

# Is hydrology kinematic?

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## Abstract:

A wide range of phenomena, natural as well as man-made, in physical, chemical and biological hydrology exhibit characteristics similar to those of kinematic waves. The question we ask is: can these phenomena be described using the theory of kinematic waves? Since the range of phenomena is wide, another question we ask is: how prevalent are kinematic waves? If they are widely pervasive, does that mean hydrology is kinematic or close to it? This paper addresses these issues, which are perceived to be fundamental to advancing the state-of-the-art of water science and engineering. Copyright © 2002 John Wiley & Sons, Ltd.

**KEY WORDS** biological hydrology; chemical hydrology; physical hydrology; kinematic wave theory; kinematics; flux laws

## INTRODUCTION

There is a wide range of natural and man-made physical, chemical and biological flow phenomena that exhibit wave characteristics. The term 'wave' implies a disturbance travelling upstream, downstream or remaining stationary. We can visualize a water wave propagating where the water itself stays very much where it was before the wave was produced. We witness other waves which travel as well, such as heat waves, pressure waves, sound waves, etc. There is obviously the motion of matter, but there can also be the motion of form and other properties of the matter.

The flow phenomena, according to the nature of particles composing them, can be distinguished into two categories: (1) flows of discrete noncoherent particles and (2) flows of continuous coherent particles. Examples of flows in the first category include traffic flow; transport of sand and gravel in pipes, flumes and rivers; evolution of ripples, dunes and bars on a river bed; settlement of sediment particles in a tank; and movement of microbial organisms. Exemplifying flows in the second category are overland flow, flood movement, baseflow, snowmelt, movement of glaciers, infiltration, evaporation, solute transport, ion exchange and chromatographic transport. Because of their wavelike behaviour these flow phenomena can be described by the hydrodynamic wave theory or one of its variants, including the kinematic wave theory, diffusion wave theory, gravity wave theory, linear wave theory, and so on.

A survey of geophysical literature reveals that the kinematic wave theory has been applied to a wide spectrum of problems, including watershed runoff modelling, flood routing in rivers and channels, movement of soil moisture, macropore flow, subsurface storm flow, erosion and sediment transport, debris flow, solute transport, ion exchange, chromatography, sedimentation, glacial motion, movement of snowmelt water, flow over porous beds in irrigation borders and furrows, river-ice motion, vertical mixing of coarse particles in gravel-bed rivers, to name but a few. Other fields, outside the realm of geophysics, applying the kinematic wave theory include traffic flow, movement of agricultural grains in a bin (Cowins and Comfort, 1982) and blood circulation. By discussing a number of illustrative hydrologic problems, it is reasoned that within certain limitations, which may vary with the problem, the hydrologic nature can indeed be approximated quite closely

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by kinematic wave theory. Outside these limitations, the theory provides only a crude approximation and the full hydrodynamic wave theory or one of its higher-order variants may be needed to mimic nature.

Before proceeding further, the question we ask is: what is meant by kinematic? Although papers and textbooks in hydrology define kinematic waves mostly from an approximation of the momentum equation and occasionally from defining flux laws, a comprehensive account of the term 'kinematic' is lacking. Therefore, a short discussion to that end is in order.

### WHAT IS KINEMATICS?

The term 'kinematic' involves consideration of such macroscopic properties of a flow phenomenon as velocity, discharge, concentration, time of travel, etc. It does not involve consideration of forces and mass. Thus, the connotation of 'kinematic' is that it is phenomenological and less physical, but its representation may be derived from or related to dynamical considerations involving forces, mass, momentum and energy.

Although studies based on kinematics had existed long before the development of Greek science, it is Ampere (1834) who is credited with having introduced the term 'kinematics' for the study of motion in his famous essay on the classification of sciences. His intent was to create a discipline for the study of motion without regard to the forces involved. In mechanical engineering kinematics is viewed as the study of the motion of particles, and it is about the motion of rigid bodies or chains of rigid bodies in engineering mechanics. Thus, kinematics is defined as the science that deals with the study of motion in the most general way. The motion can be translatory, dilatational or rotational (angular) (Singh, 1996a).

It may be instructive to view kinematics in the context of geometry and dynamics. From the standpoint of dimensions, geometry implies only length, whereas kinematics involves length and time, and dynamics considers length, mass and time. Thus, kinematics is somewhere between geometry and dynamics, perhaps closer to the latter. Although kinematics is much related to geometry, it can also be studied without geometry by means of numerical description of the functions as the Babylonians seem to have done—a precursor to curve fitting.

Theoretically, kinematics is about

$$x_i = x_i(X, t) \quad (1)$$

where  $x_i$  is the position (coordinates) of a material point  $i$ ,  $X$  is the position vector that identifies each point and  $t$  is time.  $X$  can be the initial position vector. Equation (1) is applicable to discrete as well as continuous systems, and to all kinds of models—deterministic as well as stochastic. This can be a representation of a particle that follows Newton's law or one that performs a random walk. It can also represent any discontinuity in the kinematical model.

Consider the motion of a single point representing either a small particle or that of a rigid body undergoing pure translation. The point could be in curvilinear motion either free or subject to some constraints as exemplified by a projectile in one case and by a pendulum in another. In general

$$x_1 = F(t), \quad x_2 = G(t) \quad (2)$$

where  $F$  and  $G$  are some functions. Theoretically, Equation (2) can be expressed as

$$x_2 = f(x_1) \quad (3)$$

where  $f$  is some function. Equation (3) is less informative than Equation (2). Once the kinematics of a point is developed, the kinematical theory of a rigid body and that of chains of rigid bodies or mechanisms may readily be developed.

In kinematics not only are we concerned with the position of a point at a certain time, but also with the displacement accomplished and the distance traversed in a certain time interval. Other functions of time

such as those of velocity and acceleration can be introduced. Higher derivatives as the rate of change of acceleration, called jerk, and even higher-order derivatives (Hartenberg and Denavit, 1964) can be included.

A set of points (of a rigid body or fluid) undergoes translation if the velocity vectors of all points are identical. This however does not exclude the existence of acceleration and higher derivatives of velocity. The pathlines of a body undergoing a translatory motion do not necessarily have to be straight lines.

It is difficult to find a science where motion does not play a major role. One might even conjecture that the history of kinematics is indeed the history of motion. Heavily dependent on kinematics are different branches of engineering including hydrology, hydraulics, snow and ice engineering, river engineering, irrigation and drainage engineering, environmental engineering, geological engineering, aeronautics, naval engineering, biomechanical engineering, mechanical engineering, and so on. As is the case in kinematics of particles and of bodies in mechanics, our interest is in the velocity and eventual acceleration. Leonardo da Vinci studied the motion of particles of all kinds of fluids (Macagno, 1991). He invented the technique of flow visualization in water. Kinematics also finds an interesting application in chemistry where once a reaction starts, the rate of production of a new compound or velocity of reaction is of interest. There are chemical processes that move at different speeds, as in electrophoresis and chromatography. Table I lists a number of fields applying kinematics.

The motions of the human body have been of much interest in the health science arena. Rehabilitation professionals need to know the kinematics of the human body. This is also needed to increase the productivity of workers (Rabinach, 1991). In human organs motion at different scales is important, as in the case of contractions along the digestive system (Macagno, 1980). In biology, studies of the motions of the heart and its valves and those of the eye, and flow in circulatory, respiratory and lymphatic systems, involve kinematics. A multitude of interesting aspects of kinematics arise in the study of wide ranging phenomena: from the study of Brownian motion to the flows in nebular stellar dynamics, from traffic flow to forest fire propagation, from

Table I. Fields applying kinematics (adapted from Macagno, 1991. Reproduced by permission of Iowa Institute of Hydraulic Research)

Sciences	Engineering
Astronomy	Materials engineering
Stellar dynamics	Mechanical engineering
Physics	Mechanisms and machines
Geography	Transportation
Geophysics	Hydraulics
Geology	Hydrology
Soil mechanics	Environmental engineering
Meteorology	Geological engineering
Oceanography	Irrigation and drainage engineering
Chemistry	Hydronautics
Biological flows	Aeronautics and astronautics
Biological motions	Transport phenomena
	Bio-engineering
	Snow and ice engineering
	Traffic engineering
	Naval engineering
Art	Exercises and sports
Compositional flow	Individual exercises
Kinetic art	Games
Animation in movies	Exercise machines
Mobiles	Human body kinematics

laminar to turbulent flow, from sediment transport to blood flow, and so on. A comprehensive history of kinematics is summarized by Macagno (1991).

It should be noted that the introduction of mass does not necessarily make some motions strictly dynamical; they may remain strongly and functionally kinematical. For instance, the formulation of equations for conservation of volume in fluid mechanics is very much in the realm of kinematics. When the equations for conservation of mass are formulated, they are still in the same domain; the two treatments are very similar. Even the discussions of momentum flux and kinetic energy flux are fundamentally kinematical. In general, there are terms in equations of dynamics that are essentially kinematical. The difference between kinematics and dynamics thus requires close scrutiny.

In a similar vein, in the study of kinetic theories of different states (solid, liquid, gas) and mass transport processes, kinematics plays an important role. In the theory of mass transport by fluid flows, the notion of velocity of a mixture of different species is introduced, taking into account different densities (Bird, *et al.*, 1960). In such a case, either the common density or the molar concentration is used, and the motion of mass flux, a kinematical concept, is invoked. Thus, mass becomes necessary to define velocity in this case. This is done in a purely geometrical manner. A question arises: when can kinematics be decoupled from dynamics and when can it not? One aspect of kinematics is the development of the theory of kinematic waves, whose discussion forms the basis of much of the discussion that follows. It should be noted that much kinematics existed before the kinematic wave theory was developed.

#### KINEMATIC WAVE THEORY

In a seminal contribution Lighthill and Whitham (1955a,b) developed in 1955 the kinematic wave theory and gave a full account of the theory for describing flood movement in long rivers and traffic flow on long crowded roads. The theory is comprised of three components: (1) theory of kinematic continuous waves; (2) theory of shock waves; and (3) theory of formation of shock waves out of continuous waves. Iwagaki (1955) developed an approximate method of characteristics for routing steady flow in open channels of any cross-sectional shape with nearly uniform lateral flow and proposed that the method would be applicable to hydraulic analysis of runoff estimation in river basins. Implicit in his method was the kinematic wave assumption. Thus, Iwagaki (1955) can also be credited to have independently conceived the kinematic wave concept and developed the method of kinematic wave routing.

Following Lighthill and Whitham (1955a,b), kinematic waves exist if it can be assumed with sufficient accuracy that there is a functional relationship between flux, concentration and position. Thus, the kinematic wave theory is mathematically expressed by the law of conservation of mass through the continuity equation and a flux–concentration relation. The coupling of these two equations leads to a first-order partial differential equation having only one system of wave characteristics. The essence of this is that the wave property is derived from the continuity equation alone. Since the governing equation (or continuity equation) is of first order only, the kinematic waves possess only one system of characteristics. This would also imply that kinematic waves travel in the downstream direction only, and the kinematic wave theory cannot accommodate waves that travel in the upstream direction as in the case of backwater flow. In the absence of any lateral inflow and/or outflow, the flux along the characteristics is constant and the kinematic waves are nondispersive and nondiffusive. However, the waves are dispersive and diffusive when there is lateral inflow and/or outflow, as for example in the case of rainfall runoff.

##### *Continuous waves*

If the volume of a quantity passing a given point  $x$  in unit time is denoted by flux,  $F$ , and the concentration is defined as the volume of the quantity per unit distance by  $C$ , then for a one-dimensional flow system without any lateral sources and sinks, the continuity equation or the mass conservation law is

$$\frac{\partial C}{\partial t} + \frac{\partial F}{\partial x} = 0 \quad (4)$$

Equation (4) is of first order and states that the volume of the quantity in a small element of length changes at a rate equal to the difference between the rates of inflow and outflow.

Fundamental to the development of the kinematic wave theory is the development of a flux–concentration relation. A general flux–concentration relation can be assumed as

$$F = F(C, x) \quad (5)$$

Equations (4) and (5) have two unknowns,  $F$  and  $C$ , and can therefore be combined into one equation having one unknown. Multiplying Equation (4) by

$$c = \left( \frac{\partial F}{\partial C} \right)_{x \text{ constant}} = c(C, x) \quad (6)$$

one obtains

$$\frac{\partial F}{\partial t} + c \frac{\partial F}{\partial x} = 0 \quad (7)$$

Equation (7) is the kinematic wave equation. The quantity  $c$  is described in various ways. In hydraulics it is called wave velocity or celerity, and it is referred to as mobility in vadose zone hydrology. It is the slope of the flux–concentration relation at a fixed  $x$ . This is not the same as the mean velocity  $u$ , which is expressed as

$$u = F/C \quad (8a)$$

The relation between  $c$  and  $u$  is thus obvious:

$$c = \frac{d}{dC}(uC) = u + C \frac{du}{dC} \quad (8b)$$

Equation (9) shows that  $c$  is greater than  $u$  if  $u$  increases with  $C$  as in the case of river flow, it is less than  $u$  if  $u$  decreases with  $C$  as in the case of sedimentation, and it is equal to  $u$  if  $u$  does not change with  $C$ . Equation (7) states that  $F$  is constant on waves travelling past the point with celerity  $c$  given by Equation (6). The kinematic wave Equation (7) has one system of characteristics travelling in the downstream direction only, given by  $dx = c dt$ , and along each of these characteristics  $F$  is fixed.

Although Equation (5) constitutes the basic building block of the kinematic wave theory, a more popular, but more restrictive, derivation of the theory in hydrology is obtained by assuming the local acceleration, convective acceleration and pressure gradient (or concentration gradient) in the momentum equation to be negligible. This assumption states an equivalence between friction and gravity forces. When only the local and convective acceleration terms are assumed negligible, the remainder of the momentum equation (i.e. the pressure gradient equalling the difference between gravity and frictional slopes) leads, in conjunction with the continuity equation, to diffusion wave theory. When the full momentum equation is used jointly with the continuity equation, the result is the dynamic wave theory. In general, the local acceleration and the convective acceleration are of the same order of magnitude, but are of opposite sign, thus counteracting each other. In a wide range of problems in hydrology, the friction and gravity terms are dominant and this is one of the reasons for the popularity of the kinematic wave theory.

#### *Formation of kinematic shocks*

A kinematic shock is a discontinuity representing a sudden rise or surge in the flow depth. For example, during wave movement faster-moving waves overtake slower-moving waves and there will, at a fixed position, be an increase in flux and concentration as functions of time, leading to shock formation. Thus, flood waves have an intrinsic, nonlinear tendency to steepen as they propagate downstream, eventually forming a shock. In the characteristic plane, shock formation results from the intersection of characteristics. After some time the shock weakens and dissolves into a region of uniform flow. Shocks arise in a variety of hydrologic processes,

and what these shocks mean and their practical relevance will be discussed in different sections of the paper. A short but general discussion of shocks is relevant however and is given here. The shocks or discontinuities are of first and second orders. A discontinuity of the first kind in the concentration is defined by a sudden change of concentration at a certain level. In such a case the differential equation of continuity [Equation (4)] no longer applies; instead it is replaced by an equation stating that the flow of mass into one side of the shock is equal to that on the other side of the shock. Denoting the velocity of discontinuity by  $U$ , the mass conservation leads to

$$U = \frac{F_2 - F_1}{C_2 - C_1} \quad (9)$$

where subscripts 2 and 1 denote the quantities ahead and behind the shock. Equation (9) shows that in general the discontinuity is not at rest but moves with velocity  $U$ , which is the slope of the chord joining the points  $(F_1, C_1)$  and  $(F_2, C_2)$  on the  $F-C$  diagram.

A discontinuity of the second kind is represented by a very small change in concentration. If  $C_2 - C_1 = dC$  is small, the expression for  $U$  becomes  $dF/dC = c$ , the celerity. In this case the velocity of a discontinuity between concentrations  $C$  and  $C + dC$  is the same as the celerity  $c$ . A small change, if maintained, is propagated through the medium of concentration  $C$  with velocity  $c$  in a manner similar to the propagation of sound through air with a definite velocity. A line of constant concentration therefore describes the motion of the boundary between media of concentrations  $C$  and  $C + dC$  so that its slope is necessarily equal to  $c$ . The complete modification of concentration at a position, as well as its profile, can be characterized as a series of small discontinuities propagated through the medium.

#### *Determination of kinematic shocks*

Many factors affect formation of shocks and these factors can broadly be distinguished to be of three types: (1) initial and boundary conditions; (2) lateral inflow and outflow; and (3) watershed geometric characteristics. A full account of the theory of shock formation was given by Lighthill and Whitham (1955a,b) and a mathematical treatment of the formation and decay of shocks was given by Lax (1972).

Carrier and Pearson (1976) developed a method for determining the enveloping curve for any region of intersecting characteristics. Croley and Hunt (1981) determined analytically boundary-dependent shocks in planar flow. Hairsine and Parlange (1987) dealt with kinematic shocks on curved surfaces. Kibler and Woolhiser (1970) investigated geometry-dependent shocks using a kinematic cascade. They derived mathematical properties of kinematic cascades and developed a criterion, based on the properties of adjacent flow-plane pairs, to predict when a shock would occur in a cascade. They also developed a numerical procedure for shock fitting. General properties of shock waves along with continuous kinematic waves were discussed. Full equations of motion were employed to investigate the structure of the kinematic shock.

Building on the work of Kibler and Woolhiser (1970), Schmid (1990) investigated the effect of lateral inflow and outflow as well as geometry on development of shocks. He derived a generalized criterion for shock formation on an infiltrating cascade that also encompasses cascades with time-dependent rates of effective rainfall. Using simulated conditions, Ponce and Windingland (1985) determined flow and channel characteristics that either tend to promote or inhibit development of kinematic shocks. Borah (1989) and Borah *et al.* (1980) developed approximate but efficient numerical methods for determining the shock path and shock fitting.

## FLUX LAWS

The term 'flux' is defined in two ways in hydraulics. First, flux denotes a volume of any quantity per unit area per unit time. Thus, its dimensions are  $L/T$  if the quantity is, say, runoff. Second, flux is defined as any quantity per unit time. For example, the volume flux is the volume of a quantity per unit time as exemplified by discharge, mass flux is mass per unit time, momentum flux (it has the same dimensions as force) is

momentum per unit time, energy flux (the kinetic energy flux has the same dimensions as power) is energy per unit time, and so on. In water and environmental engineering both definitions are used.

Flux laws are fundamental to the development of transport theories. The flux laws, most common in environmental and water sciences, are of two types: (1) power laws and (2) gradient laws. The kinematic wave flux laws are perhaps the most popular flux laws of power type, and Darcy's law and the Darcy–Buckingham law are the most popular gradient-type flux laws. Generalized flux laws which specialize into power and gradient laws are also used. An example of a generalized flux law is the Burgers law or a generalized version thereof (Singh and Prasana, 1999). Power-type flux laws lead to kinematic wave equations (see Table II), whereas the gradient-type flux laws lead to diffusion waves governed by elliptic or parabolic partial differential equations.

#### *Algebraic laws*

A general expression for the flux–concentration relation is given by Equation (5). One of its special forms is the popular kinematic wave flux law expressed as

$$F = \alpha C^n \quad (10)$$

where  $\alpha$  is a parameter and  $n$  is an exponent. The meaning and interpretation of  $\alpha$  and  $n$  may vary with the problem to which Equation (10) is applied. Equation (10) leads to the famous Chezy and Manning equations which are popular in studies on overland flow and flood routing in open channels and rivers. At a fixed location  $x$ ,  $(\partial F/\partial C)$  defines the wave celerity  $c$ :  $c = (\partial F/\partial C)|_{x \text{ fixed}}$  which is not the same as the average velocity of flow. When  $n = 1$ , Equation (10) becomes

$$F = \alpha C \quad (11)$$

This is a linear flux–concentration law, used for describing the movement of meltwater runoff from snowpack.

Beven (1979) employed two somewhat uncommon three-parameter forms of the flux–concentration relation for his channel network routing model:

$$F = \frac{C(a + bk) - k}{1 - bC} \quad (12)$$

and

$$\frac{F}{a(1 - \exp(-kF)) + bF} = C \quad (13)$$

where  $a$ ,  $b$  and  $k$  are parameters.

Another flux law is defined as

$$F = \alpha C^2(C_0 - C) \quad (14)$$

where  $C_0$  is the maximum value of  $C$  and  $\alpha$  is a parameter. Equation (14) is used for describing the settlement of particles in a dispersion. Still another flux law used for describing the movement of sediment in flumes and pipes is

$$F = u_0 C \left(1 - \frac{C}{C_0}\right) \quad (15)$$

where  $u_0$  is the velocity of a single particle when there are no other particles in flow. All of the above flux laws lead to kinematic waves.

#### *Gradient laws*

A gradient-type flux law is expressed as

$$F = G(\partial C/\partial x) \quad (16)$$

Table II. Kinematic flux laws applied to different branches of hydrology

Branch of hydrology	Flux law	Symbols and definitions	Problems to which flux law is applied
Surface water hydrology; river and coastal hydrology	$Q = \alpha h^n$	$Q$ = discharge per unit width, $h$ = flow depth, $n$ = exponent, $\alpha$ = friction parameter	Overland flow; channel flow; depth–discharge rating curve, storage–discharge relation, dam break flow, sediment transport in upland areas, etc.
Irrigation hydrology	$Q = \alpha A^n$	$A$ = flow cross-sectional area, $Q$ = discharge per unit width, $n$ = exponent, $\alpha$ = kinematic wave friction parameter	Furrow irrigation and border irrigation
Vadose zone hydrology	$q = K(\theta)$ ; $q = bw^a$	$K$ = unsaturated hydraulic conductivity, $\theta$ = moisture content, $q$ = volumetric moisture flux, $a$ = exponent, $b$ = conductance parameter, $w$ = macropore water content	Soil water flow, unsaturated flow and macropore flow
River and coastal hydrology	$Q_s = kv_0 \left(1 - \frac{k}{k_0}\right)$	$Q_s$ = sediment flux (rate of transport), $v_0$ = speed of a single particle on the bed of a flume of flowing water, $k$ = mean linear concentration or density, $k_0$ = maximum linear concentration	River channel bars and dunes
	$T = v_0 w \left(1 - \frac{w}{w_0}\right)$	$T$ = transport rate, $v_0$ = velocity of a single particle, $w$ = areal concentration, $w_0$ = areal concentration when transport ceases	Transport of sand by water
Snow and ice hydrology	$q = \alpha k S^{*n}$	$q$ = snowmelt water flux, $k$ = permeability, $S^*$ = saturation, $n$ = exponent, $\alpha$ = constant	Vertical movement of water in snowpacks
	$q = \alpha \theta$	$\alpha$ = constant, $\theta$ = saturated hydraulic conductivity	Basal runoff from snowmelt
	$q = \left(\frac{\rho g}{A}\right)^m h^{m+1} \sin^m s$	$q$ = volume of ice passing a given point in unit time per unit width, $s$ = slope, $m$ = exponent, $h$ = glacial thickness, $A$ = a measure of roughness of the bed, $g$ = acceleration due to gravity, $\rho$ = density	Glacial motion
	$Q = \frac{h^{n+2}}{n+2}$	$Q$ = glacial flux, $h$ = glacial thickness, $n$ = exponent	Surface wave motion in glacial movement
Water quality hydrology	$S = \rho v$	$\rho$ = number of particles crossing a horizontal section per unit area per unit time (local concentration), $v$ = velocity of a particle, $S$ = particle flux	Sedimentation and settlement of particles
	$S = a\rho^2(\rho_0 - \rho)$	$S$ = flux, $\rho$ = local concentration, $\rho_0$ = maximum concentration	Sedimentation and settlement of particles
	$Q = Mf(C)$	$C$ = concentration, $M$ = amount of adsorbant per unit length of the column, $f(C)$ = isotherm, and $Q$ = flux	Chromatography

where  $G$  is some function. As an example

$$F = \beta(\partial C/\partial x)^m \quad (17)$$

where  $\beta$  is a parameter and  $m$  is an exponent. Equation (17) is a nonlinear version of Darcy's law. When  $m = 1$ , Equation (17) becomes

$$F = \beta(\partial C/\partial x) \quad (18)$$

Equation (18) is Darcy's law with  $\beta$  describing the negative of the saturated hydraulic conductivity and  $C$  the hydraulic head.

