

# Climate, Soil, and Vegetation

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*Reprinted from*  
WATER RESOURCES RESEARCH  
VOL. 14, NO. 5 OCTOBER 1978

# Climate, Soil, and Vegetation

## 1. Introduction to Water Balance Dynamics

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A statistical dynamic formulation of the vertical water budget at a land-atmosphere interface is outlined. Physically based dynamic and conservation equations express the infiltration, exfiltration, transpiration, percolation to groundwater and capillary rise from the water table during rainstorms and interstorm periods in terms of independent variables representing the precipitation, potential evapotranspiration, soil and vegetal properties, and water table elevation. Uncertainty is introduced into these equations through the probability density functions of the independent climatic variables and yields derived probability distributions of the dependent water balance elements: surface runoff, evapotranspiration, and groundwater runoff. The mean values of these quantities give a long-term average water balance which, to the first order, defines the annual water yield and water loss in terms of the annual precipitation and potential evapotranspiration and in terms of physical parameters of the soil, vegetation, climate, and water table. This analytical framework provides physical insight into the dynamic coupling of climate-soil-vegetation systems. Details are presented in a series of subsequent papers.

### INTRODUCTION

It has long been recognized that the atmospheric and soil-vegetation systems are dynamically coupled through the physical processes which produce transport of thermal energy and water mass across the land surface.

At land surfaces the soil column responds dynamically to the climatic sequence of precipitation and evapotranspiration events and accepts part of the moisture during periods of precipitation, pumps some of this back to the surface during evaporative periods, and rejects the remainder to the water table more or less continuously. This surface moisture exchange and thus the surface heat exchange, to a large degree, depend critically upon the physical properties of the soil and vegetation as well as upon the weather conditions during the alternate periods of precipitation and evapotranspiration. The quantitative relation among the long-term averages of this partition of precipitation is called the 'water balance.'

The land surface alone returns to the base of the atmosphere by evapotranspiration about 12% of the entire earth's precipitation and through latent heat about 17% of all its absorbed solar radiation [Budyko *et al.*, 1962; Houghton, 1954]. It is reasonable to conclude therefore that large-scale patterns of climate may be influenced in significant measure by regional variability of these surface processes. We seek here to establish this climate-land surface coupling in a form which provides insight into the physical basis for man-induced change in both climate and water balance.

Such insight can come only through retention in our model of the underlying (and well-known) physical determinism. At the same time, however, uncertainty plays such a large role in nature that our approach must deal with probability distributions. This uncertainty exists in the configuration of the physical system, including the values of its parameters, in the values of the independent (i.e., input) variables to be expected, and, of course, in the information (i.e., observations) with which we evaluate the system behavior. It is seldom practical to deal with all sources of uncertainty simultaneously.

Putting aside the question of observational error, we can often deal successfully with spatially complex dynamic natural systems in the manner illustrated schematically in Figure 1. In

this approach the spatial complexity is replaced by a highly idealized deterministic configuration of physical elements performing in such a way as to preserve the essential relationships among characteristic times, volumes, rates, etc. The relationships are obtained from the process physics, and their parameters may be either fixed or uncertain. Of course, the greater the physical idealization in a particular dimension, the more difficult is the determination of the appropriate effective values of the respective parameters.

The input variables are considered to be stochastic, and their probability distributions are transformed into the probability distribution of the output variable by using the deterministic physical process.

From the viewpoint of applied statistics this is classified as a problem in 'derived distributions,' while in applied mechanics it falls in the realm of 'statistical mechanics.' By this formulation we have traded off some fidelity in physical behavior for the ability to include in the derived output statistics an explicit representation of both the essential system dynamics and the input statistics. This has great utility in environmental impact analysis and in other situations where insight into uncertain behavior takes precedence over quantitative accuracy.

An example of this approach which illustrates its power is the derivation of flood frequency from rainfall statistics [Eagleson, 1972] and from snowmelt statistics [Carlson and Fox, 1976] by using the kinematic wave model of surface runoff. We shall use the same technique here to look at the annual water balance.

Our objective in this work is to gain insight into the physical basis for uncertainty (and for changes in uncertainty) in the elements of the annual water balance as a first step toward understanding man-induced alterations in the hydrologic cycle. In so doing, we will strive for analytical as opposed to numerical solutions, for it is our personal belief that where it is feasible this approach holds greater promise for realization of these objectives.

Only an outline is presented here. Details appear in a series of subsequent papers [Eagleson, 1978a, b, c, d, e, f].

