of $300\ \mathrm{W\ m^{-2}}$ (Magilligan, 1992). In the context of DHG relations, the main question is how will the exponents differ with flow frequency. By comparing DHG exponents for different exceedance probabilities for several New Zealand gravel bed rivers, Ibbitt (1997) found that the exponents did not change significantly with streamflow frequency. This suggests that the spatial distribution of mean P_a would remain invariant with regard to streamflow frequency, although its magnitude would change. However, the effect of the low flow condition on the scaling of the variance of P_a may still be large and remains unresolved.

The results of this study show that concepts of optimality in energy expenditure may be used to describe both the spatial regularity, as well as variability in channel properties in the Ashley and Taieri Rivers. Here, the issue of spatial adjustment was addressed, but it is also temporal adjustment that defines channel geometry. For instance, it has been demonstrated on river data that streams adjust to minimize the rate of energy expenditure with time, specially in response to events that dramatically reshape channel geometry, such as large floods, volcanic eruptions, river engineering, etc. (Simon, 1992). It is the continuous spatial and temporal geomorphic adjustment of channels that, in conjunction with local lithologic and other landscape controls, promotes the variability in channel shape and form observed in nature. The possibility that there may be an underlying principle, such as uniformity in energy expenditure per unit channel area, in the mutual adjustment between alluvial channel forms and fluvial processes is appealing, because it may provide a basic measure for evaluating the state of fluvial systems and their possible future adjustment.

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