A more general form of Equation (18) is when  $\beta$  depends on  $C$ :

$$F = \beta(C)\partial C/\partial x \quad (19)$$

Equation (19) is the Darcy–Buckingham law with  $C$  describing the hydraulic head and  $\beta$  the negative of the unsaturated hydraulic conductivity which varies with the moisture content. When a gradient flux law is coupled with the continuity equation, the resulting partial differential equation turns out to be of parabolic type under transient flow conditions and of elliptic type under steady state conditions.

#### *Generalized flux laws*

A generalized flux law is obtained by combining the power and gradient-type flux laws. In general

$$F = G(C, \partial C/\partial x, x) \quad (20)$$

where  $G$  is some function with its argument defined by  $C$ ,  $\partial C/\partial x$  and  $x$ . One form of Equation (20) is

$$F = \alpha C^n + \beta(\partial C/\partial x)^m \quad (21)$$

where  $m$  and  $n$  are exponents. Equation (21) specializes into (Singh and Prasana, 1999)

$$F = \alpha C^n + \beta(\partial C/\partial x) \quad (22)$$

and

$$F = \alpha C + \beta\partial C/\partial x \quad (23)$$

Equation (23) is the Burgers flux law (Burgers, 1948) used in turbulence modelling and when used in the continuity equation the resulting partial differential equation is the Burgers equation. Equations (21) and (22) have been employed for routing of flows in open channels (Singh and Prasana, 1999).

## APPLICATION OF KINEMATIC WAVE THEORY TO SURFACE WATER HYDROLOGY

### *Depth–discharge relation: rating curve*

The depth–discharge relation, which forms the basis of much of the hydrologic literature on overland and channel flow, is a special case of Equation (10) and can be expressed as

$$q = \alpha h^n \quad (24)$$

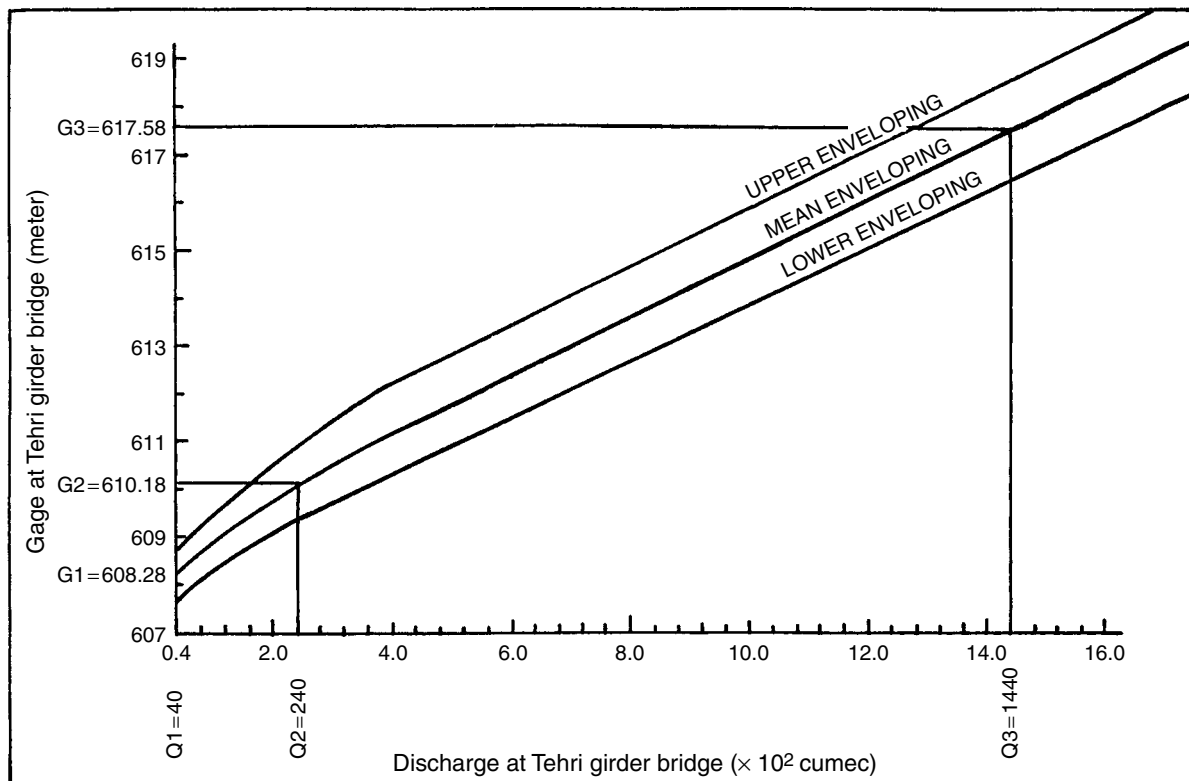


Figure 1. Stage–discharge curve at Tehri Girder bridge in Uttar Pradesh, India, within range of observations

where  $q$  is the discharge per unit width,  $h$  is the depth of flow,  $\alpha$  is a resistance parameter and  $n$  is an exponent indicating the regime of flow. A typical curve for a river is shown in Figure 1. The value of  $n$  varies from 1 (highly turbulent) to 3 (laminar). The flow is transient when  $n$  is greater than 1.67 but less than 3. Thus, the  $n$  exponent reflects the scale of turbulence in flow from the state of no turbulence (when  $n = 3$ ) to one of extreme turbulence (when  $n = 1$ ). Geometrically,  $n$  is a measure of nonlinearity. When  $n = 1$ , Equation (24) becomes linear and when  $n = 3$ , it is highly nonlinear. These two limits of  $n$  correspond to the two extremes of nonlinearity. Relating the degree of nonlinearity to the scale of turbulence, it can be inferred that turbulence tends to linearize the flow, i.e. the higher the degree of turbulence the less the degree of nonlinearity and *vice versa*. On the other hand, laminarity represents a case of extreme nonlinearity. This interpretation is important in that surface water hydrologic processes are nonlinear but their degree of nonlinearity is conducive to applicability of kinematic wave theory.

Parameter  $\alpha$  reflects the interaction between the fluid particles themselves and between fluid and the conduit boundaries within which the fluid is flowing. It therefore varies in space accounting for spatial heterogeneities and in time reflecting the temporal variability of flow and the conduit. Normally,  $\alpha$  is space-dependent but is taken as constant in practice.

The origin of Equation (24) can be traced to the works of Manning and Chezy, but perhaps a more meaningful and generalized formulation was advanced by Lighthill and Whitham (1955a). In hydraulics, Equation (24) is expressed as

$$q = Kh^a S_e^{1/2} \quad (25)$$

where  $S_e$  is the slope of the energy line or frictional slope and  $K$  and  $a$  are empirical constants. Equation (25) specializes into the Manning equation when the exponent  $a$  takes on the value of  $5/3$  and  $K$  equals  $1/n_m$ .

with  $n_m$  being the Manning roughness coefficient. Likewise, Equation (25) specializes into the Chezy equation when exponent  $a$  equals  $3/2$  and  $K$  equals  $C_z$ , the Chezy coefficient.

In Equation (25) the exponent of  $h$  is variable and the justification for its variability can be found in Kouwen *et al.* (1981), Maheshwari (1994) and Singh (1996a), when one balances the force causing the flow (i.e. gravity force) with that resisting the flow (i.e. drag force) in a section of flow. This means that the Manning roughness coefficient in Equation (25) is allowed to vary with the depth of flow as

$$n_m = (1/K)h^{(5/3)-a} \quad (26a)$$

Similarly, the Chezy coefficient follows

$$C_z = Kh^{a-1.5} \quad (26b)$$

Because the flow depth varies in space and time, so do the coefficients  $n_m$  and  $C_z$ .

#### *Interaction between parameters $\alpha$ and $n$*

By definition,  $\alpha$  is a resistance parameter and varies in time and space, reflecting the spatial and temporal variability of flow and the hydraulic system. Furthermore,  $\alpha$  depends on  $n$ . In an experimental study, Singh (1976) found, as shown in Figure 2, for a converging section:

$$\alpha = a(10)^n \quad \text{or} \quad \log \alpha = \log a + n \quad (27)$$

where  $a$  is an empirical constant. Equation (27) shows that  $\alpha$  is highly sensitive to exponent  $n$ . Therefore, an accurate determination of  $n$  is essential for accurately determining  $\alpha$ . Because  $n$  is reflective of the flow regime which changes with the evolution of flow, its dependence on the flow depth or Reynolds number needs to be quantified. The value of  $a$  varies with the nature of the surface. Some typical values of parameters are shown in Table III.

In hydrologic modelling it is usually assumed that parameters  $\alpha$  and  $n$  are independent of each other, that parameter  $\alpha$  may or may not be a constant, and that the exponent  $n$  is taken as 1.5 or 1.67. The reason is that Equation (24) is then expressible either as Chezy's equation ( $n = 1.5$ ) or Manning's equation ( $n = 1.67$ ). Forcing  $n$  to take on a fixed value translates Equation (24) into a one-parameter kinematic wave flux equation. The advantages of so doing are simplicity, reduced need for data and familiarity with the use of Chezy's or Manning's equation. The above assumption is clearly an approximation, not a true representation of flow characteristics.

#### *Storage–discharge relation*

In systems hydrology a fundamental relation is the storage–discharge relation which is frequently invoked for routing excess rainfall. This relation is represented as

$$S = kQ^m \quad (28)$$

where  $S$  is storage,  $Q$  is discharge of runoff and  $k$  and  $m$  are empirical parameters. If  $m$  is assumed to equal one,  $k$  becomes the lag time. This assumption implies that the watershed is linear and forms the basis of the unit hydrograph theory. The value of  $k$  varies with watershed characteristics and  $m$  varies with the flow regime, namely laminar, turbulent or mixed (called mixed flow). The value of  $m$  is not constant and may have a pronounced effect on the predicted hydrograph (Weeks, 1980; Bates and Pilgrim, 1983; Bates and Townley, 1985; Pilgrim, 1986; Singh, 1988; Kuczera *et al.*, 1989; Maheshwari, 1994). Equation (28) is a variant of the kinematic wave flux law given by Equation (24). As in the case of Equation (24), parameters  $k$  and  $m$  in Equation (28) are also interdependent. If  $m$  is not equal to one, the meaning of  $k$  changes and it no longer remains the lag time.

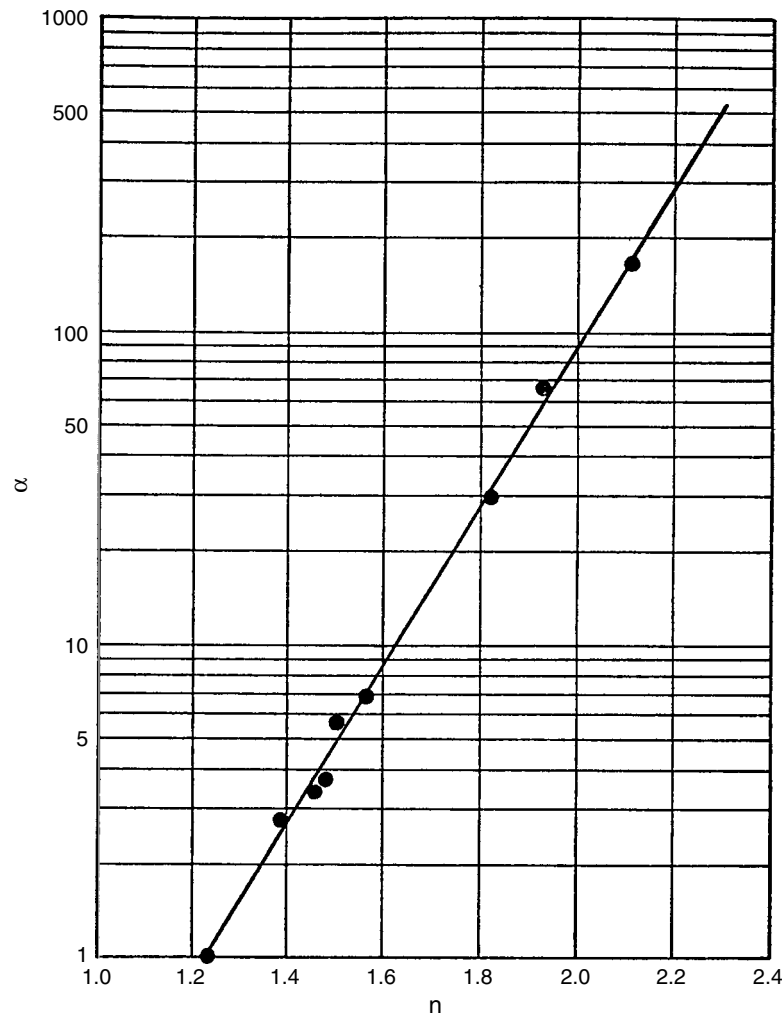


Figure 2. Relation between  $\alpha$  and  $n$  for a converging section on an experimental facility (after Singh, 1976)

### Overland flow

Overland flow is dominant on small watersheds, parking lots, rooftops, highways, airport runways, artificially constructed wetlands, agricultural plots, lawns, and so on. Such basins are bereft of a channel network and their surface flow is essentially overland flow. Woolhiser and Liggett (1967) derived a criterion for judging the goodness of kinematic wave approximation in modelling flow over a sloping plane subject to rainfall or lateral inflow. By expressing the St. Venant equations of continuity and momentum they derived what is now referred to as the kinematic wave number,  $K$ , which reflects the effects of length and slope of the plane as well as normal flow depth and velocity. The kinematic wave number is expressed as

$$K = S_0 L_0 / (h_0 F_0^2) \quad (29)$$

where  $S_0$  is the slope of the plane,  $L_0$  is the length of the plane,  $h_0$  is the normal flow and  $F_0$  is the Froude number for normal flow at  $x = L_0$  at a discharge of  $i_{\max} L_0$ ,  $i_{\max}$  = maximum rainfall intensity.

For  $K \rightarrow \infty$ , the dimensionless momentum equation reduces to the Chezy relation. Based on numerical experimentation on rising hydrographs at the downstream end of the plane, Woolhiser and Liggett (1967)

Table III. Values of  $n$  and  $\alpha$  for butyl, gravel and butyl plus gravel surfaces (after Singh, 1976)

Surface	$n$	$\alpha$ (ft <sup>(2-n)</sup> /s)
Butyl	1.54	11.92
Butyl	1.952	138.26
Butyl	2.243	1147.87
Butyl	1.835	49.29
Gravel	1.707	14.35
Gravel	1.947	40.49
Gravel	1.828	18.19
Butyl plus gravel	2.149	165.54
Butyl plus gravel	2.368	312.61
Butyl plus gravel	1.713	15.16

showed that for  $K > 20$  the kinematic wave approximation would be very good. For highly subcritical flows, Morris and Woolhiser (1980) showed that for  $KFo^2 \geq 5$  the kinematic wave approximation would be good. For many natural slopes  $K$  is usually greater than 50, and the kinematic wave theory applies. In smooth urban surfaces the value of  $K$  is normally between 5 and 20, the kinematic wave approximation would be a crude approximation. Since Woolhiser and Liggett (1967) showed that for large values of the kinematic wave number (greater than 20) the kinematic wave approximation was sufficiently accurate for overland flow, there have been numerous studies applying the kinematic wave theory to modelling overland flow (Eagleson, 1970; Singh, 1996a).

For overland flow on a wide rectangular plane the flux–concentration relation is expressed as

$$q = \alpha h^n \quad (30)$$

where  $q$  is the discharge per unit width of the plane,  $h$  is the depth of flow and  $\alpha$  and  $n$  are parameters. Equation (30) is the same as Equation (10), where the flow depth is a measure of flow concentration and the discharge per unit width a measure of flux. A one-dimensional continuity equation taking into account the lateral inflow to and outflow from the plane can be written as

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = i(x, t) - f(x, t) \quad (31)$$

where  $i(x, t)$  is the intensity of rain falling on the plane and  $f(x, t)$  is the rate of infiltration of water into the plane bed. Combining Equations (30) and (31), the kinematic wave equation for overland flow on a plane is obtained:

$$\frac{\partial h}{\partial t} + c \frac{\partial h}{\partial x} = i(x, t) - f(x, t) \quad (32)$$

where

$$c = (\partial q / \partial h)_{x \text{ constant}} = n\alpha h^{n-1} = nu \quad u = q/h \quad (33)$$

where  $u$  is the average velocity and  $c$  is the wave celerity.

Wooding (1965a,b, 1966) was probably one of the first to apply the kinematic wave theory to model surface runoff. There has since been a proliferation of its application in watershed hydrology, and there is a general consensus that the kinematic wave theory is reasonably accurate for modelling overland flow (Smith and Woolhiser, 1971; Kibler and Woolhiser, 1972; Singh and Woolhiser, 1996; Woolhiser, 1996; Singh, 1996a).

The question arises: is overland flow (or surface runoff) kinematic? The answer to the question depends on the nature of the surface and the space–time distribution of rainfall. In most cases, overland flow is subcritical (Froude number less than one) and turbulent. In rare cases it is laminar as, for example, it may

develop for a short time on urban and highway pavements and in wetlands. The discussion in the preceding section shows that the kinematic wave is the dominant wave in surface runoff, especially during the rising part of the hydrograph and during much of the recession part. The dynamic and diffusion waves do exist but are short lived and play a minor role. Thus, overland flow can essentially be construed as kinematic, with full recognition that diffusion and dynamic waves may dominate in some cases.

### Channel flow

Flow routing in channels involves calculation of a downstream hydrograph from a known upstream hydrograph. The use of kinematic wave theory is quite popular for flow routing in storm sewers for urban drainage and in channels. The movement of flood waves in a river is dominated by a balance between the friction of the bottom and the component of gravity in a direction which is downstream and parallel to the free surface (Lighthill and Whitham, 1955a). Denoting the downward slope of the free surface by  $S$ , the gravitational force per unit length of the river is  $\rho g S C$ , where  $C$  is the concentration (volume of water per unit length),  $g$  is the acceleration due to gravity and  $\rho$  is the density. The frictional force per unit length can be expressed as  $f \rho u^2 P$ , where  $P$  is the wetted perimeter of the flow cross-section and  $f$  is the coefficient of friction. Equating these forces one obtains

$$u = \left(\frac{g}{f}\right)^{0.5} \left(\frac{C}{P}\right)^{0.5} S^{0.5} \quad (34)$$

The quantity  $(C/P)$  is equal to the hydraulic radius  $R$ . If  $(g/f)^{0.5}$  is equated to Chezy's coefficient, Equation (34) is the famous Chezy formula. Equation (34) can be written as

$$u = \alpha R^{0.5} \quad \alpha = (g/f)^{0.5} S^{0.5} \quad (35)$$

The coefficient of friction  $f$  varies with the ratio of the size of typical roughness elements in the bed to the hydraulic mean depth  $R$ . Taking  $f$  as  $f = \beta R^{-1/3}$ ,  $\beta = \text{constant}$ , Equation (34) becomes for constant  $S$  and  $P$ :

$$u = \alpha R^{2/3} \quad \alpha = (g/f)^{0.5} S^{0.5} \quad (36)$$

which is the famous Manning relation. Thus, the flux-concentration relation for flood movement in rivers is kinematical.

In his fundamental study Seddon (1900) emphasized that great rivers, unlike man-made conduits, do not have a uniformly sloping bed, nor do they in any way approximate to this condition. The bed slope exhibits enormous variations (including changes of sign) across the width of the river as well as in the downstream direction over distances comparable to the width. Furthermore, the large-scale variation of the bed frequently resembles a series of pools and bars. The flow control may thus vary with flow depth. At low flow, the flow from one pool to the next is more like the flow over a submerged weir—that is one of the bars. In the neighbourhood of the bars the value of  $S$  would be large. At high flow, the part of a pool may become the narrowest section of the river and control flow like an orifice with vertical walls.

In the case of alluvial rivers the bed constantly changes with time. Variations of 3 m in depth about its mean are common on the lower Mississippi. At the same time the elevation of the water surface changes only slightly, say from  $-5$  cm to 10 cm, for the same value of specific discharge  $q$ . The elevation of the surface varies far more smoothly in time as well as in space than the depth of the bottom. Denoting the elevation of the free surface at each point above a certain datum by stage  $h$ , the flow  $q$  can be regarded as a function of  $h$  rather than the cross-section area  $A$ . For constant  $x$

$$dA = Bdh \quad (37)$$

where  $B$  is the local width of the river. Thus, Equation (6) becomes

$$c = \frac{1}{B} \left( \frac{\partial q}{\partial h} \right)_{x \text{ fixed}} \quad (38)$$

Equation (38) is less susceptible to variation in time. Thus, in general one can write

$$q = q(h, x) \quad (39)$$

With the knowledge of the width

$$B = B(h, x) \quad (40)$$

as a function of stage and position, the wave celerity  $c$  can be predicted. According to Seddon (1900), rivers conform to Equation (38).