### PROBLEM FORMULATION

The history of fluid mechanics in general and of hydraulic engineering in particular is replete with examples of the advances in understanding that dimensionless formulation can

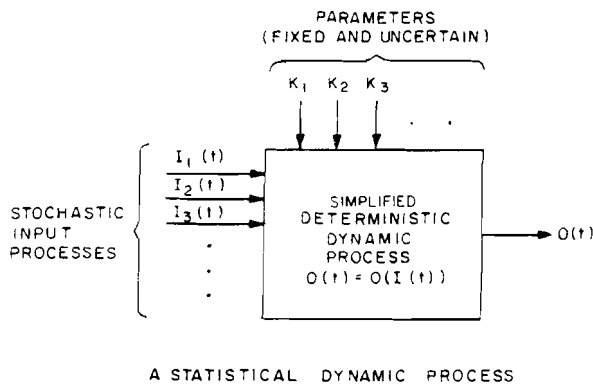


Fig. 1. A simple statistical dynamic process.

bring to bafflingly complex physical problems. Perhaps because it involves the dynamic interaction of atmosphere, surface water, soil moisture, vegetation, and groundwater, the overall hydrologic cycle has not yielded to such generalized analysis. Solutions are usually obtained to specific problems by using specific parametric values and the simulation methods used do not lend themselves readily to the extraction of general conclusions or to the generation of physical insight.

It is our hope in this work to produce a dimensionless analytical representation of the one-dimensional annual water balance which is based upon physically realistic dynamic models of hydrological subprocesses yet which is simple enough to allow analytical derivation of the probability distributions of the critical hydrologic variables from parameterized distributions of the climatic input variables.

It is realized at the outset that the achievement of this goal will require many simplifications of the true system (some apparently 'gross'). Although there is always the danger of destroying model validity through such simplifications, we believe that the potential yield of understanding is worth the risk.

*One-dimensional representation.* We will consider the dynamics of only those processes which operate in the vertical direction, namely, precipitation, infiltration, and evapotranspiration. They will act at the upper surface of a vertically sided control volume illustrated by the dashed lines in Figure 2a. In this figure,  $i(t)$  is the precipitation intensity,  $e_T(t)$  is the evapotranspiration rate,  $v_{ss}(t)$  is the rate of capture of precipitation in surface storage,  $r_s(t)$  is the surface runoff rate,  $r_g(t)$  is the groundwater runoff rate,  $y(t)$  is the yield rate,  $V_{ss}(t)$  is the volume of storage on the surface, and  $V_{sg}(t)$  is the volume of storage in the ground. Snow, ice, and the movement of soil moisture in its vapor state will not be considered.

The volumetric water balance per unit of surface area over time  $t$  is given by

$$\int_0^t \{i(t) - e_T(t) - \frac{\partial}{\partial t} [V_{ss}(t) + V_{sg}(t)]\} dt = \int_0^t [r_s(t) + r_g(t)] dt = \int_0^t y(t) dt \quad (1)$$

The dependence of all terms (except  $i(t)$  and  $V_{ss}(t)$ ) upon the hard to assess soil moisture level and distribution makes (1) very difficult to evaluate for an arbitrary period of time. To do so, one must develop a soil moisture accounting scheme and then find  $y(t)$  through (1) by simulation [Crawford and Linsley, 1966]. For a particular time period the result is critically

dependent upon the elusive storage terms. To circumvent this problem, we will first assume the system to be stationary in the mean. If one lets the integration interval be a full year and takes the expected value of (1) term by term, the change of storage terms must then vanish and give the average annual water balance equation:

$$E[P_A] - E[E_{T_A}] = E[R_{s_A}] + E[R_{g_A}] = E[Y_A] \quad (2)$$

where

- $P_A$  annual total precipitation, equal to  $\int_0^1 y^r i(t) dt$ ;
- $E_{T_A}$  annual total evapotranspiration, equal to  $\int_0^1 y^r e_T(t) dt$ ;
- $R_{s_A}$  annual total surface runoff, equal to  $\int_0^1 y^r r_s(t) dt$ ;
- $R_{g_A}$  annual total groundwater runoff, equal to  $\int_0^1 y^r r_g(t) dt$ ;
- $Y_A$  annual total yield, equal to  $\int_0^1 y^r y(t) dt$ ;