In flow in channels, dynamic waves always occur. The friction and slope terms modify the amplitude of waves. In some cases, slope is the dominant controlling factor, as for example flow over a spillway, flow in a steeply sloping gutter, flow on an embankment surface, to name but a few. Friction dominates in such cases as flat to gently sloping forest, grass and cropped lands. On the other hand, geometric features, such as bars, dunes, antidunes, pools, riffles, meandering, etc., exercise local control on flow and modify wave characteristics. Then, there are man-made controls, such as dams, bridges, levees, etc. which modify flow characteristics. These modifications are not conducive to kinematic waves.

In floods, the wave modification is made to such a degree that dynamic waves rapidly become negligible and the kinematic waves assume the dominant role. When the Froude number is less than 1, dynamic waves decay exponentially and are readily damped out. For  $F = 2$ , the kinematic waves merge with dynamic waves and for  $F > 2$ , the kinematic waves no longer exist. For  $1 < F < 2$ , dynamic waves dominate kinematic waves. In channel flow, all waves—kinematic, diffusion and dynamic as well as their variants—exist. At a given position, the relative significance of these waves changes with the changing nature of flow in time. For most river flow without artificial structures,  $F < 1$ , kinematic waves are dominant and hence the kinematic wave theory applies.

For flood movement in rivers the flux–concentration Equation (39) is of the type given by Equation (24). Its special forms, given by Equations (35) and (36), are commonly used in channel flow routing. Coupling it with the one-dimensional continuity equation, the kinematic wave equation becomes

$$\frac{\partial h}{\partial t} + n\alpha h^{n-1} \frac{\partial h}{\partial x} = 0 \quad (41)$$

where the wave celerity is

$$c = n\alpha h^{n-1} = nu \quad (42)$$

which is the same as Equation (33). Since the pioneering work of Lighthill and Whitham (1955a), the kinematic wave theory has been popular for routing flows in channels and channel networks (Singh, 1996a).

#### *Dam break flow*

Dams can break instantaneously or over a finite period of time. A flood wave generated by dam rupture propagates downstream. Such floods are usually large and cause huge damage in terms of property, animal and human life and environment. A large number of approaches have been employed to determine the movement of a dam-break flood wave down a river. Hunt (1982, 1983, 1984a,b, 1987) pioneered the use of kinematic wave theory for modelling flood wave propagation under different conditions: (1) on a dry sloping bed, (2) on a wet sloping channel, and (3) based on a reservoir, dam breach and downstream channel, all of different widths.

The equations for dam-break flood wave propagation are the same as Equations (7) and (24). In this case, however, when water is released upon dam break, a front wall of water advances down the channel. This wall of water is the shock front, that is, the interface between (newly) water-covered and dry (or previously existing water) parts of the channel. Let  $x = s(t)$  or inversely  $t = w(x)$  be the time history of that advancing front; this gives the space–time history of the shock front. The front is a free boundary that must be determined along with the solution. The equation for the free boundary is obtained from the expression for the flow velocity  $u(x, t)$  by observing that the shock front moves with the speed of water immediately behind it. Thus, the shock velocity takes the form of  $u(s(t), t)$  or  $u(x, w(x))$ . This yields

$$\frac{ds(t)}{dt} = \alpha h^{n-1}(s(t), t) \quad (43)$$

A comparison with previously published solutions and experimental results showed the kinematic wave solution was asymptotically valid after the advance of the flood wave downstream by about four reservoir lengths. Katapodes and Schamber (1983) found in a comparative study of five dam-break flood wave models ranging from the complete dynamic equations to a simple normal depth kinematic wave equation that the kinematic wave model was sufficiently accurate.

The flow profile at a given time upon dam break has three regions from the upstream to the downstream: region I is close to the dam where the dynamic wave is the dominant wave; region III is the region close to the right end of the profile or shock front where the dynamic wave is also dominant; and region II is between these two regions, which actually occupies much of the channel longitudinally and where the kinematic wave is the dominant wave. Thus, dam-break flood waves can be reasonably approximated by kinematic waves. With progression of time, the dominance of kinematic waves grows and dynamic waves dissipate.

#### *Comparison of kinematic wave theory with diffusion wave and dynamic wave theories*

Lighthill and Whitham (1955a) have shown that for Froude numbers below 1 (appropriate to flood waves) the dynamic waves are rapidly attenuated and the kinematic waves become dominant. Using a dimensionless form of the St. Venant (SV) equations, Woolhiser and Liggett (1967) obtained the kinematic wave number  $K$  as a criterion for evaluating the adequacy of the kinematic wave approximation. For  $K$  greater than 20, the kinematic wave approximation was considered to be an accurate representation of the SV equations in modelling overland flow. Morris and Woolhiser (1980) modified this criterion with an explicit inclusion of Froude number corresponding to normal flow,  $Fo$ , and showed, based on numerical experimentation, that  $Fo^2K \geq 5$  was a better indicator of the adequacy of the kinematic wave approximation.

Using a linear perturbation analysis, Ponce and Simons (1977) derived properties of the kinematic wave, diffusion wave and dynamic wave representations in modelling open channel flow. Menendez and Norscini (1982) extended the work of Ponce and Simons (1977) by including the phase lag between the depth and velocity of flow. Based on propagation characteristics of a sinusoidal perturbation, they derived criteria to evaluate the adequacy of kinematic wave and dynamic wave approximations. Daluz Vieira (1983) compared solutions of the SV equations with those of the kinematic wave and dynamic wave approximations for a range of  $Fo$  and  $K$ , and defined regions of validity of these approximations in the  $K-Fo$  space. Fread (1985) developed criteria for defining the range application of the kinematic wave and dynamic wave approximations. Ferrick (1985) defined a group of dimensionless scale parameters to establish the spectrum of river waves, with continuous transitions between wave types and subtypes.

The comparative studies, cited above, show that many cases satisfy the conditions for the validity of kinematic wave theory and that in surface water hydrology kinematic waves dominate in such cases. Other wave types may exist but either they are short lived or play a minor role. Thus, it may be concluded that surface hydrology is kinematic under certain restrictions. This is further elaborated by considering two special cases: (1) uniform, unsteady flow and (2) steady, non-uniform flow.

*Unsteady uniform flow.* In a series of papers Singh (1993, 1994a–c, 1995, 1996a,b) derived, under simplified conditions, error equations specifying error as a function of time for kinematic wave approximation for space-independent flows. He considered four different types of scenarios depending upon the presence of lateral inflow or rainfall and infiltration: (1) lateral inflow is constant and there is no infiltration and if there is, it is included in lateral inflow; (2) both lateral inflow and infiltration are considered as constant; (3) both lateral inflow and infiltration are included but their difference is zero; (4) there is no lateral inflow but infiltration is included. These scenarios were analysed under two types of initial conditions: (1) the plane or channel is initially wet and (2) the plane or channel is initially dry. In all 18 cases were analysed.

For space-independent flow the continuity equation takes the form

$$\frac{dh}{dt} = i - f \quad (44)$$

and the momentum equation takes the form

$$\frac{du}{dt} = g(S_0 - S_f) - \frac{iu}{h} \quad (45)$$

where  $g$  is acceleration due to gravity,  $i$  is lateral inflow (or rainfall intensity),  $f$  is infiltration rate,  $S_0$  is bed slope,  $S_f$  is slope of the energy line. For kinematic wave approximation, Equation (45) becomes

$$S_0 = S_f \quad (46)$$

which can be expressed as

$$u = \left(\frac{S_0}{\beta}\right)^{0.5} h^{0.5} \quad (47)$$

where  $\beta$  is a parameter. Singh (1994a,b, 1995, 1996a,b) derived for all cases error equations which turned out to be Riccati equations of the form

$$\frac{dE}{d\tau} = C_0(\tau) + C_1(\gamma, \tau)E + C_2(\gamma, \tau)E^2 \quad (48)$$

where  $E$  is error defined as (kinematic wave solution – dynamic wave solution)/dynamic wave solution,  $\tau$  is dimensionless time,  $\gamma$  is a dimensionless parameter and  $C_i$ ,  $i = 1, 2, 3$  are Riccati coefficients. The parameter  $\gamma$  is analogous to the kinematic wave parameter and is defined as

$$\gamma = \frac{4g^2\beta S_0 h_0}{i_0^2} \quad (49)$$

or

$$\gamma = 4\beta g S_0 \quad (50)$$

depending upon whether lateral inflow ( $i_0$ ) is included or not. In general, the kinematic wave approximation is very good if  $\gamma$  is equal to or greater than 10. As time progresses, i.e.  $\tau$  is greater than or equal to 10, the kinematic wave approximation converges to the dynamic wave solution, as shown for a typical case in Figure 3.

*Steady non-uniform flow.* Steady non-uniform flows are encountered in a variety of natural situations. In overland flow, the steady state is attained for constant rainfall after the flow depth at the outlet has reached equilibrium. This same is true for channel flow subject to constant lateral inflow. For a channel receiving a constant inflow of long duration at the upstream boundary, the flow at the downstream end would reach equilibrium. The steady state solution aids in understanding the nature of water surface profile. It may

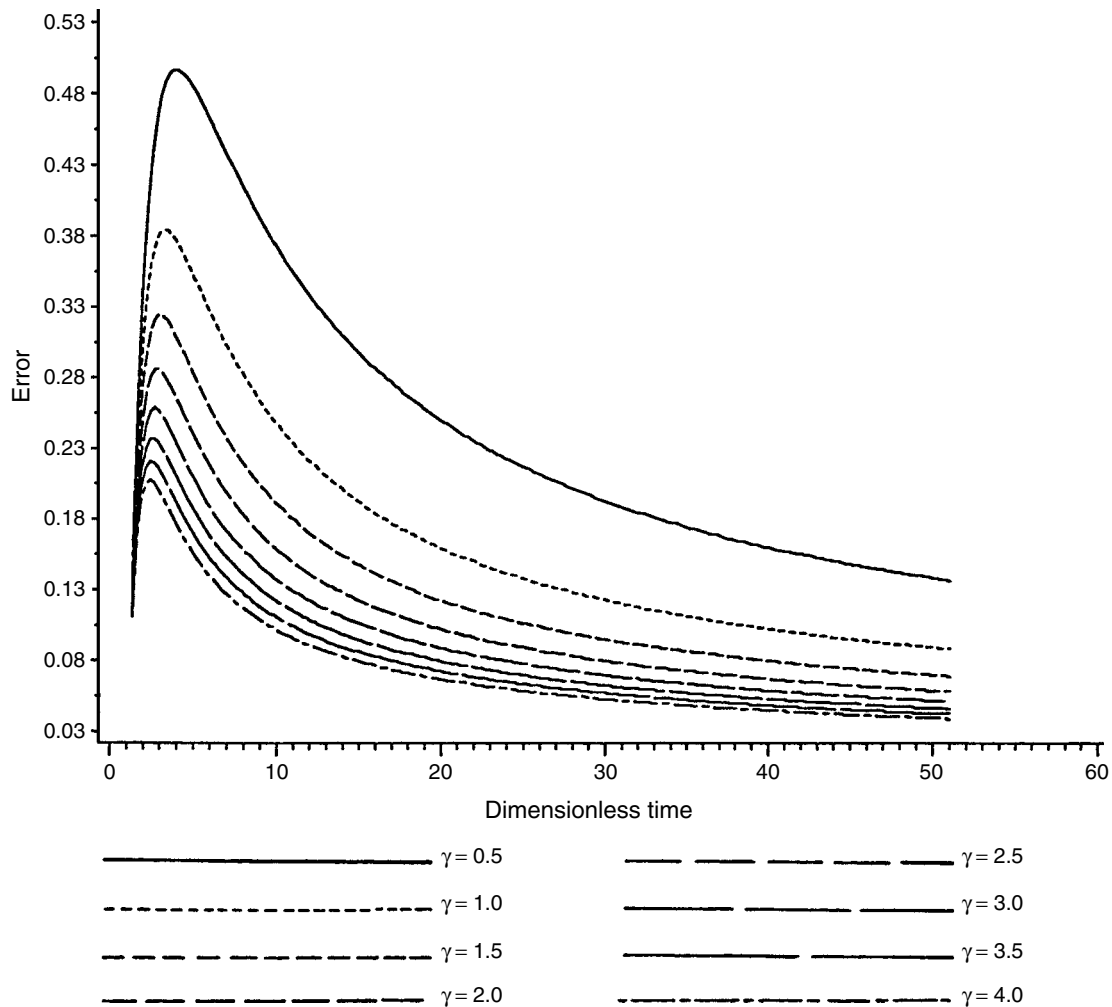


Figure 3. Error in kinematic wave approximation as a function of dimensionless time for a space-independent case for different values of  $\gamma$ , with initial condition defined by constant inflow, constant velocity or constant flow depth and zero infiltration

help define the condition for use of zero depth in place of zero inflow at the upstream boundary. When rainfall duration is much longer than the time of equilibrium, the steady state water surface profiles are very useful. Pearson (1989) examined the criteria for using the kinematic wave approximation to the SV equations for shallow water flow. For steady state one-dimensional flow over a plane he derived a new criterion as  $K \geq 3 + 5/Fo^2$  where  $K$  is the kinematic wave number and  $Fo$  is the Froude number corresponding to normal flow.

Parlange *et al.* (1990) investigated errors in the KW and DW approximations by comparing their predictions with the numerical solution of the SV equations under steady state conditions. They suggested splitting the solution in two regions, one near the downstream end of the plane and the other covering most of the plane. Singh and Aravamuthan (1995a,b, 1996, 1997) derived errors in the kinematic wave approximations under four conditions: (1) zero flow at the upstream boundary, (2) finite depth at the upstream boundary, (3) critical flow depth at the downstream end, and (4) zero-depth gradient at the downstream boundary. Depending on the inclusion of lateral inflow and infiltration, 21 cases were analysed for the four different scenarios mentioned

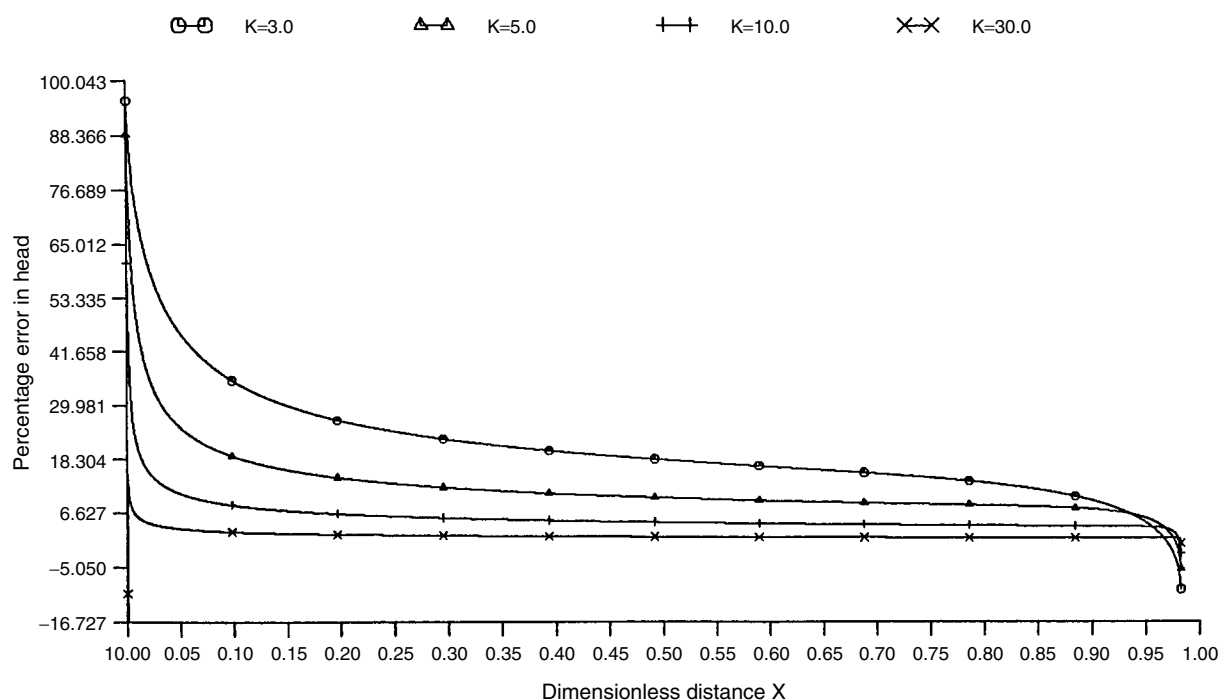


Figure 4. Percentage error in kinematic wave approximation as a function of dimensionless distance for different values of kinematic wave number ( $K$ ), with critical depth downstream boundary condition, constant inflow at the upstream boundary ( $q = 1$ ), zero infiltration and Froude number as 1

above. By comparing the kinematic wave solution with the dynamic wave solution, error equations were derived for all cases.

For time-independent flow, the governing equations are

$$\frac{dh}{dx} = i - f \quad (51)$$

and

$$\frac{d}{dt} \left( \frac{1}{2} u^2 + gh \right) = g(S_0 - S_f) - \frac{iu}{h} \quad (52)$$

The error equations were generalized Riccati-type equations:

$$\frac{dE}{dx} = C_0E + C_1E^2 + C_2E^3 + C_3E^4 + C_4E^5 \quad (53)$$

where the parameters  $C_i$ ,  $i = 1, 2, 3, 4$  are nonlinear functions of  $x$ ,  $h$ ,  $K$  and  $Fo$  which are as defined before. In most cases it was found that the downstream boundary exercised significant influence on the adequacy of the kinematic wave approximation. Away from the boundaries, i.e.  $0.1 \leq x/L \leq 0.9$ , where  $L$  is the length of the plane, the kinematic wave approximation was a good approximation, as shown for a typical case in Figure 4.

#### APPLICATION OF KINEMATIC WAVE THEORY TO IRRIGATION HYDROLOGY

Surface irrigation involves movement of water as shallow flow over planes or in channels. When an inflow stream is introduced at the upstream end of the plane, water advances with a sharply defined wetting front

down the slope toward the downstream end in what is referred to as the advance phase of the irrigation flow process. The wetting front is a shock and constitutes a free boundary. This phase is characterized by downfield movement of the advancing water front and continues until the water reaches the lower end of the field. Assuming continued inflow after water has advanced to the downstream end of the field, water will, if there is no downstream dam, flow out at the end of the field and continue to accumulate in the field in the storage phase. In this phase, water exists on the entire field, no other boundary moves and inflow continues at the upper end of the field. The storage phase ends and the depletion phase begins when the inflow ceases. The depletion phase continues until the depth of surface water at the upstream end is reduced to zero. This phase differs from the storage phase only in the absence of inflow into the field. The recession phase begins when the depth of surface water at the upstream end decreases to zero. This marks the formation of the recession or drying front. The downfield movement of the drying front characterizes the recession phase of the inflow. This phase continues until no water remains in the field and irrigation is complete. This is the schematic of the irrigation cycle (Sherman and Singh, 1978; Maheshwari, 1988).

It should be pointed out that irrigation hydrology has many similarities with surface water hydrology of the preceding section. Thus, only those aspects that are peculiar to irrigation hydrology will be dealt with here. In a study on errors in the kinematic wave solution by comparison with full dynamic wave solution in irrigation borders and channels, Reddy and Singh (1994) and Singh (1994a–c) found the kinematic wave approximation to be reasonably accurate. Irrigation hydrology is not kinematic in a strict sense, but as will be shown in what follows kinematic wave theory is a good approximation.

#### *Flow in irrigation borders*

For flow in irrigation borders, the flux–concentration relation of Equation (30) holds. However, because irrigation borders have porous beds, the continuity equation requires modification and can be expressed as

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = -f(\zeta) \quad (54a)$$

where  $f$  is the infiltration rate (volume per unit area) and  $\zeta$  is the elapsed time after the front wall of water covers a position  $x$  and is defined as

$$\zeta = t - \xi(x) \quad (54b)$$

where  $t = \xi(x)$  is the time history of the advancing front and constitutes a free boundary of the solution domain. The equation for the free boundary  $x = s(t)$  or  $t = \xi(x)$  can be obtained by replacing  $x$  by  $s(t)$  and observing that  $u(s(t), t)$  is the velocity of the front wall of water, that is

$$u(s(t), t) = \frac{ds(t)}{dt} = \alpha h^{n-1}(s(t), t) \quad s(0) = 0 \quad (55)$$

Chen (1970), Smith (1972) and Cunge and Woolhiser (1975) were among the first to apply the kinematic wave theory to model some aspects of flow in irrigation borders. Sherman and Singh (1978, 1982) and Singh and Ram (1983) presented a complete mathematical treatment of the irrigation cycle based on the kinematic wave theory and showed that the theory was sufficiently accurate for irrigation modelling.

Flow in irrigation borders is similar to flow in infiltrating channels or over porous beds. The kinematic wave theory is a good approximation but is not capable of simulating the entire irrigation cycle, as exemplified by its inability to simulate the depletion phase. This is because the kinematic wave assumption gives zero flow depth at the upstream end as soon as the flow is terminated, whereas in reality the depth becomes zero after some time. However, in most irrigation cases this limitation is not severe, for the depletion phase is minor compared with other phases of the irrigation cycle. There is another crude approximation that kinematic wave theory makes, i.e. theory yields that the shock front moves as a vertical wall whereas in actuality it moves as a front with finite slope ( $\partial h/\partial x < 0$ ). These limitations notwithstanding, the border irrigation flow, for the most part, does satisfy the conditions for validity of the kinematic wave approximation.

*Flow in irrigation furrows*

Irrigation with furrows is similar to that with borders but its hydraulics and intake phenomena are complicated by the geometry of the furrow cross-section and by the occurrence of unsteady and non-uniform flow. Walker and Humpherys (1983) discussed three modifications that are required to develop border irrigation models into furrow irrigation models. First, the geometry of the furrow cross-section must be prescribed. Although furrows are not prismatic, their hydraulic parameters, such as the depth of flow, wetted perimeter, cross-sectional area, etc. can be described using simple power functions. Second, infiltration must be described by a simple function that incorporates both a time-dependent rate and a basic or steady rate term (such as the Kostiakov–Lewis equation) rather than a simple time-dependent rate term (such as the Kostiakov equation). Elliott and Walker (1982) found that the advance could be simulated reasonably well without the addition of the basic rate but not runoff. Third, infiltration is affected by wetted perimeter and thereby flow rate (Fangmeier and Ramsey, 1978). A commonly used assumption is that infiltration is only a function of intake opportunity time and can therefore be expressed as a function of time. However, infiltration depends on discharge, thereby making it a function of time as well as flow rate. The furrow geometry is expressed using rigid perimeter models, non-rigid perimeter models or constant shape models.

The kinematic wave equations for furrow irrigations are similar to those for border irrigation. Walker and Humpherys (1983) applied the kinematic wave approximation to furrow irrigation and solved the equations using first-order Eulerian integration. Rayej and Wallender (1987, 1988) reformulated the model of Walker and Humpherys (1983) by using distance as input and solving for advance times. This permitted infiltration characteristics to vary at any point along the furrow. Wallender and Yokokura (1991) developed an explicit kinematic wave model in which the time step was adjusted to have computational nodes to fall at specified locations along the furrow. In all of these modelling efforts the kinematic wave approximation was found to be sufficiently accurate.

In furrow irrigation, the kinematic wave theory is not as good an approximation as it is in border irrigation. One reason is that the depletion phase may be significant and lasts much longer. Because furrows are temporary channels, there may be strong interaction between flow and furrow geometry. Furrow geometry changes in time due to erosion and sediment transport. Consequently, the hydraulic gradient of flow depth may be significant and cannot be neglected. Thus, waves tend to be more diffusive. Nonetheless, as a first approximation the kinematic wave theory is a good approximation.

## APPLICATION OF KINEMATIC WAVE THEORY TO VADOSE ZONE HYDROLOGY

Much of the mathematical treatment of flow in unsaturated porous media has dealt with capillary-induced flow (Smith, 1983). However, there exists a multitude of cases where gravity dominates vertical movement of soil moisture. Some examples include drainage following infiltration, water percolation deeper into the soil, vertical movement of moisture in relatively porous soils when rainfall or surface fluxes are typically of the order of, or less than, the soil-saturated hydraulic conductivity, to name but a few. For treatment of such cases, the kinematic wave theory is simple yet reasonably accurate.

*Unsaturated flow*

For gravity-dominated unsaturated flow, it can be postulated that the moisture flux is a function of only the moisture content  $\theta$ . This postulate permits application of kinematic wave theory. The moisture volume flux (or Darcy flux)  $q$  can be expressed as

$$q = K(\theta) \quad (56)$$

where  $K$  is the unsaturated hydraulic conductivity. A popular model relating  $K$  to the moisture content  $\theta$  is (Brooks and Corey, 1964)

$$q = K = K_s \left( \frac{\theta - \theta_0}{\eta - \theta_0} \right)^a = K_s S_e^a \quad S_e = \left( \frac{S - S_0}{1 - S_0} \right), \quad S = \frac{\theta}{\eta} \quad (57)$$

where  $K_s$  is the saturated hydraulic conductivity,  $\theta_0$  is the moisture content at field capacity,  $S_e$  is the effective saturation,  $S_0$  is the saturation at  $\theta = \theta_0$ ,  $\eta$  is the porosity and  $a$  is an exponent related to the pore size distribution index. Equation (57) is a kinematic wave flux law. One can also write Equation (57) as

$$\theta = \theta_0 + (\eta - \theta_0) \left( \frac{q}{K_s} \right)^{1/a} \quad (58)$$

In the kinematic wave approximation the dissipative terms (second-order spatial derivatives) are neglected and hence the character of the transport equation is changed from parabolic to hyperbolic. In many applications, the role of dissipation decreases with time while that of advection remains constant, thus validating the use of the kinematic wave approximation.

The one-dimensional continuity equation can be written as

$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = -e(z, \tau) \quad (59)$$

where  $z$  is the vertical distance positive in the downstream direction,  $e(z, \tau)$  is the evaporation rate,  $\tau = t - s(z)$  is the time history of the wetting front and  $s(z)$  is the space-time history of the advance front. Coupling Equations (56) and (59) one gets the kinematic wave equation for soil moisture movement in the vadose zone:

$$\frac{\partial \theta}{\partial t} + M \frac{\partial \theta}{\partial z} = -e(z, \tau) \quad (60)$$

where  $M = (\partial q / \partial \theta)_{z \text{ constant}}$  is referred to as the mobility of water in soil (Irmay, 1956).

Although Sisson *et al.* (1980) applied the kinematic wave theory to internal drainage, Smith (1983) was probably the first to apply the theory to develop a complete kinematic wave model for soil moisture movement. Charbeneau (1984) extended Smith's work. In a series of papers, Germann and coworkers (Germann, 1987; Germann and Beven, 1985, 1986; Germann *et al.*, 1987) extended the application of the theory to infiltration and drainage into and from soil macropores, as well as microbial transport. Yamada and Kobayashi (1988) discussed the kinematic wave characteristics of vertical infiltration and soil moisture, with the aid of field observations on tracers. They concluded that the vertical infiltration of soil moisture had the characteristics of kinematic waves. Singh and Joseph (1994) extended the theory to model soil moisture movement with plant root extraction.

The question arises: is unsaturated flow kinematic? The answer to the question depends on the status of soil moisture. At low moisture contents, capillary forces dominate and flow is dominated by the diffusion wave, although the kinematic wave also exists. With increasing moisture content, the diffusion wave diminishes in value and the kinematic wave begins to dominate. Thus, at a fixed position in soil both waves exist at a time but their relative magnitudes are governed by the moisture content at that time.

### Macropore flow

Structural soil pores constitute important pathways for water and may carry water before the finer pores of the soil matrix are fully saturated. Macropores may be characterized as animal burrows, such as worm and insect tunnels or structures built by small mammals; as channels formed by plant roots; as desiccation cracks; or as cracks and joints formed by various physical and chemical geological processes. The permeability of a soil is controlled by the presence of large pores that do not exert capillary forces on water flowing in them. Water is more efficiently transmitted through structural pores and not all smaller pores are filled with air prior to the infiltration event participating in the flow process. Flow may occur in two domains: (1) structural pores and (2) soil matrix. In structural pores, referred to as macropores or the macropore system, the flow of water is primarily driven by gravity and is only faintly impeded by capillary forces. In the soil matrix or soil micropores, the flow of water is subject to capillary forces. Micropores are often not completely saturated with water before structural pores can actively transport water, because the time required for saturation of

micropores may exceed that required to establish flow in macropores (Germann, 1990a,b). When the minimum width is 3 mm, macropores are assumed to exert negligible capillarity on water flowing in them and the capillary potential is assumed greater than  $-0.01$  m (Beven and Germann, 1981). Macropore systems may also include channels of considerably smaller diameter, as small as 0.03 mm, provided the channels extend markedly in the general direction of flow.

To deal with the flow in macropores, the entire soil or rock volume, not the individual flow path, is considered. This is a macroscopic definition of the flow system and does not require a detailed description of the actual flow paths, including morphological descriptions and detailed geometrical and spatial definitions of the macropore systems. Fundamental to this approach is the flux–concentration relation defined as

$$q = bw^a \quad (61)$$

where  $q$  (m/s) is the volume flux density, defined as the volume of water in the macropore system flowing across a unit area of the entire porous medium per unit time;  $w$  is the macropore water content ( $\text{m}^3/\text{m}^3$ ) defined as the volume of water in the macropore per unit volume of the entire porous medium; and  $a$  and  $b$  are parameters. Parameter  $b$  (m/s) is called conductance and parameter  $a$  is a dimensionless exponent. An appropriate value of parameter  $a$  is 2.5. Germann (1990a,b) has presented solutions for flow in macropores under different conditions: (1) Flow in a homogeneous macropore system that is surrounded by a water-saturated matrix; (2) flow in a homogeneous macropore system that is surrounded by a water-sorbing matrix; and (3) flow in a heterogeneous macropore system that is surrounded by the water matrix. He found the value of parameter  $a$  to vary between 2.2 and 3.76.

The one-dimensional continuity equation is

$$\frac{\partial w}{\partial t} + \frac{\partial q}{\partial z} = 0 \quad (62)$$

Coupling Equations (61) and (62), one obtains the kinematic wave equation for flow in a homogeneous macropore system surrounded by a water-saturated matrix as

$$\frac{\partial q}{\partial t} + c \frac{\partial q}{\partial z} = 0 \quad (63)$$

where  $c$  is the wave celerity defined by

$$c = \frac{dq}{dw} = ab^{1/a} q^{(a-1)/a} \quad (64)$$

If the macropore system is surrounded by a water-sorbing matrix then a sink term has to be added to Equation (62) to account for the loss of water into the matrix. Thus, the kinematic wave equation becomes

$$\frac{\partial q}{\partial t} + c \frac{\partial q}{\partial z} + crw = 0 \quad (65)$$

where  $r$  (l/s) is the sorbance—the rate of water loss per unit time and per unit  $w$  such that

$$\frac{\partial w}{\partial t} = -rw(t) \quad (66)$$

Germann (1987, 1990a,b), Germann *et al.* (1987), DiPietro and Lafolie (1991) among others have demonstrated that the kinematic wave theory fits the experimental data reasonably. When rain intensities are low, however, there is high dispersion and the kinematic wave theory does not produce flow hydrographs accurately. The question arises: is macropore flow kinematic? In macropores flow is governed by gravity and capillarity plays a minor role, unless rainfall intensities are very low. Thus, it can be concluded that macropore flow is kinematic under a broad range of conditions but is diffusive outside this range.

## APPLICATION OF KINEMATIC WAVE THEORY TO SUBSURFACE STORM FLOW

*Interflow*

In many parts of the world storm runoff is produced by subsurface runoff (Dunne, 1978). Subsurface flow, throughflow, saturated interflow and interflow are other designations used somewhat interchangeably in hydrological literature. Beven (1989) defined interflow as the near-surface flow of water within the soil profile resulting in seepage to a stream channel. It may involve both saturated and unsaturated flows in both vertical and downslope directions, and may encompass all surface contributions to the storm hydrograph. When the seepage face extends onto the soil surface the inflow process may transform into or encompass the process of return flow by which the subsurface flow can contribute to overland flow. In some watersheds, there may exist a steep hydraulic gradient either on account of the slope of the hillslope (Beven, 1981) or the buildup of groundwater mounds on shallow slopes (Sklash and Farvolden, 1979). Under such conditions, the watershed or hillslope response to rainfall may be dominated by subsurface stormflows involving both saturated and unsaturated flows. The characteristics of such a response depend on the depth and hydraulic conditions of the unsaturated zone existing at the time of rainfall storm (Dunne, 1978). The kinematic wave theory can be applied to model stormflow, provided certain requirements relative to the physical nature of the watershed or hillslope are met. According to de Vries (1987) these requirements are a thin saturated zone, a steep bed and a low recharge rate. If the zone of saturation is thin, water table and flow line can be assumed to be approximately parallel to the bed; this assumption, however, requires a high value of saturated hydraulic conductivity in this zone.

On a sloping soil mantle with constant hydraulic conductivity Beven (1981) found that the kinematic wave approximation was a good representation of the extended Dupuit–Forchmmer theory in terms of predicting both the water table profiles and subsurface stormflow hydrographs, if the nondimensional parameter  $\lambda$  was less than 0.75. This parameter is defined as

$$\lambda = \frac{4i \cos S_0}{K \sin S_0} \quad (67)$$

where  $i$  is the input per unit area parallel to the impermeable bed,  $S_0$  is slope of the hillslope and  $K$  is the hydraulic conductivity. By using the critical value of  $\lambda$ , a range of values of slope angle and saturated hydraulic conductivity for which the kinematic wave approximation is valid was specified. Beven (1981) found the kinematic wave approximation to be useful in cases of practical interest. Beven (1982a) examined the response times of unsaturated and saturated flows on hillslopes using the kinematic wave approximation. He showed that for many combinations of hillslope and soil parameters subsurface response times were too long to be considered as stormflow and the combinations of high hydraulic conductivities, shallow soils and wet initial conditions leading to fast response were quite reasonable and are expected in field conditions. For such cases the kinematic wave approximation yields useful results.

The question arises: is subsurface flow kinematic? The answer is not always so, depending on the rate of recharge, hydraulic conductivity of the soil, bed slope and the thickness of the saturated zone. One case where the kinematic wave theory does not do well is where the porous medium is cut by the bed of a stream or lake. The kinematic wave assumption does not accommodate the lower boundary condition. As a result it will always overpredict the length of the seepage face at the downstream boundary, since it takes no account of downstream effects in the vicinity of the boundary (Beven, 1981). Away from the seepage face to the upstream, the theory would yield acceptable results, provided  $\lambda$  is less than 0.75. The farther the distance from the lower boundary, the more accurate the theory, since the effect of the downstream boundary will decline.

*Hillslope response*

Three types of approach can be distinguished for the study of storm runoff generation: antecedent moisture-based, runoff type-based and generation mechanism-based. The generation mechanism-based approach forms

the basis of the most widely accepted theories of storm runoff generation. This approach examines the manner in which the runoff water travels over the last several tens of metres to the stream. It accounts for both the rapid response of the stream to runoff-producing events and the observed increase in stream discharge. The dominant mechanisms in streamflow generation are Hortonian overland flow, partial area-overland flow, variable source area-overland flow, variable source area-subsurface storm flow (Wigmosta *et al.*, 1994) and deep aquifer or groundwater flow. Freeze and Harlan (1969) sketched a blueprint for investigating the dynamic response of a hillslope. Smith and Hebbert (1983) described the interaction of surface and subsurface hydrologic processes. They modelled surface flow using the kinematic wave approximation, the subsurface flow by a simplified but analytical approach and the subsurface saturated flow by the Dupuit approximation. Beven (1982b) employed the kinematic wave approximation to model flow from both the unsaturated and saturated zones and showed that the kinematic wave model produced results that were in good agreement with field observations. Sloan and Moore (1984) obtained similar results for subsurface stormflow on steeply sloping forested watersheds. In hillslope hydrology, many complications arise from variable rainfall intensities, variable initial conditions downslope, changes in soil depth and slope angle along the slope, topographic convergence and divergence, existence of a saturated zone prior to the onset of rainfall, macropore channelling through unsaturated zone, etc. The hillslope hydrology is not kinematic but, as argued by Beven (1987), the kinematic wave theory can be considered as a good approximation to a more rigorous, physically based description. Thus, there is a strong argument in favour of the kinematic wave theory capable of mimicking the important features of streamflow generation.

#### APPLICATION OF KINEMATIC WAVE THEORY TO RIVER AND COASTAL HYDROLOGY

In river and coastal hydraulics, kinematic wave theory has received much less attention. That may be because historically laboratory experimentation has played a significant role in development of hydraulics and there is a good deal of both laboratory and field data on hydraulic phenomena for model parameter calibration. The spatial scales at which hydraulic problems, in general, are dealt with are much smaller than those in hydrology. As a result, there occur much less averaging and lumping. Nevertheless, many hydraulic phenomena can indeed be approximated by the kinematic wave theory as shown in what follows.

##### *Sediment transport in upland areas*

Erosion and transport of soil by water from upland areas represents a complex interaction between the kinematics of the falling rain, the hydrodynamics of flow and the dynamics of granular materials. Sediment transport involves: (1) erosion by rainfall input; (2) erosion by flowing water; (3) transport by rainfall splash; (4) transport by flowing water; and (5) deposition during flow. The amount of sediment available for transport is the sum of the sediment inflow from contributing flow units, initially loose soil left from previous storms, the amount of soil detached by raindrop impact and the amount of soil detached by flow of water. The erosion and sediment flux due to rainfall impact,  $E_I$ , is expressed as

$$E_I = BI^a \quad (68)$$

where  $B$  is a coefficient and  $I$  is rainfall intensity;  $a$  is an exponent; and  $E_I$  represents interrill erosion or sheet erosion and varies with rainfall intensity, soil characteristics, vegetative cover and slope. Usually, the exponent  $a$  is taken as unity. Interestingly, Equation (68) is a kinematic relation.

The sediment flux,  $E_R$ , representing rill erosion and accounting for deposition can be expressed as

$$E_R = \gamma Bh^n - C\gamma Q \quad (69)$$

where  $\gamma$  and  $B$  are coefficients,  $Q$  is water discharge,  $h$  is the flow depth and  $C$  is sediment concentration. Equation (68) is a kinematic relation, meaning that both aggradation and degradation are kinematic.

The sediment discharge  $Q_s$  can be expressed as

$$Q_s = CQ \quad (70)$$

where  $Q$  is water discharge and is obtained from the kinematic wave Equation (32).

A one-dimensional form of sediment continuity equation is

$$\frac{\partial(Ch)}{\partial t} + \frac{\partial Q_s}{\partial x} = E_I + E_R \quad (71)$$

Equation (71) is the kinematic wave equation and has been the basis of a number of studies on soil erosion. Smith (1981), Singh (1980, 1983), Singh and Prasad (1982), Singh and Regl (1983), Hjelmfelt *et al.* (1975), Hjelmfelt (1976), Croley (1982), Croley and Foster (1984), Laguna and Giraldez (1993), Rose *et al.* (1983a,b) among others applied the kinematic wave theory to model soil erosion by water from watersheds and found reasonable agreement between theory and field or experimental data. It is concluded that soil erosion and sediment transport on upland watersheds are kinematic for the most part. This is also because on upland watersheds overland flow, which is dominant, is kinematic and is primarily responsible for erosion of soil and subsequent transport of eroded soil. Therefore, there is little surprise that upland watershed erosion is dominantly kinematic.

#### *Bed load transport*

The transport of larger and denser sediments, such as sand and aggregates, takes place as bed load. This is usually the case in furrows, rills and other flow concentrations which exist on intensively cropped land. Sediment transport occurs in the form of a densely packed moving layer near the bottom of the channel. In developing a hydrodynamic model for sediment transport in rill furrows, Prasad and Singh (1982) employed continuum mechanics, taking into account the interaction between suspension region and bed layer. When sediment transport is assumed to occur mainly due to the action of shear stress, the resulting equation turns out to be the kinematic wave equation:

$$\frac{\partial b}{\partial t} + Kb \frac{\partial b}{\partial x} = \frac{I}{C_b} \quad (72)$$

where  $b$  is the bed layer thickness,  $x$  is the distance,  $K$  is a nonlinear function of shear stress at the bed,  $I$  is the net sediment deposited on the channel bed which is transported in the bed layer and  $C_b$  is the concentration of sediment in the bed.

Using measurements in the East Fork River in Wyoming, Weir (1983) showed that the wave equation for the bed was the kinematic wave equation which adequately described the gravel wave translation:

$$\frac{\partial z}{\partial t} + c \frac{\partial z}{\partial x} + cS = 0 \quad (73)$$

where  $z$  is the bed thickness above some fixed datum,  $c$  is characteristic velocity of bed waves found previously by Gradowczyk (1968),  $x$  is horizontal distance and  $S$  is friction slope. Both  $c$  and  $S$  can be functions of  $x$  and  $t$ . The term  $c$  is associated with the net rate of gravel flow. Weir (1983) extended his analysis to predict annual bedload transport and annual variations in riverbed cross-sectional area. Song (1983) extended the kinematic wave theory to explain various aspects of bed form phenomena, including identification of the forces that are related to the migration speed and the growth or decay of sand waves. He suggested that the migration speed of sand waves was determined by the mean hydrodynamic condition that caused the bed load to be affected by bed elevation changes.

Bed load, under certain conditions, is kinematic. Outside those conditions, it is not. For example, the exchange of sediment back and forth between suspension zone and bed load zone does not occur. However, the conditions for the validity of the kinematic wave theory have not yet been delineated.

*River channel bars and dunes*

River beds are made up of grains or cobbles and exhibit wavelike undulations. They are neither smooth nor regular in form. The irregularity of a river bed may be either patterned or textured. Sand beds are typically characterized by ripples and dunes except when there is high flow. A majority of river beds have alternating deeps and shallows, called pools and riffles. These bed forms are caused by accumulation of gravel in bars along the stream at distances equal to five to seven times the channel width. Forming all of the bed forms—ripples, dunes and gravel bars—are groups of noncoherent particles which are deposited in some characteristic manner. Langbein and Leopold (1968) recognized that ripples, dunes and bars of a river exhibit characteristics similar to those of kinematic waves wherein these waves are composed of bed particles that concentrate temporarily in bed form and are later transported downstream. River channel bars and dunes are not kinematic but do exhibit kinematic wavelike properties.

*Flow of beads in a flume.* To establish a relation between the speed of discrete particles moved by water and the distance between them, Langbein and Leopold (1968) derived a flux–concentration relation for this kind of transport in a flume study. For glass spheres of 0.185 inch diameter introduced at the upper end of the flume, they found a relation

$$\frac{v}{v_0} = 1 - \frac{C}{C_0} \quad (74)$$

where  $v$  is the mean speed of beads,  $v_0$  is the speed of a single particle on the bed of flume of flowing water,  $C$  is the mean linear concentration or density (beads per unit length, say an inch) and  $C_0$  is the maximum linear concentration or density. This relation shows that the mean speed decreases linearly from  $v_0$ , when a single bead is moving, to zero, when beads are in contact, which is the case when  $C/C_0 = 1.0$ . According to Langbein and Leopold (1968) the particles do not move uniformly but form and move in groups rather quickly. Different beads move at different speeds depending on the size of their group as they leave one group and move ahead to overtake another. When the rate of feed at the upstream end is increased, the size of the groups increases. This leads to decrease in the average speed and ultimately a point is reached when a group becomes so large that a jam is formed and all motion is halted. This situation is analogous to a traffic jam and constitutes a shock. The close spacing of particles has the effect of retarding the motion by shielding one particle from its neighbours downstream. Taking account of Equation (74) the flux–concentration relation is expressed as

$$Q_s = C v_0 \left( 1 - \frac{C}{C_0} \right) \quad (75)$$

Viewing the flux–concentration relation, the peak represents the maximum rate of transport. The decreasing flux represents a kind of statistical average between two cases: (1) when the concentration of beads breaks up or becomes more open and transport takes place at the maximum speed and (2) when the beads jam and transport is zero. The greater spatial concentration results in greater frequency of jamming. The effect of close spacing does not become very large until the particles are about seven diameters apart. Equation (75), based on laboratory experiments, is clearly kinematic and therefore the flow of beads in a flume can be inferred to be kinematic.

*Transport of sand in pipes.* Considering the movement of sand in pipes and flumes, it is observed that sand particles start, stop, exchange positions or readily pass one another in a complex manner. Thus, rather than considering the movement of individual particles, it is advisable to consider bulks of particles. In this case concentration should be interpreted in terms of the weight of particles per unit space rather than the number of particles per unit space and includes moving sediment as well as sediment deposited temporarily at rest. This definition is different from the one used in sediment transport where concentration is defined as the mass of sediment per unit mass of water. Grains grouped together move less readily than those widely spaced, and

one type of group may include grains lying in a ripple or dune and may thus be covered temporarily by other grains. When the pipe is clogged,  $v = 0$  and when sand transport is light,  $v = v_0$ . For medium sand, Langbein and Leopold (1968) obtained linear (or spatial) concentration as

$$C = \left[ 0.7 \left( 1 - \frac{v}{v_0} \right) + C_a \frac{v}{v_0} \right] Aw \quad (76)$$

where  $C_a$  is the per cent of sand divided by 100,  $A$  is the cross-sectional area of pipe,  $v_0$  is the mean velocity for clear water and  $w$  is the unit weight of sediment (sand) material.