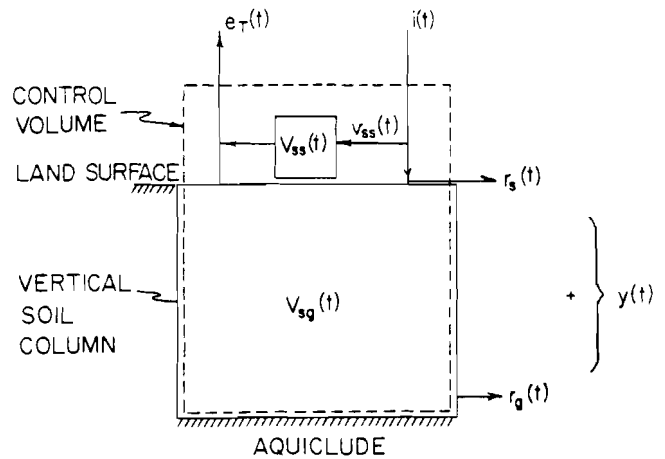
$E[ ]$  expected value of [ ].

Equation (2) is illustrated by the fluxes external to the steady state control volume of Figure 3.

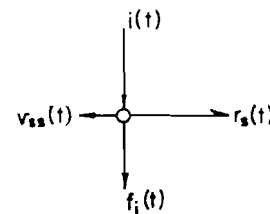
Looking at the instantaneous partition of precipitation as shown in Figure 2b, where  $f_i(t)$  is the infiltration rate, we can write the auxiliary conservation equation

$$\int_0^t i(t) dt - \int_0^t v_{ss}(t) dt - \int_0^t f_i(t) dt = \int_0^t r_s(t) dt \quad (3)$$

If we introduce the crude approximation that evaporation losses from surface runoff are negligible, then averaging over a



a. CONTROL VOLUME FOR WATER BALANCE



b. PARTITION OF PRECIPITATION AT SURFACE

Fig. 2. Instantaneous water balance.

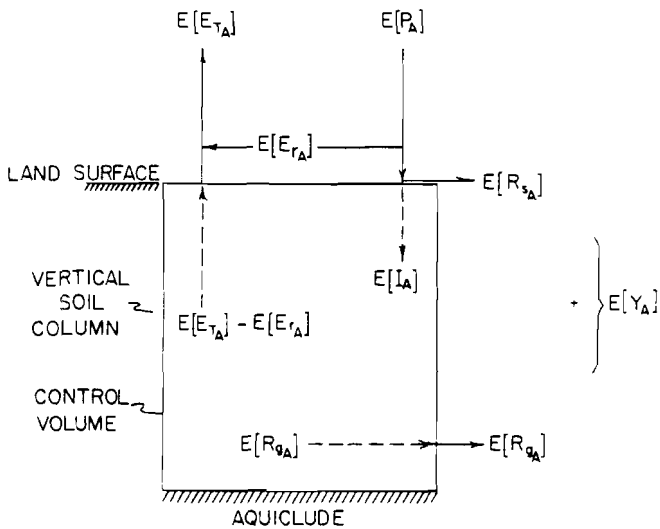


Fig. 3. Average annual water balance.

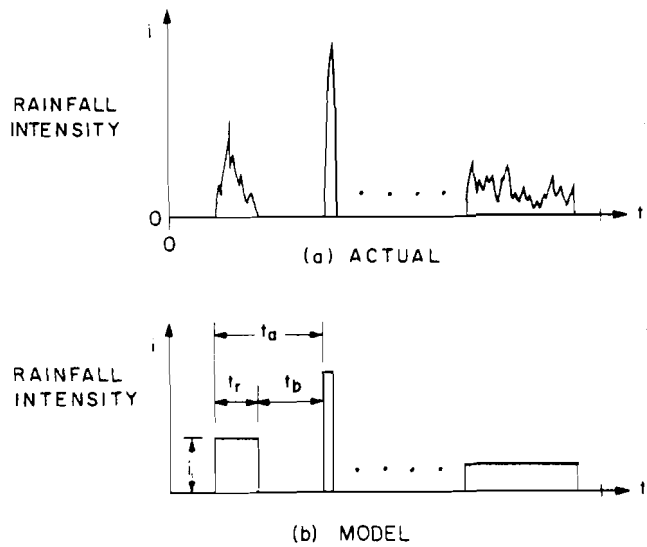


Fig. 5. Model of precipitation event series.

large number of water years gives for a stationary system,

$$E[P_A] - E[I_A] = E[R_{sA}] + E[E_{rA}] \equiv E[R_{sA}^*] \quad (4)$$

where

- $I_A$  annual total infiltration, equal to  $\int_0^{yr} f_i(t) dt$ ;
- $E_{rA}$  annual total evaporative loss from surface retention, equal to  $\int_0^{yr} v_{sa}(t) dt$ ;
- $R_{sA}^*$  annual total rainfall excess, equal to  $R_{sA} + E_{rA}$ .

When evaluating this relationship it must be remembered that we are considering processes that operate in the vertical dimension only. Thus evapotranspiration and infiltration are being neglected during lateral movement of the quantity called surface runoff here. The surface retention losses, which are included here, are normally small but may nevertheless be significant in arid climates.

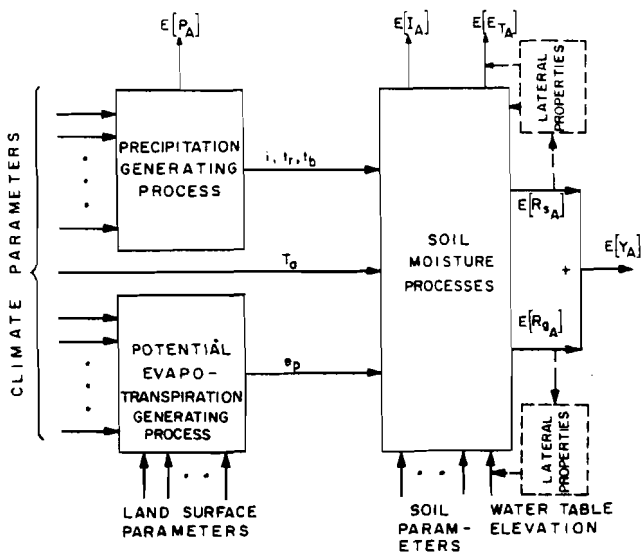


Fig. 4. Computational flow chart for annual water balance.

Subtracting (2) and (4) gives the useful average annual water balance for soil moisture as

$$E[I_A] = E[E_{TA}] - E[E_{rA}] + E[R_{sA}] \quad (5)$$

which is illustrated by the fluxes internal to the steady state control volume of Figure 3.

*System structure.* We represent the physical processes by the schematic flow chart shown in Figure 4. In this representation the climate provides a set of independent random variables which are shown as inputs to the soil moisture processes. The output of these processes is the set of average annual water balance components.

In the true three-dimensional system the lateral properties such as surface physiography and medium transmissivity will provide a coupling between the groundwater flow and the water table elevation. A similar feedback results from the evaporation and infiltration of surface runoff as it is conveyed away from its point of generation. These feedback links are shown by the dashed lines in Figure 4 but are not included in the present model.

Precipitation is generated phenomenologically by Poisson arrivals of independent and identically distributed rectangular pulses such as are shown in Figure 5. This model is chosen for the following reasons:

1. It has the ability to represent the distribution of total precipitation in any time period (e.g., a year) in terms of a few parameters of the storm process [Todorovic, 1968]. The relatively large number of storms in even a few years of record make estimation of these parameters fairly accurate.
2. It can also represent conveniently the distributions of the two critical time periods, the duration  $t_r$  of precipitation during which infiltration and/or surface runoff occurs and the interval between storms  $t_b$  during which evapotranspiration occurs. The distribution of storm inter-arrival time  $t_a$  of storm depth  $h$  and hence that of its average intensity  $i$  is also important.

The atmospheric temperature  $T_a$  is assumed to have a small coefficient of variation and hence is replaced by its mean (over the rainy season).

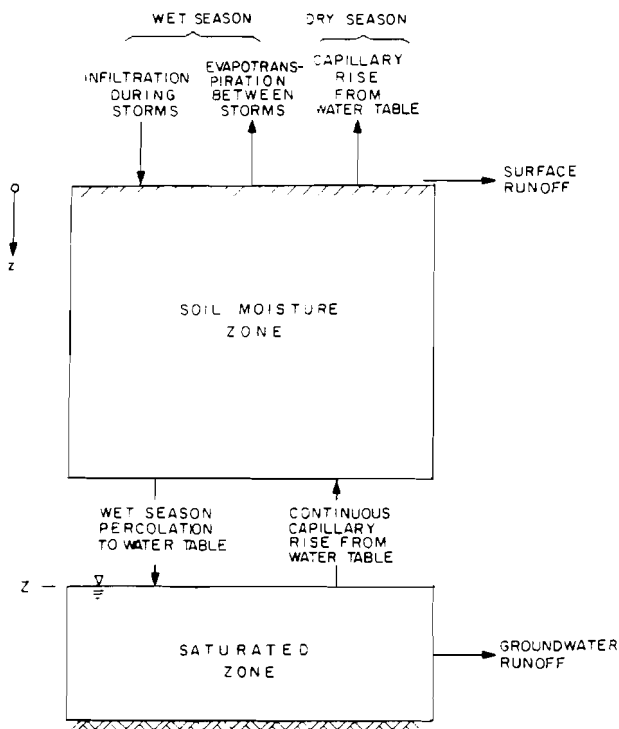


Fig. 6. Schematic representation of soil column.