For data from Blatch's experiment (Blatch, 1906) on transport of 0.6 mm sand in a 1-inch pipe for a head loss of 30 ft per 100 ft of pipe, the value of  $v_0$  was 9.25 ft/s for clear water ( $C_a$ ). The concentration was measured. In turbulent suspension  $C = C_p Aw$ , where  $C_p$  is the proportion of sand in the effluent per unit volume of the sediment material (mixture). At the other extreme of the pipe being clogged, sediment concentration is  $(1 - \eta)Aw$ , where  $\eta = \text{porosity} \simeq 0.3$  for sand particles. Within these limits concentration varies linearly with  $(1 - v/v_0)$  where  $v$  is the mean velocity and  $v_0$  is the mean velocity for the same rate of head loss for clear water. The average speed of particle movement decreases with an increase in concentration.

The flux is obtained as the product of per cent sand (volume of sand transported in ratio to the total mixture discharge) times mean velocity times cross-sectional area of pipe:

$$Q_s = C_p v A \quad (77)$$

The flux-concentration relation takes the form

$$Q_s = v \frac{C}{w} = \left[ 0.7 \left( 1 - \frac{v}{v_0} \right) + C_a \frac{v}{v_0} \right] Av \quad (78)$$

This relation is shown in Figure 5. According to Blatch (1906), the rising limb of the curve corresponds to complete suspension of the transported sand. The falling limb indicates that sand is being dragged along the bottom of the pipe. A layer of sand is formed on the bottom and transport of sand occurs in much the same manner as in an open flume. With increasing load, there is spasmodically a blockage and transport occurs in an average sense. As spatial concentration approaches its maximum of 0.63 lb/ft, transport ceases entirely. The maximum transport occurs when concentration is at about 45% of its maximum value.

Equation (78) is kinematic, for Equations (76) and (77) are kinematic, and is based on experimental data. Thus, transport of sand in pipes can be referred to as kinematic. This is also because flow is kinematic for the most part.

*Transport of sand in flumes.* For transport of sand in flumes, one can invoke Equation (74) where  $v_0$  is the velocity of a single particle,  $k$  is linear concentration or particles per unit length and  $k_0$  is the reciprocal of the linear dimension of a single particle. Since the mean rate of transport  $T$  equals the product of mean speed and mean linear concentration, we obtain

$$T = vk \quad (79)$$

where  $v$  is given by Equation (74).

For transport of sand by flowing water in an open flume, the transport rate and concentration are defined a little differently.  $T$  is in pounds of sand carried per second per foot of width and  $W$ , in terms of areal concentration, is in pounds of sand in motion per square foot. Thus, transport is effectively the product of two factors—mean particle velocity and mean areal concentration. Thus the flux-concentration relation is defined as

$$T = v_0 W \left( 1 - \frac{W}{W_0} \right) \quad (80)$$

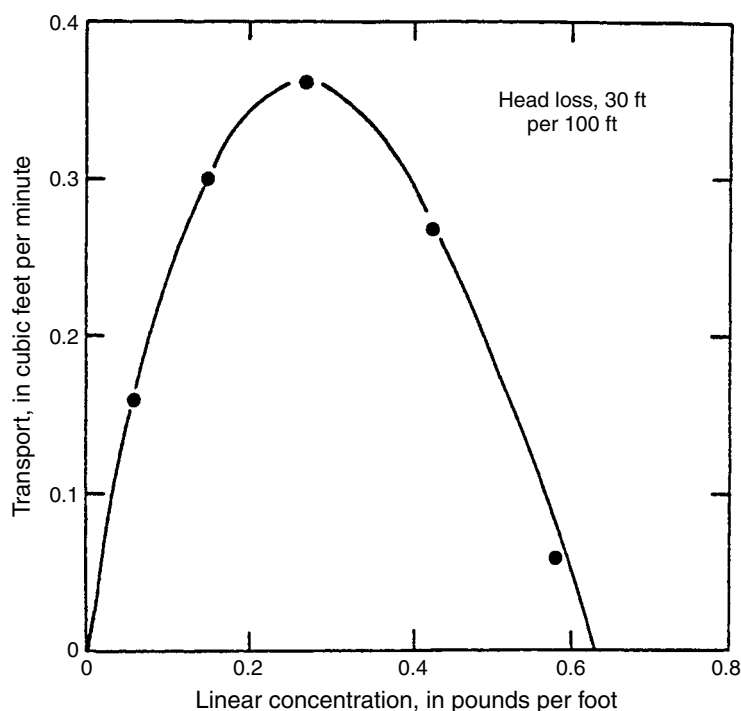


Figure 5. Flux-concentration relation for movement of sand in a pipe (data from Blatch, 1906)

where  $v_0$  is the velocity of particles as the concentration approaches zero (meaning clear water) and is a function of the velocity of water,  $W$  is the areal concentration (weight of sand per square foot of channel) and  $W_0$  is the areal concentration when transport ceases ( $T = 0$ ).

There is a suggested maximum for a value of  $W$  of about  $25 \text{ lb/ft}^2$  and  $W_0$  may be estimated at  $50 \text{ lb/ft}^2$ . The quantity  $(1 - W/W_0)$  decreases as  $W$  increases, reaching zero when  $W = W_0$ . Conceptually,  $W_0$  corresponds to a state represented by dunes whose height is so great that the flow in the flume is blocked. As in the case of pipe flow, concentration is the weight of the sediment in motion and the sediment deposited temporarily at rest. It includes sediment moving in suspension or in bed forms. In flumes, moving bed forms account for nearly all transport. An estimate of  $W$  can be made from the reported height of dunes or sand waves on the flume bed. Taking the average depth of sand in motion as about half the dune height and the weight volume ratio as about  $100 \text{ lb/ft}^3$ ,  $W$  in pounds per square foot numerically equals 50 times the dune height in feet. Equation (80) shows that transport would be zero when  $W = 0$  and when  $W = W_0 = 50$ . Transport for a given depth and size of sediment varies with the water velocity and with areal concentration. The mean particle speed  $T/W$  decreases linearly with  $W$  to a value of zero when  $W = W_0$ . It should be noted that in a sand-channel flume, water velocities are different at troughs and crests of dunes. This means that different flux-concentration curves apply at such points. In these cases the flow is unsteady and uniform, and the transport equation takes the form

$$\frac{\partial C}{\partial t} + \frac{\partial Q_s}{\partial x} = 0 \quad (81)$$

which is the kinematic wave equation. Thus, the kinematic wave theory, based on laboratory data, provides an adequate qualitative description of the transport processes pertaining to sand in flumes. The transport of sand in flumes is approximately kinematic, for the flow in flumes is kinematic.

*Vertical mixing of coarse particles in gravel bed rivers*

Vertical mixing of particles leads to fluvial dispersion of sediment and is part of the evolution of the streambed. A proportion of active particles deposited on the bed surface gets buried by the sediment deposited later. The vertical location of particles in the streambed is controlled by a layer, called the active layer. This is the layer of episodically mobilized material of the streambed. The thickness of this layer is determined by the flow conditions, sediment characteristics, and bed and channel morphology, as well as interactions among these factors (Hassan and Church, 1994). Buried particles begin moving again when all particles covering them have been removed. The movement of particles depends on their relative location within the scouring layer. The particles buried more deeply are less mobile than those on the surface. The particle exchange is concentrated primarily in top layers. The movement of particles on river beds was studied by Hassan and Church (1994). In sand-bed rivers vertical mixing is dominated by wavelike bedforms, such as dunes and ripples, whereas in gravel-bed rivers the movement is sporadic, and the mixing results primarily from local scour and fill.

Hassan and Church (1994) modelled vertical mixing of coarse particles in gravel-bed rivers using a kinematic model. They examined distributions of burial depth of particles by considering observed burial depth of individual size groups of all mobile particles and of all tagged particles (including static ones). When a group of tracer particles was released simultaneously on the bed surface of the stream channel, the tracer material moved and dispersed downstream from the source area and vertically from the bed surface to the lower limit of the mobile layer. Initially the vertical distribution of the tracers in the active layer was not uniform because of the progressively limited exposure of the bed at increasing depth. After the initial dispersion, further movement of a particle depended on, among other factors, its vertical location in the bed. To account for the variations of frequency of burial depth of particles, the model consisted of  $m$  layers of constant thickness equal to the median size of the bed material. The vertical distribution of the particles after the first event was taken as

$$f(y) = k \exp(-ky) \quad (82)$$

where  $f(y)$  is the frequency of the proportion of particles in a given layer,  $y$  is the layer number and  $k$  is the mixing parameter equal to the reciprocal of the mean scaled burial depth. It should be noted that Equation (82) is essentially a relation between concentration and position. The concentration is expressed in terms of the proportion of particles. The proportion of particles which changed vertical position from a given layer was assumed constant from event to event for a series of discrete flow events of equal magnitudes. This is a key assumption, meaning a constant flux of particles in the vertical plane. Although crude, this assumption appears to be supported by data.

Using the same distribution for both the movement and the vertical mixing, Hassan and Church (1994) derived the proportion of moved particles in layer  $y_m$ ,  $f(y_m)$ , in the  $n$ th step as

$$f_n(y_m) = \sum_{i=0}^m f_{n-1} * f_1(y) \quad (83)$$

where the operator asterisk denotes convolution. The stationary particles,  $f(y_s)$ , are considered to remain in the same layer and their proportion may be expressed as

$$f(y_s) = f_{n-1}(y) * [1 - f_1(y)] \quad (84)$$

This calculation is based on the assumption that there is no net change in the elevation of the bed surface over the entire area and hence in layer assignment for the stationary particles. The kinematic model was applied to the Carnation Creek in British Columbia, Canada and the model results were consistent with the overall distributions obtained from several events.

Thus, it can be concluded that vertical mixing of coarse particles in gravel-bed rivers approaches the kinematic wave behaviour if there is no change in bed elevation over the entire area and the proportion of

particles changing vertical position from a given layer is constant. However, vertical mixing is complicated by bed features and other geometric complexities resulting in nonkinematic behaviour of vertical mixing of sediment in general.

### *Debris and mud flow*

Rainfall or snowfall may induce debris flow. Indeed every canyon mouth in the Wasatch Front range in north-central Utah has an alluvial fan built by debris flows (Anderson *et al.*, 1984). A prolonged late snowmelt can cause slope failure. The failure can mobilize into a debris flow which can accumulate and deposit debris and bury houses. The slope failure in a canyon may be caused by saturated conditions resulting from heavy rains and heavy snowfall followed by heavy snowmelt. The mass of material moves rapidly downslope, mobilizing great amounts of material such that debris flow can increase in volume 50-fold by the time it reaches the canyon mouth.

Santi (1989) summarized mobilization (erosion), transport and depositional characteristics of debris flow. By plotting the total volume of debris as a function of distance from the failure scar in the Lightning Canyon in north-central Utah, he delineated four geomorphic regions: scar, upper, middle and lower canyon areas. The scar area (average slope 23°) is transport dominated; the upper canyon area (average slope 20°) is dominated by mobilization of colluvium; the middle portion (average slope 16°) is a transition between a deep, debris-confining canyon above and a wider, more gently sloping canyon below and is mobilization dominant but to a less pronounced degree; and the lower canyon area (average slope 12°) is transport-dominant and exhibits the first occurrence of deposition in the form of levees.

Santi's data showed two thresholds which may be used to describe debris flow behaviour. The first threshold divided the scar area and the upper canyon area and corresponded to a rapid increase in the rate of scour of colluvium from the channel. The second threshold divided the middle and lower canyon areas and corresponded to the first instance of debris deposition. Santi (1989) identified thresholds between the mobilization (erosion), transport and deposition characteristics of debris flows.

In the scar area, debris is not sufficiently thick to cause significant colluvium mobilization because the flow is not well channelized and the total amount of debris is small. This area can be conjectured to be kinematic to a large degree. In the upper canyon area, the amount of debris increases significantly and is sufficiently concentrated by a deepening channel such that a large amount of debris is possible. This area is kinematic to a significant degree. In the middle canyon area the mobilization is still dominant but is less pronounced. In this area, other waves may also be significant. The lower canyon area shows the first occurrence of deposition in the form of levees. The debris is quite thick (3 m approximately) in the centre of the channel. Nevertheless the wide, flat morphology of the channel permits debris to further thin out in the banks, causing deposition. A few metres further down, transport is entirely dominant and a few metres further down still a significant amount of mobilization takes place. In this area, the kinematic wave is still significant but may not be dominant.

In summarizing the kinematics of debris flow down a canyon, Santi (1989) identified thresholds between the mobilization (erosion), transport and deposition characteristics of debris flows. He derived empirically an expression for the critical minimum debris thickness,  $d_m$ , necessary to cause mobilization of the underlying colluvium:

$$d_m = \frac{h\gamma \cos S_0 \tan S_f + A}{\gamma \sin S_0} - h \quad (85)$$

where  $h$  is the thickness of the colluvium,  $S_0$  is the slope of the base,  $S_f$  is the internal friction slope,  $\gamma$  is the unit weight of the colluvium and  $C$  is a parameter. The values of  $A$  and  $S_f$  are chosen for drained and undrained conditions. Santi (1989) also derived, using the rigid plug model of Johnson (1970), a factor of safety,  $FS$ :

$$FS = \frac{A}{\gamma d_d \sin S_0} \quad (86)$$

where  $S_0$  is channel slope and  $d_d$  is the thickness of the plug. Deposition will occur when the factor of safety is greater than one, made possible by thin debris (small  $d_d$ ) or low channel slope (small  $S_0$ ). Deposition usually occurs in the form of mounded levees high up on the gently sloping channel banks. Lang and Dent (1987) summarized kinematic properties of mudflows on Mt. St. Helens which erupted on May 18, 1980. The properties were derived from application of a computer model, SMAC, which is an incompressible, linear viscous model. By calibrating the viscosity term, the model was verified by simulating mudflow processes. In a discussion of SMAC, Bradley (1981) emphasized that mudflows are non-Newtonian in nature and must be modelled accordingly.

In debris and mud flow, the kinematic wave behaviour is weak and dynamic waves are more dominant. Nevertheless, at certain scales the debris flow is kinematic. Indeed, the debris behaviour in mobilization, transport and deposition can be modelled kinematically to a first degree reasonably accurately. However, because of the non-Newtonian nature of mud flows a more complete dynamic description is needed.

#### *Wave phenomena near coastal structures*

Physical processes near coastal structures are influenced by the hydrodynamics of flow due to a combination of incident and partially reflected wave trains. Wave kinematics are developed as a summation of many irregular incident and reflected wave velocity components. Observations show that extreme single waves—referred to as freak waves—have become increasingly substantial during the last decade. These waves occur in both deep water as well as in shallow water. The waves are too high, too asymptotic and too steep. An analysis by Klitting and Sand (1987) of some of the freak waves from a 13-h storm in September 1983 at the Gorm field in the Danish sector of the North Sea concluded that asymmetry, steepness, crest height and directional composition were unique.

Hughes and Fowler (1995) estimated wave-induced kinematics, including root mean squared (RMS) velocity parameters associated with irregular waves that have been reflected by coastal structures. Wave kinematics were developed as a summation of many regular incident and reflected wave velocity components, and the resulting RMS horizontal and vertical velocity expressions were functions of the incident wave spectrum, water depth, location in the water column relative to the structure toe and reflection coefficient and reflection phase angle associated with each wave component. For most tested conditions, good correspondence was found between velocity measurements and estimates obtained using measured wave spectra as input.

To conclude, hydrodynamics of wave phenomena is complicated and is not kinematic but certain wave features are when incident and reflected wave velocity components are summed. There is a high degree of scale dependence. At certain frequencies, the kinematic wave approximation is reasonably accurate but at other frequencies a more dynamic description is needed.

## APPLICATION OF KINEMATIC WAVE THEORY TO SNOW AND ICE HYDROLOGY

### *Movement of meltwater in deep snowpacks*

A snow cover is deposited by a sequence of discrete snowstorms and is usually a layered medium. After each snowfall event, the snow surface may be rearranged by snow drifting which tends to break down the snow grains and repack them into a higher density wind crust. Snowpacks in temperate regions generally experience freeze–thaw cycles, which lead to formation of stratigraphic features, known as ice layers. These layers form preferentially at windcrust and have more ice content and are much less permeable than the surrounding snow.

Before any meltwater can leave a snowpack, the snow temperature must be at the melting point, and the snow in a vertical drain must be saturated to at least the liquid water content held by the snowpack after free drainage. For a homogeneous snowpack the whole snowpack must be saturated to this irreducible liquid content, before any meltwater can reach the base of the snowpack.

Gerdel (1965) discussed vertical flow in channels in the snowcover. These channels appear to form from an accelerated melt metamorphosis in the presence of liquid. The channels consist of coarse grains and behave as drains, conducting away a disproportionate share of water.

To consider percolation of water in a snowpack, one can first assume the snowpack to be homogeneous. Then the effects of ice layers and time variations can be superimposed. Wet snow in many ways is analogous to water bearing soils, sands or glass beads. In case of snow, small ice grains are quickly eliminated by metamorphosis and rapid growth continues until grains of diameter 1 to 2 mm are achieved (Wakahama, 1968). In mature snow there is a narrow distribution of grain sizes and grains are large enough to provide high permeability and reduced capillary potential. Thus, water moves rapidly through a mature snowcover with gravity forces dominating flow. This suggests applicability of the kinematic wave theory.

When water flows into a snow cover for the first time, small grains increase capillary attraction of the snow matrix. Once the small grains disappear, the liquid water held immobile by capillarity is released. This storage mechanism is essentially responsible for delay of water runoff during early stages of snow melt. Fundamental to describing the movement of snowmelt water in a snowpack is the law relating the water flux to concentration. Colbeck and Davidson (1973) formulated this flux law as

$$q = \alpha k S^{*n} \quad (87)$$

where  $\alpha$  for water is  $5.47 \times 10^6/\text{m-s}$ ,  $k$  is permeability,  $S^*$  is effective saturation  $= (S - S_i)/(1 - S_i)$ ,  $S$  is saturation,  $S_i$  is irreducible saturation,  $n$  is an exponent and  $q$  is volume flux. The value of  $n$  depends on the type of snow with different stages of metamorphosis (Denoth *et al.*, 1978): type A, surface snow after snowfall, aged for two days of fair weather, single crystals of about 0.5 mm in diameter; type B, snow several weeks old from a depth of about 1 m, after a long period of fair weather, coarse grained, 1 to 2 mm in diameter. For type A,  $n = 1.5$  and for type B,  $n = 3.7$ . For 17 experiments, depending upon the porosity and grain size at the beginning of the experiment, and  $k$  and grain size, the calculated value of  $n$  was found to vary from 1.4 to 4.6.

The one-dimensional continuity equation for meltwater flow can be written as

$$\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z} = 0 \quad \theta = \eta S \quad (88)$$

Combining Equations (87) and (88), one gets

$$\eta(1 - S_i)(\alpha k)^{-1/n} \frac{\partial}{\partial t} q^{1/n} + \frac{\partial q}{\partial z} = 0 \quad (89)$$

which is the kinematic wave equation. Here  $\eta$  is porosity assumed constant.

The theory of water percolation in snow based on Equations (87) and (88) or Equation (89) was developed by Colbeck and is reported in a series of papers (Colbeck, 1972, 1974, 1977, 1978; Colbeck and Davidson, 1973). Colbeck (1972) has shown that under normal conditions of gravity drainage in snow, the flow occurs under the influence of gravity with little capillary influence. Jordan (1983a,b) applied the theory to a watershed in British Columbia, Canada. Marsh and Woo (1984a,b, 1985) investigated the movement of meltwater in snowpacks. Using field observations they showed that in a naturally stratified snow cover the movement of meltwater was complicated by the interaction of the wetting front with stratigraphic horizons. Wankiewicz (1978a-c) considered Equation (88) to incorporate the effect of pressure gradient in snow. Singh *et al.* (1997a) provided a comprehensive mathematical treatment of vertical movement of snowmelt water in snow.

The question we ask is: is flow of meltwater in snowpacks kinematic? The answer to this question is straightforward. In homogeneous snowpacks the flow is close to being kinematic. However, in stratified snowpacks the flow is complicated by the presence of stratigraphic horizons and is not kinematic. Nevertheless, even under these conditions, the kinematic wave theory provides a good first-order approximation and this is borne by field data.

### Runoff from snowmelt

A snowpack can be idealized to be comprised of two layers over an impermeable boundary. The vertical flow occurs through the unsaturated layer and flow along the boundary occurs in the boundary layer. The vertical flow is described in the foregoing section. In the basal layer, the intrinsic permeability is much greater than in the unsaturated upper layers. Colbeck (1978) has shown that flow in the saturated layer (horizontal) is directly amenable to description by kinematic wave theory. Colbeck (1974) has shown that the flux is

$$q = \alpha\Theta \quad (90a)$$

where  $\Theta$  is the moisture content. Coupling Equation (90a) with the continuity equation

$$\varphi \frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = I(x, t) \quad (90b)$$

results in the kinematic wave equation. Here  $\varphi$  is the effective porosity of snow,  $I(x, t)$  is the net input, i.e. the influx at the top minus the seepage into the underlying ground,  $h$  is the thickness of the saturated zone,  $u$  is the volume flux in the saturated layer (per unit cross-sectional area) and  $x$  is the coordinate along the base of the snowpack. The experiments conducted by Colbeck (1978) confirm the validity of the theory for modelling saturated basal flow in a snowpack. Singh *et al.* (1997b) extended the Colbeck theory by including infiltration at the snowpack base.

The question arises: is runoff from a snowpack kinematic? If the meltwater is moving horizontally over an impervious barrier, the flow is definitely kinematic and this is borne out by field data.

### Glacial motion

Glaciers are extremely sensitive to climatic variations. Even a slight climatic variation is sufficient to cause a significant advance or retreat of ice. This suggests that glacial advances and retreats form a record of climatic changes of the past. This record can be interpreted by knowing how a glacier responds to a change in climate. To that end, a glacier can be treated as an essentially one-dimensional flow system which continuously throughout its length either receives new material by snowfall or loses material by melting or evaporation. When there is a sudden change in the rate of accumulation (snowfall), all parts of the glacier thicken, but the lower parts thicken unstably until the wave of ice arrives to restore stability. The thickening of lower parts and the advance of glaciers can be very large even for only a small change in accumulation.