The potential rate of evaporation  $e_p$  provides an upper bound to the actual rate of bare soil evaporation  $e_t$ . It also is assumed to have a small coefficient of variation during the rainy season and is thus replaced by its seasonal average value  $\bar{e}_p$ . This rate, and hence the average annual potential evapotranspiration  $E[E_{p_a}]$ , is calculated from climate and land surface parameters by using the modified Penman equation [Van Bavel, 1966].

The soil is assumed to be homogeneous, unsaturated vertical flow of soil moisture being based for the bare soil fraction upon the Philip [1969] equation to which a distributed sink has been added to account for moisture extraction by plant roots. The vegetated fraction is modeled after Van den Honert [1948], with the added assumption that a vegetal canopy in equilibrium with its environment must operate in the unstressed state on the average.

Actual evapotranspiration is given by the net upward flow to the composite surface as bounded by the potential value. Groundwater is generated by the difference between wet season gravitational percolation to and continuous capillary rise from a fixed water table at elevation  $Z$ . These subprocesses are illustrated schematically in Figure 6.

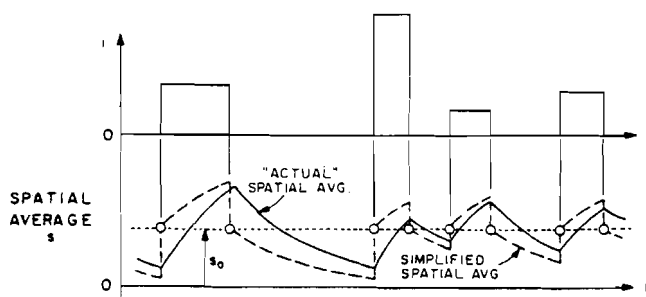


Fig. 7. Soil moisture representation.

When analyzed separately the respective rates of soil moisture movement are superimposed as though the system were linear.

*Soil properties.* The permeability  $k(s)$  and matrix potential  $\Psi(s)$  of the homogeneous soil are represented by functions of the following forms:

$$k(s) = k(1)s^c \quad (6)$$

and

$$\Psi(s) = \Psi(1)s^{-1/m} \quad (7)$$

where

- $s$  effective soil moisture concentration (i.e., effective volume of soil moisture divided by effective volume of voids);
- $k(s)$  effective intrinsic permeability, square centimeters;
- $\Psi(s)$  matrix potential, centimeters (suction);
- $c$  pore disconnectedness index;
- $m$  pore size distribution index.

Also  $s = 1$  signifies effective saturation. The behavior of soil moisture is described [Eagleson, 1978b] in terms of three independent soil parameters:  $n$ ,  $k(1)$ , and  $c$ , where  $n$  is the effective porosity (i.e. the effective volume of voids divided by the total volume).

*Soil moisture representation.* A principal climatic effect upon the water budget results from the random intermittency of the processes by which the soil moisture storage is discharged and recharged through the land surface. The forces causing this are (1) the unidirectional gravitational force and (2) a reversing capillary force. The latter is the result of a space and time variable matrix potential gradient which is itself a function of the soil moisture concentration.

In response to saturation of the surface soil a wave of soil moisture moves downward through the dry soil column. Similarly, following surface drying of the wet soil column, a negative wave of soil moisture propagates downward. This complex wavelike process demands computer-based numerical representation for faithful reproduction of internal soil moisture variations. Such detailed analysis requires, for consistency, equally detailed knowledge of the heterogeneity of the soil properties, information which is seldom available outside the laboratory. Here, where we are attempting to establish aggregate system behavior by a one-dimensional model, it hardly seems appropriate or necessary to include such detail.