Glacial motion is of three types: surface waves, seasonal waves and surges. Surface waves are undulations of the glacial surface profile which travel down glaciers at speeds (typically) of three to four times the surface speed, which itself is of the order of 100 m/year. These are analogous to ordinary flood waves and are dominated by the kinematic wave. Fundamental to describing the glacier motion is the relation between glacial flow  $q$  and glacial thickness  $h$ . The glacial flow is the volume of ice per unit width passing a given point in unit time. The glacial thickness varies in time and space and in a manner such that  $(\partial h/\partial x)_t \ll 1$ . Thus the surface slope  $s(x, t)$ , downhill in the  $x$ -direction, is given by

$$s = b - \partial h/\partial x \quad (91)$$

where  $b$  is the slope of inclination upon which the glacier rests and flows down in the  $x$ -direction.

A one-dimensional continuity equation per unit width for glacial motion is

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a(x, t) \quad (92)$$

where  $a(x, t)$  is the rate of accumulation at the surface to the upper surface by snowfall and avalanching measured as thickness of ice per unit time. A negative value of  $a$  implies melting or evaporation of ice (ablation) from the upper (or lower) surface of the glacier.

If there is a unique relation between  $q$  and  $h$  at a fixed  $x$ , then

$$c = (\partial q / \partial h)_x \quad (93)$$

where  $c$  is the celerity of the glacier wave. If this relation is unique then coupling it with the continuity equation gives the kinematic wave equation. This approach was pioneered by Nye (1960, 1963a,b) and Palmer (1972), amongst others.

Many relations have been proposed to relate glacial thickness to glacial flow. For small perturbations, Nye (1960) took

$$q_1 = C_0 h_1 + D_0 s \quad (94)$$

$$C_0 = (\partial q / \partial h)_0 \quad D_0 = (\partial q / \partial s) \quad (95)$$

where

$$q = q_0 + q_1, \quad h = h_0 + h_1, \quad s = s_0 + s_1 \quad (96)$$

Then Equation (92) becomes

$$\frac{\partial h_1}{\partial t} + \frac{\partial q_1}{\partial x} = a_1 \quad (97)$$

The derivatives are evaluated at the steady values  $h_0$ ,  $a_0$ , and  $s_0$ . The function  $q(x, h, s)$  was considered by Weertman (1958). Accordingly

$$q = \left(\frac{\rho g}{A}\right)^m h^{m+1} \sin^{-m} s \quad (98)$$

$$c_0 = \left(\frac{\partial q}{\partial h}\right)_0 = (m+1) \frac{q_0}{h_0} = (m+1)u_0 \simeq 3u_0 \quad (99)$$

Thus

$$c_0 = \left(\frac{\partial q}{\partial h}\right) = \varepsilon x \quad (0 \leq x \leq 1/2); \quad C_0 = \varepsilon(1-x), \quad (1/2 \leq x \leq 1) \quad (100)$$

where  $A$  is some constant. Assuming that  $a$  is such that  $q$  is small, the kinematic wave velocity is fixed at each point by the steady state configuration.

Another relation is

$$q = Fh^{n+1} \sin^{-n} s + Gh^{m+1} \sin^m s \quad (101)$$

where  $F$  and  $G$  are constant. The first term arises from differential motion and the second term arises from sliding over the bed;  $n$  is about 3 or 4 and  $m$  is about 2. Thus

$$c = (n+2)u_d + (m+1)u_b \quad (102)$$

where  $u_d$  = mean velocity due to shear motion and  $u_b$  is mean sliding velocity. The mean forward velocity of the ice  $u = u_d + u_b$ . Thus  $c$  always lies between  $(m+1)u$  and  $(n+2)u$  or between  $3u$  and  $6u$ , depending on the two forms of motion.

Seasonal waves manifest themselves as fluctuations in the surface velocity which propagate down-glacier at speeds in the range 20 to 150 times the surface speed. They can then travel at speeds of the order of magnitude of 20 km/year. The speed can fluctuate greatly but there is virtually no depth perturbation. For surface waves, the flux-concentration relation (Fowler, 1982) can be approximated as

$$Q = \frac{H^{n+2}}{n+2} \quad (103)$$

where  $H$  is the glacial thickness.

Glacier surging is a manifestation of instability and is characterized by large-scale relaxation oscillations of a large portion of glacier ice mass. Surging is a self-regulating mechanism due to internal glacier flow dynamics. Meir and Post (1969) described a surge on the Tikke Glacier in the Alsek range of British Columbia, and summarized the main characteristics of this type of motion. A surge-type glacier exhibits a quiescent phase lasting 10 to 100 years (typically 10 to 20 years) during which the ice mass increases owing to accumulation of ice on its upper portions (the accumulation area). The thickness of the ice increases in the reservoir area and when it reaches a critical depth, a surge is initiated. In surging mode, the ice in the reservoir area moves very quickly and there is substantial displacement of ice into the receiving area. This can lead to rapid advances of the glacier and velocities up to 10 km/year have been recorded. Field data show that the rapid flow zone is preceded by an acceleration zone, so that as a wave passes a point on the ice the velocity of the point increases by a factor of four. The zone of rapid flow is followed by a deceleration wave moving at about the same speed as the accelerating wave and as it passes the ice returns to its original velocity. The velocity of acceleration and deceleration waves, called surge velocity, is about five times the velocity behind the acceleration wave.

Since glacial surge is a self-induced instability, it can be explained by the kinematic wave theory. Indeed this can be explained, as discussed by Palmer (1972), by considering a relation between the flow of ice and the ice depth, which is different when ice is accelerating from when ice is decelerating. This then takes care of the apparent hysteresis in the flow–concentration relation. Undeniably there is diffusion, but its effect is qualitatively similar to that of the two-way flux concentration relation. Diffusion can be expected to cause sharp shock fronts to decay rapidly, and exerts a generally stabilizing influence. As reasoned by Palmer (1972), surges exhibit some of the features of kinematic waves.

The above flux relations point to the use of kinematic wave theory for modelling glacial flow. Nye (1960, 1963a,b) pioneered kinematic wave analysis to model the response of glaciers and ice-sheets to seasonal and climatic changes as well as to changes in the rate of nourishment and wastage. Meier and Johnson (1962) described the kinematic wave on Nisqually Glacier in Washington. Palmer (1972) described a kinematic wave model of glacier surges. Lliboutry (1971) presented a comprehensive treatment of the glacier theory and discussed the various aspects under which the kinematic wave theory would be applicable. Fowler and coworkers (Fowler and Larson, 1978, 1980; Fowler, 1982; Fowler and Walder, 1993) presented comprehensive mathematical treatments of glacial motion including the use of the kinematic wave theory. Hutter (1983) presented a comprehensive discussion of theoretical glaciology including kinematic wave modelling.

The question we ask is: is glacial motion kinematic? Based on field evidence, it appears that surface waves and seasonal waves are not exactly kinematic but are so to a high degree. Although glacial surging is not kinematic, the kinematic wave theory does provide a good first-order approximation to the otherwise complicated phenomenon of glacial surging.

#### *River ice motion*

When there occurs an ice break upon a river, complex interactions between the flow of water and ice motion take place in both the movement of water and ice. The interactions cause rapid changes. Ferrick *et al.* (1993) recognized an analogy between ice motion during dynamic breakup and the flow of traffic on crowded roads and proposed to describe the river ice and flow processes using the kinematic wave theory. Treating the dynamics of ice motion as kinematic, they identified four fronts—breaking front, convergence front, stoppage front, and release front—and related their speeds with ice discharge and volume per unit surface area on either side of each front.

Different fronts move through the ice field in response to changing conditions. The breaking front travels downstream and separates the stationary ice ahead and the moving ice behind. The convergence front moves with the upstream limit of an ice accumulation, demarcating it from thinner ice upstream. The stoppage front moves upstream following the cessation of ice motion and separates the moving ice upstream from ice at rest downstream. The release front moves upstream through the ice field, triggering motion in response to an ice release downstream.

The conservation of mass can be expressed for ice near each front that connects the speed of the front to the change in ice thickness and accumulation porosity across the front. The ice discharge on either side of the front can also be obtained. Since ice does not accumulate in any arbitrary moving control volume, there is no time–rate-of-change term to consider. By balancing the fluxes, the ice continuity equation is

$$(c - u_{i1}) = R(c - u_{i2}) \quad (104)$$

where  $c$  is the front speed in the downstream direction,  $u_{i1}$  is the velocity of ice moving in,  $u_{i2}$  is the velocity of ice moving out and  $R$  is the dimensionless ratio of effective control volume surface areas defined as

$$R = \frac{B_2 t_{i2} (1 - E_{c2})}{B_1 t_{i1} (1 - E_{c1})} \quad (105)$$

where subscripts 1 and 2 correspond to entry downstream surface and exit upstream control surface, respectively,  $E_c$  is the porosity of ice accumulation,  $t_i$  is the thickness of the ice sheet or accumulation,  $B$  is the river width and  $Bt_i$  is the area of the control surface. The sign convention yields negative speed,  $-c$ , for fronts moving upstream. The ice discharge at a point  $p$ ,  $Q_p$ , can be expressed as

$$Q_p = B p u_{ip} t_{ip} (1 - E_{cp}) \quad (106)$$

Each of the fronts is now considered.

*Breaking front.* For the breaking front, we consider a reach of broken ice jammed upstream of intact ice cover. A flow surge moving downstream arrives at the accumulation and ice motion is triggered. The ice movement downstream keeps adding ice to the front of the ice pack, and the speed of the breaking front  $c_b$  is always greater than the local ice velocity  $u_{i2}$ . With appropriate substitutions the dimensionless speed of the breaking front is obtained from Equation (104) as

$$\frac{c_b}{u_{i2}} = \frac{R_b}{R_b - 1} \quad (107)$$

where  $R_b$  is  $R$  at the breaking front,  $R_b \geq 1$ . If the breaking front comes across an unbroken ice sheet of thickness  $t_{is}$  in place of ice rubble,  $t_{i1} = t_{i2}$ . If the sheet has negligible open water area, the incoming sheet porosity  $E_{c1} = 0$ .

The ice discharge upstream of the breaking front,  $Q_b$ , is obtained from Equations (105) and (107) as

$$Q_b = c_b D_b \quad (108)$$

where  $D_b$  is the value of  $D$  associated with the breaking front:

$$D = B_2 t_{i2} (1 - E_{c2}) - B_1 t_{i1} (1 - E_{c1}) \quad (109)$$

Ferrick *et al.* (1992) analysed river ice motion near a breaking point in detail.

*Convergence front.* The movement of a breaking front produces ice convergence which decreases the surface area occupied by ice. In the ice accumulation immediately upstream of the breaking front ice velocities are lower due to high resistance. Farther upstream, the ice velocities are typically higher due to lower resistance. This difference in velocities causes ice convergence at the upstream limit of a moving accumulation. All ice converges to the convergence front, located at the upstream limit of the accumulation. The dimensionless speed of the ice convergence front is obtained from Equation (104) as

$$\frac{c_c}{u_{i2}} = \frac{(u_{i1}/u_{i2}) - R_c}{1 - R_c} \quad (110)$$

where  $c_c$  is the speed of the convergence front,  $R_c$  is  $R$  at the convergence front,  $u_{i1}$  is the accumulation velocity and  $u_{i2}$  is the velocity of the ice immediately upstream.  $R_c < 1$  indicates convergence of thinner or more porous ice from upstream of the accumulation and  $c_c$  is positive for front moving downstream and negative for front moving upstream.

The ice discharge upstream of the convergence front,  $Q_c$ , is obtained from Equations (105) and (110) as

$$Q_c = Q_b = c_c D_c \quad (111)$$

where  $D_c$  is the value of  $D$  as in Equation (111).

*Stoppage front.* The initial stoppage occurs upstream of the breaking front. Stoppage of ice motion progresses upstream following the arrest of a breaking front or of the moving ice somewhere behind this front. The dimensionless speed of the stoppage front is obtained from Equation (104) as

$$\frac{c_s}{u_{i2}} = \frac{-R_s}{1 - R_s} \quad (112)$$

where  $R_s$  is  $R$  at the stoppage front and  $R_s \leq 1$ , indicating that the unit ice volume downstream of the front is greater than that upstream. As  $R_s$  approaches 1, the front progresses upstream very rapidly. The breaking and stoppage fronts are opposites. A stoppage front is the limiting case of a convergence front with a velocity ratio of zero when the speed of ice accumulation approaches zero,  $c_c$  becomes negative and the convergence front becomes a stoppage front. The stoppage front travels upstream through the rigid body accumulation.

The ice discharge upstream of the stoppage front,  $Q_s$ , is obtained from Equations (105) and (112):

$$Q_s = c_s D_s \quad (113)$$

where  $D_s$  is the value of  $D$  associated with the stoppage front.

*Release front.* When there occurs a failure of the resistance at a point, the ice motion is initiated. With the release of ice and initial motion, the release front moves upstream. The dimensionless speed of the ice release front is obtained from Equation (104) as

$$\frac{c_i}{u_{i1}} = \frac{-1}{R_i - 1} \quad (114)$$

where  $R_i$  is  $R$  at the release front, with  $R_i \leq 1$ . The ice release causes divergence because the water surface area available to an ice accumulation is increased. From the point of release a breaking front travels downstream and a release front travels upstream. Both fronts initiate ice motion, with convergence occurring upstream of the breaking front and divergence occurring downstream of the release front.

The ice discharge downstream of a release front,  $Q_i$ , is obtained from Equations (106) and (113):

$$Q_i = C_i D_i \quad (115)$$

where  $D_i$  is the value of  $D$  associated with the ice release front.

Ferrick *et al.* (1993) simulated the ice and front motion through time for a reach of the Connecticut River, with the assumption that accumulation thickness and porosity are uniform and changes in ice conditions and motion occur only at a front. The kinematic wave model simulated results consistent with observations.

The question we ask is: is river ice motion kinematic? Based on the work pioneered by Ferrick *et al.* (1993), the kinematic wave theory provides a good approximation to ice motion, provided the following assumptions are valid: the changes in ice motion and conditions occur only at the front and these changes are small. The accumulating thickness and porosity are uniform. Otherwise, the full dynamic wave theory would be the logical theory for describing the river ice motion.

## APPLICATION OF KINEMATIC WAVE THEORY TO WATER QUALITY HYDROLOGY

*Solute transport in surface runoff*

Non-point source pollution often has low strength but high volume and causes significant environmental degradation in both urban and rural watersheds. During certain storm events runoff from street surfaces contributes significantly more biochemical oxygen demand to the area's waters than the effluent from sewage treatment plants. While point source pollutants originate from waste products, many non-point source pollutants (nutrients, pesticides, de-icing salts, etc.) result from chemicals deliberately applied for beneficial use, as for example from debris and contaminants from streets, contaminants from open land areas, publicly used chemicals, and dirt and contaminants washed from vehicles. In most cases, overland flow, rather than surface flow is the primary transport of non-point source pollutants. Thus, the rate and amount of pollution abatement are closely tied to the rate and volume of surface runoff.

Postulating the analogy between the bulk transport of conservative soluble pollutants and the convective movement of water particles in overland flow, the pollutant transport in surface water becomes amenable to the use of the kinematic wave theory (Brazil *et al.*, 1979; Buchberger and Sanders, 1982; Akan, 1987). The implication of this analogy is that the fundamental principles of overland flow may be employed to estimate the discharge of soluble non-point source pollutants.

Pollutant transport involves four mechanisms: (1) dilution, (2) convection, (3) diffusion and dispersion, and (4) chemical reactions. During a rainfall event the depth of flow increases in the direction of flow. Dilution must occur as the conservative soluble pollutant mass is transported downstream into progressively deeper flow. The effect of convection can be visualized by considering the movement of two pollutant particles at different distances from the watershed outlet. The non-uniform convection spreads the pollutant mass as it moves down the watershed, because the downstream particle moves faster than the upstream particle. Diffusion occurs because the solute particles moving from the upstream boundary of the watershed are followed by unpolluted runoff. This causes a concentration gradient which then causes a diffusive mass flux in the upstream direction.

When pollutants are transported in shallow overland flow over a short period, the effects of dispersion and biochemical reactions can be neglected (Nakamura, 1984). Thus the transport becomes convective and the kinematic wave theory applies. The fundamental premise is that the velocity of a soluble particle equals the velocity of a water particle at the same location. Thus, everywhere the pollutant particles are transported at the local convective velocity. Hence, the pollutant mass discharge per unit width is equal to the product of the convective particle velocity and pollutant mass per unit area. The rate is given by the pollutant concentration and runoff per unit width.

Defining concentration  $C$  as the mass of solute  $M$  per unit volume of solution  $V$ ,  $C = M/V$ , the mean flow velocity  $u = q/h$ ,  $q$  = discharge per unit width,  $h$  = depth of flow, mass per unit area  $w = M/A$ ,  $A$  = watershed area and  $M$  = total initial pollutant mass, the pollutant discharge flux is

$$uw = Cq \quad (116)$$

where  $q$  is obtained using the kinematic wave theory.

The one-dimensional continuity equation for solute transport can be written as

$$\frac{\partial(CQ)}{\partial t} + \frac{\partial(CQ)}{\partial x} = G(x, t) \quad (117)$$

where  $G$  is some function depending upon the injection of solute in flow. For example, for instantaneous mixing of soluble pollutant in runoff water

$$G(x, t) = w\delta(t) \quad (118)$$

where  $\delta(t)$  is the instantaneous unit flux of the solute.

If the pollutant mixes in a finite period of time

$$G(x, t) = C_r i - \frac{\partial w}{\partial t} \quad (119)$$

where  $C_r$  is the mass of pollutant per unit volume of rainwater (concentration in rainwater) and  $i$  is rainfall intensity. The quantity  $\partial w/\partial t$  specifies the wash rate and depends on the depth of flow and slope of the plane.

If the plane is infiltrating,  $G(x, t)$  takes the form

$$G(x, t) = Kh - C(i - f) + C_r i - C f \quad (120)$$

where  $f$  is the rate of infiltration and  $K$  is a constant. Buchberger and Sanders (1982), Brazil *et al.* (1979), Ingram and Woolhiser (1980), Akan (1987), Havis *et al.* (1992), among others, have applied the kinematic wave theory to model solute transport by surface runoff and found that the theory-based predictions were in good agreement with laboratory as well as field data.

To conclude, solute transport by surface water is kinematic for the most part. This is because the surface runoff is kinematic.

#### *Solute transport in unsaturated flow*

Solute transport closely parallels soil moisture transport in unsaturated or vadose zone. In case of solute transport a complication arises, however, because many solutes interact strongly with soil. As a result, there occurs a partitioning of the solute between the mobile solution phase and the immobile soil surface phase. To that end, frequently an equilibrium sorption is used which with a general nonlinear sorption isotherm can be expressed as

$$s = F(C) \quad (121)$$

where  $s$  is the sorbed phase concentration in mass sorbed per unit mass of the soil and  $C$  is the solute concentration in mass per unit volume of soil water.

The solute mass flux,  $q_s$ , can be expressed as

$$q_s = C q \quad (122)$$

With  $\rho$  as the bulk density of the soil and the solute mass flux, the solute mass continuity is

$$\frac{\partial(\theta C)}{\partial t} + \frac{\partial(\rho s)}{\partial t} + \frac{\partial q_s}{\partial z} = 0 \quad (123)$$

The mass flux includes both the effects of advection and mixing. Using Equation (122) with the right side set to zero, Equation (123) reduces to

$$R \frac{\partial C}{\partial t} + v \frac{\partial C}{\partial z} = 0 \quad (124)$$

where  $v = q/\theta$  is the seepage velocity,  $R = 1 + \rho F'/\theta$  is the retardation factor and  $F'$  is the slope of the sorption isotherm at concentration  $C$ . Equation (124) is the governing equation for solute transport in saturated flow. With knowledge of  $v$  given by the kinematic wave equation for flow of water this equation can be solved for  $C$ . Charbeneau (1984) was probably the first to have applied the kinematic wave theory to model solute transport in soils. Charbeneau *et al.* (1989) extended the theory to model multiphase solute transport in the vadose zone. They included transient hydrologic phenomena such as evaporation and infiltration and a model for stochastic generation of rainfall. Based on laboratory experimentation they suggested that the kinematic rainfall and oily pollutant transport model would be preferred for most applications. Weaver *et al.* (1994) used Green–Ampt and kinematic wave theory to develop a screening model for non-aqueous phase liquid

transport in the vadose zone. They verified their model using results from a simple column experiment and concluded that the model was capable of capturing the qualitative behaviour of the experimental system.

Solute transport in unsaturated zone is not kinematic under all conditions but is so under a good range of conditions. This is borne out by both field and experimental data. These conditions are related to the conditions under which moisture transport is kinematic.

#### *Solute transport in macropores*

In undisturbed soils with macropores, there is only partial displacement of resident water and solutes by incoming water and solutes. This means that the interaction between incoming water due to rain or irrigation and the relatively immobile soil water is limited. This behaviour has important bearing on nitrate leaching, desalinization and contamination of groundwater and streams. Because the effect of soil macropores is to increase or even control the maximum rate at which water infiltrates, contaminants and fertilizers move more deeply into the soil profile and at an accelerated rate (German, 1988; Levy and Germann, 1988). Solute loss from the macropore system into various solute storages due to mixing can be considered in a manner similar to water sorbance. Germann (1990a,b) considered solute transport in macropores surrounded by (1) water-saturated matrix and (2) water-sorbing matrix. In the first case, the solute transport equation is

$$\frac{\partial(Cq)}{\partial t} + c \frac{\partial(Cq)}{\partial z} + crwC + csw(C - C_s) = 0 \quad (125)$$

where  $C$  and  $C_s$  ( $\text{kg/m}^3$ ) are the solute concentrations in the water moving as kinematic wave and in the storage zone prior to the passage of the wave, respectively,  $s$  (1/s) is the exchange coefficient of the solute,  $\gamma$  (1/s) is sorbance = loss of water per unit time and per unit  $w$  (macropore water content). With  $q$  given by the kinematic wave equation for soil water flow, Equation (125) is the governing equation for solute transport in macropores and is the kinematic wave equation.

Germann (1987) modelled the transport of *Escherichia coli* in the vadose zone taking into account the macropore flow. Germann (1988) discussed restrictions imposed by convective–dispersive approaches to water flow and solute transport in porous media. Levy and Germann (1988) applied the theory to model solute transport in soils, accounting for the loss of a conservative solute tracer from preferred paths during macropore flow. Hornberger *et al.* (1990) conducted experiments under steady conditions on flow and solute transport in macroporous forest soils. Hornberger *et al.* (1991) investigated throughflow and solute transport in an isolated sloping soil block in a forested catchment. There is general consensus that under certain limitations the theory is an adequate approximation to simulate solute transport in macropore flow. These limitations fortunately are not severe and solute transport in macropores can be concluded to be kinematic.

#### *Solute transport in snowpacks*

Isotopes, salts and dyes are used extensively for identifying particle flow paths in wet snow, determining atmospheric conditions at the time of deposition of dry snow. Snowmelt water contains some or all of the constituents normally found in surface water and ground water. Because snow cover is influenced by ground level air masses, it is regarded as a temporary depot of air pollutant. Furthermore, there are interactions between air pollution and the chemical composition of a snow cover. Snowpacks receive pollutants from natural sources, atmospheric as well as man-made sources. As a result, snowpacks contain a range of impurities. The pollutants whether coming from precipitation or deposited later into the snow surface are accumulated over a period of time during winter and transmitted to water channels in the spring when snowpack melts. Colbeck (1977) noted that early in the melt season, melt–freeze cycles concentrated soluble pollutants in the lower portion of the snowpack, thus preparing the contaminants for rapid removal. These pollutants are quickly removed at high concentrations within the first fraction of the snowmelt runoff. Colbeck (1977) was probably the first to have applied the kinematic wave theory to model tracer movement. The percolation of melt water is not a homogeneous process; meltwater often flows in preferred channels through the snowpack rather than

percolating as a uniform front of water. The effect of such inhomogeneities is to cause pollutant and melt water to penetrate the snowpack more quickly within the preferred channels, resulting in an earlier and less abrupt start to the meltwater runoff. Hibberd (1984) extended the theory for pollutant concentrations during snow melt and found the results of the model to be in agreement with experimental studies.

The pollutant transport through snow involves four components: movement of meltwater through snowpack, (2) movement of pollutant in meltwater through snowpack, (3) movement of meltwater at the base of snowpack and (4) movement of pollutant in snowmelt runoff. The movement of meltwater through snow and at the base of a snowpack was already described in an earlier section. The movement of pollutant can be described using the kinematic wave equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial z} = 0 \quad (126)$$

where  $C$  is concentration of pollutant in snowmelt water,  $u$  is snowmelt water flow velocity and  $z$  is distance measured positive downward from the top of the snow surface. In a similar fashion, the movement of pollutant in melt water runoff can be expressed as

$$\frac{\partial C}{\partial t} + \frac{KS_s}{\varphi_e} \frac{\partial C}{\partial x} = \frac{1}{\varphi_e} [I(C_I - C) - f(C_f - C)] \quad (127)$$

where  $C_I$  is pollutant concentration in the lateral inflow of meltwater ( $I$ ),  $C_f$  is the pollutant concentration in the melt water entering the soil,  $\varphi_e$  is effective porosity of the snowpack,  $f$  is the rate of infiltration,  $S_s$  is the slope angle of the base and  $K$  is the saturated hydraulic conductivity.

Solute transport in snowpacks is kinematic for a good range of conditions, especially early in the melt season. As noted by Colbeck (1977), snowmelt water flows in preferred channels and follows kinematic wave theory reasonably accurately. This means that solute transport in snowpacks can be accurately described by the theory.

#### *Microbial transport*

Germann *et al.* (1987) applied kinematic wave theory to establish threshold values for macropore flow parameters critical to transport of microbial suspensions through macroporous soils. These thresholds were used to assess the maximum depth of microbial translocation under various conditions of input rates and durations. It was found that hardly any microbes were transported deeper than 2–3 m in soils under frequently occurring natural precipitation intensities and durations. However, prolonged or intensified water input to the soil surface was capable of efficiently carrying microbes deeper than 100 m into the vadose zone. The sizes and shapes of micro-organisms in suspension and their affinity to the soil particles need, however, to be included in such an analysis.

Much of the work on microbial transport in soils has been based on advection–dispersion theory and there is little work reported on microbial transport in surface water systems. In vadose zone, it can be concluded that if the flow of water is conducive to description by kinematic wave theory, as for example in macropores and saturated or near-saturated soils, it is reasonable to surmise that the kinematic wave theory would be a good approximation. The theory would not be accurate under all conditions but would be under certain conditions which are quite prevalent.

#### *Sedimentation*

Kynch (1952) developed a theory of sedimentation assuming that the fall of particles in a dispersion is determined by the local particle density only. The theory has implications in design of biological clarifiers, settling basins, naval ships, etc. For settling of a dispersion of similar particles, the velocity of any particle is a function of the local concentration of particles,  $\rho$ , in its immediate vicinity. Concentration is defined as the

number of particles per unit volume of dispersion. For particles of the same shape and size, it is proportional to the volume fraction. The flux–concentration relation is

$$S = \rho v \quad (128)$$

where  $\rho$  is the volume concentration,  $S$  is the number of particles crossing a horizontal section per unit area per unit time or the particle flux and  $v$  is the velocity of the fall. The concentration is assumed to be the same everywhere across any horizontal layer. An example of a flux–concentration relation is

$$S = a\rho^2(\rho_0 - \rho) \quad (129)$$

where  $a$  is constant and  $\rho_0$  is the maximum value of  $\rho$ . Equation (129) states that flux increases with concentration up to a certain maximum value and then decreases with increasing concentration.

Assuming that at any point in a dispersion the velocity of fall of a particle depends only on the local concentration of particles, the settling process can then be determined from the continuity equation alone, without knowing the details of the forces on the particles. Changes in particle density are propagated through a dispersion just as sound is propagated through air. A one-dimensional continuity equation is

$$\frac{\partial \rho}{\partial t} - \frac{\partial S}{\partial x} = 0 \quad (130)$$

Combining Equations (130) and (128), one gets

$$\frac{\partial \rho}{\partial t} + V(\rho) \frac{\partial \rho}{\partial x} = 0 \quad (131)$$

where

$$V(\rho) = -\frac{dS}{d\rho} \quad (132)$$

which is nothing but the kinematic wave equation. Thus this theory of sedimentation is based on the kinematic wave theory.

Schneider (1982) extended the kinematic wave theory to investigate two-phase flow in settling basins with walls that are inclined to the vertical. The theory predicted sedimentation processes with centred waves which emerge from the bottom if the initial concentration was such that a simple concentration jump from the suspension to the sediment was not possible. Although interparticle forces, that are important in the high-concentration region near the bottom, are ignored by the theory, centred waves have been observed in one-dimensional flow in vessels with vertical walls and good agreement has been found between the experimental observations and the theory-predicted results.

Is sedimentation kinematic? It is surprising that Kynch's original formulation more than four decades ago has not been followed up and therefore, experimental evidence backing the kinematic wave theory is lacking and it is difficult to argue about its validity. However, based on the work in chemical and naval engineering, the theory appears to hold great promise. From a conceptual viewpoint, the theory should provide a good first approximation to the process of settlement of particles in dispersion.

### *Chromatography*

When a solution containing a mixture of coloured solutes is allowed to run through a vertical glass tube filled with a suitable powdered adsorbing material, the material adsorbed in the tube appears as a series of coloured bands, exhibiting that a partial separation of components has occurred. This series of coloured bands is referred to as a chromatogram. The separation can be accomplished by a procedure known as development of the chromatogram in which a suitable solvent is poured through the tube, which washes the coloured bands down the tube at different rates. The lowest lying band moves the fastest. The chromatographic methods have been popular methods for separation, purification and identification of small amounts of complex, naturally occurring organic compounds.

Wilson (1940) proposed a theory of chromatography which is based on two assumptions: (1) there exists instantaneous equilibrium between the solution and adsorbant; (2) the effects of diffusion are neglected. DeVault (1943) and Weiss (1943) extended Wilson's theory. The flux law assumed was

$$Q = Mf(C) \quad (133)$$

$$q/m = f(C) \quad (134)$$

where  $q$  is the number of millimoles of solute adsorbed on  $m$  grams of adsorbant in equilibrium with a solute whose concentration is  $C$  moles per litre,  $Q$  is adsorbed millimoles of solute per unit length (cm) of the column and  $M$  is the amount of adsorbant per unit length of the column in grams.  $f(C)$  is the isotherm which represents the adsorption of the given solute on the adsorbant. Thus the flux-concentration relation is through the isotherm.  $Q$  is a function of time;  $C$ , as a function of time and position in the column, is the amount of solute in solution per unit volume of solution.

A one-dimensional mass conservation equation for a single solute is

$$\frac{\partial Q}{\partial V} + \frac{\partial C}{\partial x} = 0 \quad (135)$$

where  $V$  is the volume of solution. Combining Equations (135) and (133), the differential equation for chromatography is obtained:

$$\frac{\partial}{\partial V}(Mf(C)) + \frac{\partial C}{\partial x} = 0 \quad (136)$$

which is essentially the kinematic wave equation.

When the amount of solute in the solution in the column is taken into account the continuity equation becomes

$$\frac{\partial C}{\partial x} + \alpha \frac{\partial C}{\partial V} + \frac{\partial Q}{\partial V} = 0 \quad (137)$$

where  $\alpha$  is the pore volume per unit length of the column. When  $\alpha = 0$ , Equation (137) reduces to Equation (135). Combining equations (137) and (133), the kinematic wave equation becomes

$$\frac{\partial C}{\partial x} + [\alpha + Mf'(C)] \frac{\partial C}{\partial V} = 0 \quad (138)$$

Cassidy and Wood (1941) compared the Wilson theory with data obtained in laboratory and found, on the whole, quite good agreement between experimental results and theory-based predictions. Bolt (1979) summarized the principles of adsorption and exchange chromatography for movement of solutes in soil. He also reported on the applicability of the kinematic wave theory, and concluded that in certain cases unsteady flow of water was likely to exercise a far greater influence on the movement and spreading of a solute front than a diffusion/dispersion term as used in steady state solutions.

The question is: is chromatography kinematic? The answer is partly yes and partly no. As noted by Bolt (1979), under certain conditions unsteady flow (convection of water) plays a far greater role in the movement and spreading of the solute front than do diffusion and dispersion. Under those conditions, the kinematic wave theory holds. This is borne out by experimental data. Under steady conditions, the diffusion and dispersion processes exercise greater influence than they do under unsteady conditions and the theory would not be a good approximation. Fortunately, chromatography is usually unsteady and therefore the theory would be a good first-order approximation.

### *Ion exchange*

When a solid is immersed in a solution, there will be an exchange of ions. The solid substance, carbonaceous, resinous or mineral, capable of exchanging ions with a solution is defined as a zeolite. The performance of

a mass of zeolite can be evaluated if the law governing the exchange is known. Thomas (1944) described heterogeneous ion exchange in a flowing system, assuming that the ion exchange was governed by a law of concentration product type. The process of ion exchange may be applicable to the problem of water softening.

Let  $C(x, t)$  and  $p(x, t)$  be the concentrations of the cation in the solution and in the zeolite, respectively, at time  $t$  after the entrance of the solution into the column and at distance  $x$  from the input end of the column. Let  $C_0$  be the initial exchangeable cation concentration of the solution. The ion exchange can be characterized by a law explicit in concentrations only of ions in solution and of exchangeable atoms or ions in the zeolite. The net rate of exchange of the cation into zeolite is given by

$$\frac{\partial p}{\partial t} = F(C, p) \quad (139)$$

where  $F$  is any function. Various forms of  $F$  may be expressed reflecting various theories of interaction between zeolite and solute. Equation (139) holds if the exchange processes are slow compared to any diffusion processes in the solution or in the zeolite particles. When the zeolite first approaches saturation, this type of law is expected to hold. The form of  $F$  may vary with the interaction between zeolite and solute. For an exchange between univalent ions a simple law is given by

$$\frac{\partial p}{\partial t} = k_1 C(a - p) - k_2 p(C_0 - C) \quad (140)$$

where  $k_1$  and  $k_2$  are velocity constants associated with two opposing second-order reactions presupposed in Equation (140), and  $a$  is the initial exchange capacity of the zeolite. The quantities  $p$  and  $a$  are measured in milliequivalents per gram of zeolite and  $C$  and  $C_0$  are measured in milliequivalents per millilitre of solution. The coefficients  $k_1$  and  $k_2$  are estimated by fitting to breakthrough curve data on a column run under known conditions.

The continuity equation expressing the conservation of exchanging cation can be expressed as

$$R \frac{\partial C}{\partial x} + \frac{\partial C}{\partial t} + m \frac{\partial p}{\partial t} = 0 \quad (141)$$

where  $m = M/v_0$ ,  $M$  is overall density of zeolite as packed in the column,  $R = (\delta V/v_0)\delta x$  = linear rate of flow of the solution,  $v_0$  is fractional free space in the zeolite column,  $x$  is the distance from the input end of the column,  $t$  is the time after entrance of the solution into the column and  $V$  is the volumetric flow of the solution. Equations (141) and (140) constitute the mathematical formulation for heterogeneous ion exchange in a flowing system. Thomas (1944) elaborated on the feasibility of application of the above theory to experimental results. The theory can be used for description of the performance of packed ion exchange columns used for water softening.

The question arises: is heterogeneous ion exchange kinematic? The answer to this question is difficult for the following reason. Since the work of Thomas more than five decades ago, no follow-up work on applying the kinematic wave theory appears to have been reported. Consequently, experimental evidence supporting the applicability of the theory to ion exchange is lacking. However, conceptually it appears that theory would provide a good approximation if the ion exchange process is slow compared to the diffusion process in the solution or zeolite particles. Clearly this is the case when the zeolite approaches saturation. Thus, at least under certain conditions the theory holds.

## SUMMARY AND CONCLUSIONS

The foregoing discussion shows that there is a full spectrum of waves exhibited in nature in general and in hydrology in particular. However, in a wide range of hydrologic processes kinematic waves occupy a large portion of this spectrum. In some extreme cases the spectrum may be approximately enveloped by kinematic

waves and in others kinematic waves may occupy little or no part of the spectrum. These are two extreme limits. In a significant number of cases, however, the spectrum is overwhelmed by kinematic waves. Thus, the answer to the question raised in the title of the paper is very definitely no, but within certain constraints and space–time scales the answer becomes yes. There is another point to be noted about the general physically based descriptions. Our knowledge about the validity of these descriptions and the physical meaning and measurability of the parameters contained in them is woefully inadequate. A close examination of these descriptions suggests that the so-called physical descriptions are not really physical after all, for we cannot determine their parameters beforehand and therefore a lot of fitting is to be undertaken. Thus, the kinematic wave theory is attractive for the reasons of being simple, flexible and conceptually appealing.

The following conclusions are drawn from this study. (1) The kinematic wave theory is a good approximation of the full dynamic wave theory for a variety of flow phenomena. (2) The kinematic wave theory has wide-ranging application in a variety of environmental and water science areas. (3) Under certain conditions, many flow phenomena are indeed kinematic and must so be dealt with. (4) The potential of the kinematic wave theory should be fully utilized in hydrologic investigations.

#### REFERENCES

- Akan AO. 1987. Pollutant washoff by overland flow. *Journal of Environmental Engineering* **113**(4): 811–823.
- Ampere AM. 1834. *Essai sur la philosophie de sciences, or exposition analytique d' une classification naturelle de toutes les connaissances humaines*. Bachelier: Paris.
- Anderson LR, Keaton JR, Saarinen TF, Wells WG. 1984. *The Utah Landslides, Debris Flows, and Floods of May and June 1983*. National Academy Press: Washington, DC.
- Bates BC, Pilgrim DH. 1983. Investigation of storage–discharge relations for river reaches and runoff routing models. *Institution of Engineers, Australia, Civil Engineering Transactions* **CE25**: 153–161.
- Bates BC, Townley LR. 1985. Estimation of parameters and uncertainty in a runoff routing model. National Conference Publication No. 85/2, Institution of Engineers: Australia; 48–52.
- Beven K. 1979. On the generalized kinematic routing method. *Water Resources Research* **15**(5): 1238–1242.
- Beven K. 1981. Kinematic subsurface stormflow. *Water Resources Research* **17**(5): 1419–1424.
- Beven K. 1982a. On subsurface stormflow: an analysis of response times. *Hydrological Sciences Journal* **4**: 505–521.
- Beven K. 1982b. On subsurface stormflow: predictions with simple kinematic theory for saturated and unsaturated flows. *Water Resources Research* **18**(6): 1627–1633.
- Beven K. 1987. Reply. *Water Resources Research* **23**(4): 749.
- Beven K. 1989. Interflow. In *Unsaturated Flow in Hydrologic Modelling: Theory and Practice*, Morel-Seytoux HJ (ed.). Kluwer Academic: Dordrecht.
- Beven K, Germann P. 1981. Water flow in soil macropores: 2. A combined flow model. *Journal of Soil Science* **32**: 15–29.
- Bird RB, Stewart WE, Lightfoot EN. 1960. *Transport Phenomena*. Wiley: New York.
- Blatch NS. 1906. Works for the purification of the water supply of Washington, DC: discussion. *Transactions, American Society of Civil Engineers* **57**: 400–408.
- Bolt GH. 1979. Movement of solutes in soil: principles of adsorption/exchange chromatography. In *Soil Chemistry, B. Physico-Chemical Models*, Bolt GH (ed.). Elsevier Science: New York; 285–348.
- Borah DK. 1989. Runoff simulation model for small watersheds. *Transactions of the ASAE* **32**(3): 881–886.
- Borah DK, Prasad SN, Alonso CV. 1980. Kinematic wave routing incorporating shock fitting. *Water Resources Research* **16**(3): 529–541.
- Bradley JB. 1981. Discussion of 'Kinematic properties of mudflows on Mt. St. Helens', T. E. Lang and J. D. Dent. *Journal of Hydraulic Engineering*: ASCE, **114**(12): 1538–1539.
- Brazil LE, Sanders TG, Woolhiser DA. 1979. Kinematic parameter estimation for transport of pollutant in overland flow. In *Surface and Subsurface Hydrology*, Morel-Seytoux HJ (ed.). Water Resources Publications: Fort Collins, CO; 555–568.
- Brooks RH, Corey AT. 1964. *Hydraulic properties of porous media*. Hydrology Paper 3, Colorado State University: Fort Collins, CO.
- Buchberger SG, Sanders TG. 1982. A kinematic model for pollutant concentrations during the rising hydrograph. In *Modelling Components of Hydrologic Cycle*, Singh VP (ed.). Water Resources Publications: Fort Collins, CO; 405–427.
- Burgers JM. 1948. A mathematical model illustrating the theory of turbulence. *Advances in Applied Mechanics* **1**: 171–199.
- Carrier GF, Pearson CE. 1976. *Partial Differential Equations: Theory and Practice*. Academic Press: New York.
- Cassidy HG, Wood SE. 1941. Chromatography of solutions containing a single solute. *Journal of the American Chemical Society* **63**: 2628–2630.
- Charbeneau RJ. 1984. Kinematic models for soil moisture and solute transport. *Water Resources Research* **20**(6): 699–706.
- Charbeneau RJ, Weaver JW, Smith VJ. 1989. *Kinematic modelling of multiphase solute transport in the vadose zone*. EPA Report EPA/600/2-89/035, R.S.K. Environmental Research Laboratory, US Environmental Protection Agency: Ada, OK; 1588 pp.
- Chen CL. 1970. Surface irrigation using kinematic wave method. *Journal of Irrigation and Drainage Division, ASCE* **96**(IR-1): 39–48.
- Colbeck SC. 1972. A theory of water percolation in snow. *Journal of Glaciology* **11**: 369–385.
- Colbeck SC. 1974. Water flow through snow overlying an impermeable boundary. *Water Resources Research* **10**(1): 119–123.