If we represent the time and space variable soil moisture by a time-varying spatial average, we might expect the time variations shown in Figure 7 in response to rectangular pulses of precipitation at the surface. This simplified representation highlights the dominant role which soil moisture plays in controlling the water budget. The soil moisture level at the beginning of precipitation sets the 'receptor' matrix potential governing the amount of water which will be infiltrated during the subsequent storm. Similarly, the soil moisture level at the cessation of precipitation sets the rejector matrix potential governing the amount of water which will be exfiltrated and transpired during the subsequent interstorm period. To account for this time variation while a spatial average is used requires bookkeeping of the additions to and subtractions from a storage volume whose lower boundary must be defined. Prescription of this fictitious limit may be avoided by assuming that the initial (spatial average) soil moisture  $s_0$  at the beginning of all precipitation and interstorm periods is a constant value determined by the climate, soil, and vegetation prop-

erties. The resulting time variation of  $s$  is illustrated qualitatively by the dashed line in Figure 5. The magnitude of  $s_0$  will be set at the time average of the spatial average and will be found by equating the expected annual additions to and subtractions from soil moisture (given by (5)).

It should be noted that this key assumption is really a neglect of the carryover effect of one storm or interstorm episode upon the following episode. This will result in a consistent overestimate of average soil moistures during periods of infiltration and a consistent underestimate of average soil moistures during periods of desorption. The net effect of this approximation will be overestimation of total yield primarily through exaggeration of the surface runoff component.

This assumption establishes the time and spatial average soil moisture  $s_0$  as a primary climate-soil-vegetation variable in terms of which all soil moisture processes will be defined.

#### OUTLINE OF SOLUTION

The following is an outline of the solution.

1. In terms of the soil and soil moisture models just described the physics of the soil moisture movement processes are used to derive the following equations

Infiltration depth during the  $j$ th storm [Eagleson, 1978d]

$$I_j = g_1[i, t_r; s_0, n, k(1), c, Z] \quad (8)$$

Evapotranspiration depth during the  $j$ th interstorm period [Eagleson, 1978c]

$$E_{T_j} = g_2[t_b, h, e_p; s_0, n, k(1), c, M, k_v, h_0, Z] \quad (9)$$

(Here  $h$  is storm depth, and  $h_0$  is surface retention capacity,  $M$  is vegetal canopy density, and  $k_v$  is plant coefficient potential transpiration efficiency.)

Evaporation from surface retention during the  $j$ th interstorm period [Eagleson, 1978c]

$$E_{r_j} = g_3[h, t_b, M, h_0] \quad (10)$$

Uniform rate of flow to water table [Eagleson, 1978b]

$$r_g = K(1)s_0^c - w[n, k(1), Z] \quad (11)$$

(Here  $K(1)$  is the saturated effective hydraulic conductivity. The first term represents steady gravitational percolation during the wet season only and  $w$  represents the steady year-round rate of capillary rise from the water table to a dry surface.)

2. Short-period observations of storm series are used to define the climatic probability density functions for storm intensity  $i$ , storm duration  $t_r$ , storm depth  $h$ , time between storms  $t_b$ , and storm interarrival time  $t_a$  [Eagleson, 1978a]. Analytically tractable marginal distributions are fitted to these observations: the gamma distribution for  $h$  and exponentials for  $i$ ,  $t_r$ ,  $t_b$ , and  $t_a$ . Independence of  $t_r$  and  $i$  and of  $h$  and  $t_b$  is assumed as an analytical expedient.

3. Expected values of  $I_j$ ,  $E_{T_j}$ , and  $E_{r_j}$  are derived by using these density functions and (8), (9), and (10).

4. Annual average values of  $I$ ,  $E_T$ , and  $E_r$  are found by summing over all events in the year in the following manner:

$$E[E_{r_a}] = E\left[\sum_{j=1}^{m\tau} E_{r_j}\right] = E[\omega\tau]E[E_{r_j}] \quad (12)$$

where  $\omega^{-1} = E[t_a]$  is the average storm interarrival time and  $\tau$  is the length of the rainy season.

For infiltration this gives [Eagleson, 1978d]

$$E[I_A]/E[P_A] = 1 - e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma} \quad (13)$$

where  $G$  is the gravitational infiltration parameter and  $\sigma$  is the capillary infiltration parameter. Both are functions of  $s_0$ , soil and climate parameters.

For soil moisture evaporation we get [Eagleson, 1978c, e]

$$E[E_{T_A}^*]/E[E_{P_A}^*] = J(E, M, k_v) \quad (14)$$

where

$J(E, M, k_v)$  evapotranspiration function;

$E$  capillary exfiltration parameter, a function of  $s_0$ , soil and climate parameters;

$E_{T_A}^*$  annual soil moisture evapotranspiration, equal to  $E_{T_A} - E_{r_A}$ ;

$E_{P_A}^*$  annual potential evapotranspiration from soil moisture, equal to  $m_r E[t_b] \bar{e}_p^* - E_{r_A}$ ;

$\bar{e}_p^*$  weighted average rate of potential evapotranspiration, equal to  $[1 - M + Mk_v]\bar{e}_p$ .