- Colbeck SC. 1977. Tracer movement through snow. *IAHS Publications* **118**: 255–262.
- Colbeck SC. 1978. The physical aspects of water flow through snow. In *Advances in Hydrosience*, Vol. 11, Chow VT (ed.). Academic Press: New York; 165–206.
- Colbeck SC, Davidson G. 1973. Water percolation through homogeneous snow. In *Proceedings of the Banff Symposium on the Role of Snow and Ice in Hydrology*, Vol. 1. UNESCO: Geneva; 242–257.
- Cowins SC, Comfort III WJ. 1982. Gravity-induced density discontinuity waves in sand columns. *Journal of Applied Mechanics* **49**: 497–500.
- Croley TE. 1982. Unsteady overland sedimentation. *Journal of Hydrology* **56**: 325–346.
- Croley TE, Foster GR. 1984. Unsteady sedimentation in nonuniform rills. *Journal of Hydrology* **70**: 101–122.
- Croley TE, Hunt B. 1981. Multiple-valued and non-convergent solutions in kinematic cascade models. *Journal of Hydrology* **49**: 121–138.
- Cunge JA, Woolhiser DA. 1975. Irrigation systems. In *Unsteady Flow in Open Channels*, Mahmood K, Yevjevich V (eds). Water Resources Publications: Fort Collins, CO; 522–537.
- Daluz Vieira JA. 1983. Conditions governing the use of approximations for the Saint-Venant equations for shallow surface water flow. *Journal of Hydrology* **60**: 43–58.
- Denoth A, Seidenbusch W, Blumthaler M, Kirchlechner P. 1978. Some experimental data on water percolation through snow. In *Proceedings, Modelling of Snow Cover Runoff*, Colbeck SC, Ray M (eds). US Army Cold Regions Research and Engineering Laboratory: Hanover, NH; 253–256.
- DeVault D. 1943. The theory of chromatography. *Journal of American Chemical Society* **65**: 532–540.
- De Vries J. 1987. Comments on 'On subsurface stormflow: prediction with simple kinematic theory for saturated and unsaturated flows' by K. Beven. *Water Resources Research* **23**(4): 747–748.
- DiPietro L, Lafolie F. 1991. Water flow characterization and test of kinematic-wave model for macropore flow in a highly contrasted and irregular double-porosity medium. *Journal of Soil Science* **42**: 551–563.
- Dunne T. 1978. Field studies of hillslope flow processes. In *Hillslope Hydrology*, Kirkby MJ (ed.). Wiley: Chichester; 227–293.
- Eagleson PS. 1970. *Dynamic Hydrology*. McGraw Hill: New York.
- Elliott RL, Walker WR. 1982. Field evaluation of furrow infiltration and advance functions. *Transactions of the ASAE* **25**(2): 396–400.
- Fangmeier DD, Ramsey MK. 1978. Intake characteristics of irrigation furrows. *Transactions of the ASAE* **21**(4): 696–700, 705.
- Ferrick MJ. 1985. Analysis of river wave types. *Water Resources Research* **21**(2): 209–220.
- Ferrick MG, Weyrick PB, Hunnewell ST. 1992. Analysis of river ice motion near a breaking front. *Canadian Journal of Civil Engineering* **19**: 105–116.
- Ferrick MJ, Weyrick PB, Nelson DF. 1993. Kinematic model of river ice motion during dynamic break up. *Nordic Hydrology* **24**: 111–134.
- Fowler AC. 1982. Waves on glaciers. *Journal of Fluid Mechanics* **120**: 283–321.
- Fowler AC, Larson DA. 1978. On the flow of polythermal glaciers: 1. Model and preliminary analysis. *Proceedings of the Royal Society London, Series A* **363**: 217–242.
- Fowler AC, Larson DA. 1980. On the flow of polythermal glaciers: 2. Surface wave analysis. *Proceedings of the Royal Society London, Series A* **370**: 155–171.
- Fowler AC, Walder J. 1993. Creep closure of channels in deforming subglacial till. *Proceedings of the Royal Society London, Series A* **441**: 17–31.
- Fread DL. 1985. *Applicability criteria for kinematic and dynamic routing models*. Laboratory of Hydrology, National Weather Service, NOAA, US Department of Commerce: Silver Spring, MD.
- Freeze RA, Harlan RL. 1969. Blueprint for a physically-based, digitally-simulated hydrologic response model. *Journal of Hydrology* **9**: 237–258.
- Gerdel RW. 1965. The transmission of water through snow. *Transactions, American Geophysical Union* **35**: 475–485.
- Germann P. 1987. Kinematic wave approximation to the transport of *Escherichia coli* in the vadose zone. *Water Resources Research* **23**(7): 1281–1287.
- Germann P. 1988. Approaches to rapid and far-reaching hydrologic processes in the vadose zone. *Journal of Contaminant Hydrology* **3**: 115–127.
- Germann O. 1990a. Macropores and hydrologic hillslope processes. In *Processes Studies in Hillslope Hydrology*, Anderson MG, Burt TP (eds). Wiley: New York.
- Germann P. 1990b. Preferential flow and generation of runoff: 1. Boundary-layer theory. *Water Resources Research* **26**(12): 3055–3063.
- Germann P, Beven K. 1985. Kinematic wave approximation to infiltration into soils with sorbing macropores. *Water Resources Research* **21**: 990–996.
- Germann P, Beven K. 1986. A distribution function approach to waterflow in soil macropores based on kinematic wave theory. *Journal of Hydrology* **83**: 173–183.
- Germann P, Smith MS, Thomas GW. 1987. Kinematic wave approximation to the transport of *Escherichia coli* in the vadose zone. *Water Resources Research* **23**: 1281–1287.
- Gradowczyk MH. 1968. Wave propagation and boundary instability in erodible-bed channels. *Journal of Fluid Mechanics* **33**: 93–112.
- Hairsine SY, Paralange JY. 1987. Kinematic shock waves on curved surfaces and application to the cascade approximation. *Journal of Hydrology* **87**: 187–200.
- Hartenberg R, Denavit J. 1964. *Kinematic Synthesis and Mechanisms*. McGraw-Hill: New York.
- Hassan MA, Church M. 1994. Vertical mixing of coarse particles in gravel bed rivers: a kinematic model. *Water Resources Research* **30**(4): 1173–1185.
- Havis RN, Smith RE, Adrian DD. 1992. Partitioning solute transport between infiltration and overland flow under rainfall. *Water Resources Research* **28**(10): 2569–2580.
- Hibberd SA. 1984. A model for pollutant concentrations during snowmelt. *Journal of Glaciology* **30**(104): 58–65.

- Hjelmfelt AT. 1976. *Modelling of soil movement across a watershed*. Completion Report, Project No. A-076-MO, Missouri Water Resources Center, University of Missouri: Columbia, MO.
- Hjelmfelt AT, Piest RP, Saxton KE. 1975. Mathematical modelling of erosion on upland areas. *Proceedings, XVIth Congress of IAHR*, Vol. 2, Sao Paulo: Brazil; 40–47.
- Hornberger GM, Beven K, Germann PF. 1990. Inferences about solute transport in macroporous forest soils from time series models. *Geoderma* **46**: 249–262.
- Hornberger GM, Germann PF, Beven KJ. 1991. Throughflow and solute transport in an isolated sloping block in a forested catchment. *Journal of Hydrology* **124**: 81–99.
- Hughes SA, Fowler JE. 1995. Estimating wave-induced kinematics at sloping structures. *Journal of Waterway, Port, Coastal, and Ocean Engineering* **121**(4): 209–215.
- Hunt B. 1982. Asymptotic solution for dam break problem. *Journal of the Hydraulics Division, ASCE* **108**(HY1): 115–126.
- Hunt B. 1983. Asymptotic solution for dam break on sloping channel. *Journal of Hydraulic Engineering, ASCE* **109**(12): 1698–1706.
- Hunt B. 1984a. Dam break solution. *Journal of Hydraulic Engineering* **110**(6): 675–686.
- Hunt B. 1984b. Perturbation solution of dam break floods. *Journal of Hydraulic Engineering* **110**(8): 1058–1071.
- Hunt B. 1987. A perturbation solution of the flood routing problem. *Journal of Hydraulic Research* **25**(2): 215–234.
- Hutter C. 1983. *Theoretical Glaciology*. Kluwer Academic: Dordrecht; 582 pp.
- Ingram JJ, Woolhiser DA. 1980. Chemical transfer into overland flow. In *Proceedings, ASCE Symposium on Watershed Management*. American Society of Civil Engineers: New York; 40–53.
- Irmay SE. 1956. Extension of Darcy law to unsaturated flow through porous media. In *Proceedings of the Darcy Symposium*. IASH–IUGG: Dijon, France.
- Iwagaki Y. 1955. *Fundamental studies on the runoff analysis by characteristics*. Bulletin 10, Disaster Prevention Research Institute, Kyoto University: Kyoto, Japan; 25 pp.
- Johnson AM. 1970. *Physical Processes in Geology*. Freeman and Cooper: San Francisco; 577 pp.
- Jordan P. 1983a. Meltwater movement in a deep snowpack: 1. Field observations. *Water Resources Research* **19**(4): 971–978.
- Jordan P. 1983b. Meltwater movement in a deep snowpack: 2. Simulation model. *Water Resources Research* **19**(4): 979–985.
- Katapodes ND, Schamber DR. 1983. Applicability of dam-break flood wave models. *Journal of Hydraulic Engineering* **109**(5): 702–721.
- Kibler DF, Woolhiser DA. 1970. *The kinematic cascade as a hydrologic model*. Hydrology Paper 39, Colorado State University: Fort Collins, CO.
- Kibler DF, Woolhiser DA. 1972. Mathematical properties of the kinematic cascade. *Journal of Hydrology* **15**: 131–145.
- Klitting P, Sand SE. 1987. Analysis of prototype freak waves. In *Proceedings, Specialty Conference on Nearshore Hydrodynamics*. University of Delaware: Newark.
- Kouwen N, Li RM, Simons DB. 1981. Flow resistance in vegetated waterways. *Transactions of the ASAE* **24**: 684–690.
- Kuczera G, Teo SY, Williams BJ. 1989. A generalized power law for RORB model. In *Hydrology and Water Resources Symposium, Christchurch, New Zealand*. National Conference Publication No. 89/19, Institution of Engineers: Canberra, Australia; 124–130.
- Kynch GJ. 1952. A theory of sedimentation. *Transactions, Faraday Society* **48**: 166–176.
- Laguna A, Giraldez JV. 1993. The description of soil erosion through a kinematic wave model. *Journal of Hydrology* **145**: 65–82.
- Lang TE, Dent JD. 1987. Kinematic properties of mudflows on Mt. St. Helens. *Journal of Hydraulic Engineering* **113**(5): 646–1539.
- Langbein WB, Leopold LB. 1968. *River channel bars and dunes—theory of kinematic waves*. Geological Survey Professional Paper 422-L, US Geological Survey: Washington, DC; 20 pp.
- Lax PD. 1972. The formation and decay of shock waves. *The American Mathematical Monthly* **79**: 227–241.
- Levy BS, Germann PF. 1988. Kinematic wave approximation to solute transport along preferred flow paths in soils. *Journal of Contaminant Hydrology* **3**: 263–276.
- Lighthill MJ, Whitham GB. 1955a. On kinematic waves: 1. Flood movement in long rivers. *Proceedings of the Royal Society London, Series A* **229**: 281–316.
- Lighthill MJ, Whitham GB. 1955b. On kinematic waves: 2. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society London, Series A* **229**: 317–345.
- Lliboutry LA. 1971. The glacier theory. In *Advances in Hydrosience*, Chow VT (ed.). Academic Press: New York; 81–167.
- Macagno E. 1980. Fluid mechanics of the duodenum. *Annual Reviews of Fluid Mechanics* **12**: 139–158.
- Macagno E. 1991. *Leonardian fluid mechanics: 1. History of kinematics and II. Inception of modern kinematics*. IIHR Monograph No. 112, Iowa Institute of Hydraulic Research, The University of Iowa: Iowa City, IA; 128 pp.
- Maheshwari BL. 1988. *Evaluation of border irrigation models for southeast Australia*. PhD thesis, University of Melbourne: Australia.
- Maheshwari BL. 1994. Values for the exponent of the storage-discharge equation in runoff-routing models. *Journal of Hydrology* **163**: 95–106.
- Marsh P, Woo MK. 1984a. Wetting front advance and freezing of meltwater within a snow cover: 1. Observations in the Canadian Arctic. *Water Resources Research* **20**(12): 1853–1864.
- Marsh P, Woo MK. 1984b. Wetting front advance and freezing of meltwater within a snow cover: 2. A simulation model. *Water Resources Research* **20**(12): 1865–1874.
- Marsh P, Woo MK. 1985. Meltwater movement in natural heterogeneous snow covers. *Water Resources Research* **21**(11): 1710–1716.
- Meier MF, Johnson JN. 1962. The kinematic wave on Nisqually Glacier, Washington. *Journal of Geophysical Research* **67**(2): 886 (abstract).
- Meier MF, Post AS. 1969. What are glacier surges? *Canadian Journal of Earth Sciences* **6**(4: 2): 807–817.
- Menendez AN, Norscini R. 1982. Spectrum of shallow water waves: an analysis. *Journal of the Hydraulics Division, ASCE* **108**(HY1): 75–93.
- Morris EM, Woolhiser DA. 1980. Unsteady one dimensional flow over a plane: partial equilibrium hydrograph. *Water Resources Research* **16**(2): 355–360.

- Nakamura E. 1984. Factors affecting the removal rate of street surface contaminants by overland flow. *Journal of Research, Public Works Research Institute* **24**: 1–47.
- Nye JF. 1960. The response of glaciers and ice sheets to seasonal and climatic changes. *Proceedings of the Royal Society London, Series A* **256**: 559–584.
- Nye JF. 1963a. On the theory of the advance and retreat of glaciers. *Geophysical Journal of the Royal Astronomical Society* **7**: 431–456.
- Nye JF. 1963b. The response of a glacier to changes in the rate of nourishment and wastage. *Proceedings of the Royal Society London, Series A* **275**: 87–112.
- Palmer AC. 1972. A kinematic wave model of glacier surges. *Journal of Glaciology* **11**(61): 65–72.
- Parlange JY, Hogarth W, Sander G, Rose C, Haverkamp R, Surin A, Brutsaert W. 1990. Asymptotic expansion for steady state overland flow. *Water Resources Research* **26**: 579–583.
- Pearson CP. 1989. One-dimensional flow over a plane: criteria for kinematic wave modelling. *Journal of Hydrology* **111**: 39–48.
- Pilgrim DH. 1986. Estimation of large and extreme design floods. *Institution of Engineers, Australia, Civil Engineering Transactions* **CE28**: 62–73.
- Ponce VM, Simons DB. 1977. Shallow wave propagation in open channel flow. *Journal of the Hydraulics Division, ASCE* **103**(HY12): 1461–1475.
- Ponce VM, Windingland B. 1985. Kinematic shock: sensitivity analysis. *Journal of Hydraulic Engineering* **111**(4): 600–611.
- Prasad SN, Singh VP. 1982. A hydrodynamic model of sediment transport in rill flows. *IAHS Publications* **137**: 293–301.
- Rabinach A. 1991. *The Human Motor: Energy, Fatigue and the Origins of Modernity*. Basic Books: New York.
- Rayej M, Wallender WW. 1987. Furrow model with specified space intervals. *Journal of Irrigation and Drainage Engineering* **113**: 536–548.
- Rayej M, Wallender WW. 1988. Time solution of kinematic-wave model with stochastic infiltration. *Journal of Irrigation and Drainage Engineering* **114**(4): 605–621.
- Reddy JM, Singh VP. 1994. Modelling and error analysis of kinematic wave equations of furrow irrigations. *Irrigation Science* **14**: 189–198.
- Rose CW, Williams JR, Sander GC, Barry DA. 1983a. A mathematical model of soil erosion and deposition processes: 1. *Soil Science Society of America Proceedings* **47**: 991–995.
- Rose CW, Williams JR, Sander GC, Barry DA. 1983b. A mathematical model of soil erosion and deposition processes: 2. *Soil Science Society of America Proceedings* **47**: 996–1000.
- Santi PM. 1989. The kinematics of debris flow down a canyon. *Bulletin of the Association of Engineering Geologists* **XXVI**(1): 5–9.
- Schmid BH. 1990. On kinematic cascades: derivation of generalized shock criterion. *Journal of Hydraulic Research* **28**(3): 331–340.
- Schneider W. 1982. Kinematic-wave theory of sedimentation beneath inclined walls. *Journal of Fluid Mechanics* **120**: 323–346.
- Seddon JA. 1900. River hydraulics. *Transactions, ASCE* **XLIII**: 179–229.
- Sherman B, Singh VP. 1978. A kinematic model for surface irrigation. *Water Resources Research* **14**(2): 357–384.
- Sherman B, Singh VP. 1982. A kinematic model for surface irrigation: an extension. *Water Resources Research* **18**(3): 639–667.
- Singh VP. 1976. *Studies on rainfall-runoff modelling: 3. Converging overland flow*. WRRRI Report No. 073, New Mexico Water Resources Research Institute, New Mexico State University: Las Graces, NM; 290 pp.
- Singh VP. 1980. Sediment yield equations for upland areas. In *Proceedings, International Symposium on Water Resources Systems*. University of Roorkee: Roorkee, India; 11-13-75–11-13-81.
- Singh VP. 1983. Analytical solutions of kinematic equations for erosion on a plane: II. Rainfall of finite duration. *Advances in Water Resources* **6**: 88–95.
- Singh VP. 1988. *Hydrologic Systems*, Vol. 1, *Rainfall-Runoff Modelling*. Prentice Hall: Englewood Cliffs, NJ.
- Singh VP. 1993. Quantifying the accuracy of hydrodynamic approximations for determination of flood discharges. *Journal of IWRS* **13**(3/4): 172–185.
- Singh VP. 1994a. Accuracy of kinematic-wave and diffusion-wave approximations for space-independent flows. *Hydrological Processes* **8**: 45–62.
- Singh VP. 1994b. Accuracy of kinematic-wave and diffusion-wave approximations for space-independent flows with lateral inflow neglected in the momentum equation. *Hydrological Processes* **8**: 311–326.
- Singh VP. 1994c. Derivation of errors of kinematic-wave and diffusion-wave approximations for space-independent flows. *Water Resources Management* **8**: 57–82.
- Singh VP. 1995. Accuracy of kinematic-wave and diffusion-wave approximations for space-independent flows on infiltrating surfaces with lateral inflow neglected in the momentum equation. *Hydrological Processes* **9**: 783–796.
- Singh VP. 1996a. *Kinematic Wave Modelling in Water Resources: Surface Water Hydrology*. Wiley: New York; 1399 pp.
- Singh VP. 1996b. Errors of kinematic wave and diffusion wave approximations for space-independent flows on infiltrating surfaces. *Hydrological Processes* **10**: 955–969.
- Singh VP. 1996c. *Dam Breach Modelling Technology*. Kluwer Academic: Dordrecht.
- Singh VP, Aravamuthan V. 1995a. Accuracy of kinematic wave and diffusion wave approximations for time-independent flows. *Hydrological Processes* **9**: 755–782.
- Singh VP, Aravamuthan V. 1995b. Errors of kinematic wave and diffusion wave approximations for time-independent flows. *Water Resources Management* **9**: 175–202.
- Singh VP, Aravamuthan V. 1996. Errors of kinematic wave and diffusion wave approximations for steady state overland flows. *Catena* **27**: 209–227.
- Singh VP, Aravamuthan V. 1997. Accuracy of kinematic wave and diffusion wave approximations for time-independent flow with momentum exchange included. *Hydrological Processes* **11**: 511–532.
- Singh VP, Joseph ES. 1994. Kinematic-wave model for soil-moisture movement with plant-root extraction. *Irrigation Science* **14**: 189–198.
- Singh VP, Prasad SN. 1982. Explicit solutions to kinematic equations for erosion on an infiltrating plane. In *Modelling Components of Hydrologic Cycle*, Singh VP (ed.). Water Resources Publications: Fort Collins, CO; 515–538.

- Singh VP, Prasana M. 1999. A generalized flux law, with an application. *Hydrological Processes* **13**: 73–87.
- Singh VP, Ram RS. 1983. A kinematic model for surface irrigation: verification by experimental data. *Water Resources Research* **19**(6): 1599–1612.
- Singh VPP, Regl RR. 1983. Analytical solutions of kinematic equations for erosion on a plane: 1. Rainfall of infinite duration. *Advances in Water Resources* **6**: 2–10.
- Singh VP, Woolhiser DA. 1996. A nonlinear kinematic wave model for watershed surface runoff. *Journal of Hydrology* **31**: 221–243.
- Singh VP, Aravamuthan V, Joseph ES. 1994. Errors of kinematic wave and diffusion-wave approximations for time-independent flows in infiltrating channels. *Irrigation Science* **15**: 137–145.
- Singh VP, Bengtsson L, Westerstrom G. 1997a. Kinematic wave modelling of vertical movement of snowmelt water through a snowpack. *Hydrological Processes* **11**: 149–167.
- Singh VP, Bengtsson L, Westerstrom G. 1997b. Kinematic wave modelling of saturated basal flow in a snowpack. *Hydrological processes* **11**: 177–187.
- Sisson JB, Ferguson AH, van Genuchten MT. 1980. Simple method for predicting drainage from field plots. *Soil Science Society of America Journal* **44**: 1147–1152.
- Sklash MG, Farvolden RN. 1979. The role of groundwater in storm runoff. *Journal of Hydrology* **43**: 45–65.
- Sloan PG, Moore ID. 1984. Modelling subsurface stormflow on steeply sloping forested watersheds. *Water Resources Research* **20**: 1815–1822.
- Smith RE. 1972. Border irrigation advance and ephemeral flood waves. *Journal of Irrigation and Drainage Division, ASCE* **98**(IR-2): 289–307.
- Smith RE. 1981. A kinematic model for surface mine sediment yield. *Transactions of the ASAE* **24**: 1508–1514.
- Smith RE. 1983. Approximate soil water movement by kinematic characteristics. *Soil Science Society of America Journal* **47**: 3–8.
- Smith RE, Hebbert RHB. 1983. Mathematical simulation of interdependent surface and subsurface hydrologic processes. *Water Resources Research* **14**(3): 533–538.
- Smith RE, Woolhiser DA. 1971. Overland flow on an infiltrating surface. *Water Resources Research* **7**(4): 899–913.
- Song CCS. 1983. Modified kinematic model: application to bed forms. *Journal of Hydraulic Engineering* **109**(8): 1133–1151.
- Thomas HC. 1944. Heterogeneous ion exchange in a flowing system. *Journal of American Chemical Society* **66**: 64–66.
- Wakahama G. 1968. Infiltration of meltwater into snow cover. III. Flowing down speed of meltwater via snow cover. *Low Temperature Science, Series A* **26**: 85–86.
- Walker WR, Humpherys AE. 1983. Kinematic wave furrow irrigation model. *Journal of Irrigation and Drainage Engineering Division, ASCE* **109**(4): 377–392.
- Wallender WW, Yokokura J. 1991. Space solution of kinematic wave model by time iteration. *Journal of Irrigation and Drainage Engineering* **116**(1): 114–122.
- Wankiewicz A. 1978a. A review of water movement in snow. In *Proceedings, Modelling of Snow Cover Runoff*, Colbeck SC, Ray M (eds). US Army Cold Regions Research and Engineering Laboratory: Hanover, NH; 222–252.
- Wankiewicz A. 1978b. Water pressure in ripe snow packs. *Water Resources Research* **14**: 593–599.
- Wankiewicz A. 1978c. Hydraulic characteristics of snow lysimeters. *Proceedings Eastern Snow Conference* **35**: 105–116.
- Weaver JW, Charbeneau RJ, Lien BK. 1994. A screening model for nonaqueous phase liquid transport in the vadose zone using Green–Ampt and kinematic wave theory. *Water Resources Research* **30**(1): 92–105.
- Weeks WD. 1980. Using the Laurenson model: traps for young players. National Conference Publication No. 80/9, Institution of Engineers: Canberra, Australia; 29–33.
- Weertmen J. 1958. Travelling waves on glaciers. *IAHS Publications* **47**: 162–168.
- Weir GJ. 1983. One-dimensional bed wave movement in lowland rivers. *Water Resources Research* **19**(3): 627–631.
- Weiss J. 1943. On the theory of chromatography. *Journal of the Chemical Society* **139**: 297–303.
- Wigmosta MS, Vail LW, Lettenmaier DP. 1994. A distributed hydrology-vegetation model for complex terrain. *Water Resources Research* **30**(6): 1665–1679.
- Wilson JN. 1940. A theory of chromatography. *Journal of the American Chemical Society* **62**: 1583–1591.
- Wooding RA. 1965a. A hydraulic model for the catchment-stream problem: 1. Kinematic wave theory. *Journal of Hydrology* **3**(3/4): 268–282.
- Wooding RA. 1965b. A hydraulic model for the catchment-stream problem: 2. Numerical solutions. *Journal of Hydrology* **3**(3/4): 268–282.
- Wooding RA. 1966. A hydraulic model for the catchment-stream problem: 3. Comparisons with runoff observations. *Journal of Hydrology* **4**: 21–37.
- Woolhiser DA. 1996. Search for physically based runoff model—a hydrologic El Dorado? *Journal of Hydraulic Engineering* **122**(3): 122–129.
- Woolhiser DA, Liggett JA. 1967. Unsteady one-dimensional flow over a plane: the rising hydrograph. *Water Resources Research* **3**(3): 753–771.
- Yamada T, Kobayashi M. 1988. Kinematic wave characteristics and new equations of unsaturated infiltration. *Journal of Hydrology* **102**: 257–266.