The average number of storms per year  $m_r$  is given by

$$m_r = E[\omega\tau] = E[P_A]/E[h]$$

For groundwater runoff we average across the variable season length to obtain [Eagleson, 1978e]

$$E[R_{gA}] = m_r K(1)s_0^c - Tw \quad (15)$$

where  $m_r$  is the mean season length in seconds and  $T$  equals  $3.15 \times 10^7$  s/yr.

5. A natural selection hypothesis is proposed which suggests that in the short term, natural vegetal systems of given  $k_v$  reach a 'growth equilibrium' density  $M = M_0$  at which the soil moisture is maximum because in this state, stress is minimized. Application of this hypothesis determines  $M$ .

An extension of this hypothesis to evolutionary time scales suggests that the species mix (e.g.,  $k_v$ ) will evolve from one growth equilibrium to another, provided nutrition or light is not limiting, until an 'evolutionary equilibrium' is reached at which the rate of production of biomass is maximized. This implies maximization of the rate of soil moisture utilization and hence maximization of the product  $M_0 k_v$ . For water-limited cases this hypothesis determines  $k_v = k_{v_0}$ .

6. Equation (5) can now be expanded to give the mean annual water balance equation [Eagleson, 1978e],

$$E[P_A][1 - e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma}] = E[E_{P_A}^*]J(E, M, k_v) + m_r K(1)s_0^c - Tw \quad (16)$$

(The term to the left of the equal sign is infiltration, the first term to the right is soil moisture evapotranspiration, and the last two terms give the groundwater runoff.)

Equation (16) serves to define the dimensionless dependent variable  $s_0$  in terms of a set of 10 independent dimensionless variables which arise from the (simplified) physics of the climate-soil-vegetation coupling and hence form the basis for the dynamic similarity of the average annual water balance [Eagleson, 1978e].

An asymptotic analysis of this equation [Eagleson, 1978e] yields expressions for the  $E[P_A] \rightarrow 0$  and  $E[P_A] \rightarrow \infty$  asymptotes of  $E[E_{T_A}^*]$  and  $E[I_A]$ , the intersections of which define, in comparison with  $E[P_A]$ , rational classifications of climate-soil-vegetation systems into arid, semiarid, subhumid, and humid categories.

Equation (16) can be solved (numerically) to find

$$s_0 = s\{E[P_A], E[E_{P_A}^*], m_r K(1)\} \quad (17)$$



interstorm period (and vice versa) is neglected with internal moisture at the start of every period being  $s_0$  the space and time average in the surface boundary layer.

4. Vegetation (natural systems only)
  - a. Transpiration occurs at the potential rate.
  - b. Rate of soil moisture extraction by the root system is a constant throughout the soil volume above the maximum root depth.
  - c. Canopy density seeks a short-term equilibrium state at which soil moisture is a maximum.
  - d. In water-limited systems, species evolve in the long term toward maximum water use.
5. Infiltration and surface runoff
  - a. No surface inflows from outside the region are considered.
  - b. Storm intensity and duration are statistically independent.
6. Evapotranspiration
  - a. Vegetation transpires at the potential rate.
  - b. Potential rate of evaporation averaged over the interstorm period has a negligible coefficient of variation during the rainy season.
7. Percolation to water table
  - a. Percolation is steady throughout rainy season at a rate determined by the average soil moisture  $s_0$ .
  - b. Percolation is zero during dry season.
8. Capillary rise from water table
  - a. Potential rate of evaporation is much greater than rate of capillary rise from water table.
  - b. Dry surface matrix potential is much greater than saturated matrix potential.
9. Miscellaneous
  - a. Water table is constant (no carryover groundwater storage from year-to-year).
  - b. Relation among annual water balance components is given to the first order by the relation among the average annual quantities.

Only the outline of this development is presented. Details and verifications are found in a series of subsequent papers.

#### NOTATION

- $c$  pore disconnectedness index.  
 $E$  exfiltration parameter.  
 $E_{p_A}$  annual potential evapotranspiration, centimeters.  
 $E_{p_A}^*$  annual potential soil moisture evapotranspiration, centimeters.  
 $E_r$  storm surface retention, centimeters.  
 $E_{r_A}$  annual evaporation from surface retention, centimeters.  
 $E_{T_A}$  annual total evapotranspiration, centimeters.  
 $E_{T_A}^*$  annual soil moisture evapotranspiration, centimeters.  
 $E_{T_I}$  interstorm evapotranspiration, centimeters.  
 $e_p$  potential (soil surface) evaporation rate, centimeters per second.  
 $\bar{e}_p$  time average potential evaporation rate, centimeters per second.  
 $\bar{e}_p^*$  weighted average potential evapotranspiration rate, centimeters per second.  
 $\bar{e}_{pv}$  time average potential transpiration rate, centimeters per second.  
 $f_i$  infiltration rate, centimeters per second.  
 $G$  gravitational infiltration parameter.

- $h$  storm depth, centimeters.  
 $h_0$  surface retention capacity, centimeters.  
 $I_A$  annual infiltration, centimeters.  
 $I_j$  storm infiltration, centimeters.  
 $i$  precipitation rate, centimeters per second.  
 $k$  effective intrinsic permeability, square centimeters.  
 $k_v$  potential transpiration efficiency (i.e., plant coefficient), equal to  $\bar{e}_{pv}/\bar{e}_p$ .  
 $k_{v_0}$  evolutionary equilibrium plant coefficient.  
 $K$  effective hydraulic conductivity, centimeters per second.  
 $m$  pore size distribution index.  
 $m_H$  mean storm depth, centimeters.  
 $m_{p_A}$  average annual precipitation, centimeters.  
 $m_v$  mean number of storms per year.  
 $m_T$  mean length of rainy season, days.  
 $M$  vegetated surface fraction.  
 $M_0$  growth equilibrium vegetated surface fraction.  
 $n$  medium porosity (i.e., volume of voids divided by total volume).  
 $P_A$  annual precipitation, centimeters.  
 $R_{g_A}$  annual groundwater runoff, centimeters.  
 $R_{s_A}$  annual surface runoff, centimeters.  
 $R_{s_A}^*$  annual rainfall excess, centimeters.  
 $r_g$  groundwater runoff rate, centimeters per second.  
 $r_s$  surface runoff rate, centimeters per second.  
 $s$  degree of medium saturation (i.e., soil moisture concentration), equal to volume of water divided by volume of voids.  
 $s_0$  time and spatial average soil moisture concentration in surface boundary layer.  
 $T$  1 yr, seconds.  
 $t_a$  storm interarrival time, days.  
 $t_b$  time between storms, days.  
 $t_r$  storm duration, days.  
 $V_{s_s}$  storage on surface, centimeters.  
 $V_{g_s}$  storage in ground, centimeters.  
 $v_{s_s}$  rate of surface storage, centimeters per second.  
 $w$  upward apparent pore fluid velocity representing capillary rise from the water table, centimeters per second.  
 $x$  value of dimensionless annual precipitation.  
 $Y_A$  annual water yield, centimeters.  
 $y$  yield rate, centimeters per second.  
 $Z$  depth to water table, centimeters.  
 $z$  value of dimensionless water balance term.  
 $\beta$  reciprocal of average time between storms, days<sup>-1</sup>.  
 $\kappa$  shape parameter of gamma distribution.  
 $\sigma$  capillary infiltration parameter.  
 $\tau$  length of rainy season, days.  
 $\Psi$  soil matrix potential, centimeters (suction).  
 $\Psi(1)$  saturated soil matrix potential, centimeters (suction).  
 $\omega$  storm arrival rate, days<sup>-1</sup>.  
 $E[ ]$  expected value of [ ].  
 $F[ ]$  cumulative distribution function.  
 $g( )$  functional notation.  
 $J( )$  evapotranspiration function.  
 $P[ , ]$  Pearson's incomplete gamma function.  
 $\Gamma( )$  gamma function.  
 $\gamma[a, x]$  incomplete gamma function.

*Acknowledgments.* This work was performed in part during the 1975-1976 academic year while the author was a visiting associate in the Environmental Quality Laboratory at the California Institute of

Technology (Caltech) on sabbatical leave from MIT. Robert C. Y. Koh at Caltech and Chiang C. Mei at MIT were particularly helpful with some of the analytical questions, while Pedro Restrepo, research assistant at MIT, performed the necessary machine computation. Publication has been made possible by a grant from the Sloan Basic Research Fund of MIT.

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(Received August 19, 1977;  
revised December 23, 1977;  
accepted February 16, 1978.)