1. Use the kinematic wave form of the de Saint-Venant equations to route the hydrograph given below in a wide rectangular prismatic channel of the following characteristics:

Bottom slope, $S_o = 0.01$; bottom width, $B = 200$ ft; Manning roughness, $n = 0.035$; length of the channel reach, $L = 50000$ ft;

The input hydrograph at the upstream end of the reach is given by the following function:

$$Q(x = 0, t) = 500 + \frac{1}{3}(t / 500)^3 e^{-t/500}$$

where $t$ is in seconds, and $Q$ is in ft$^3$/s.

The lateral inflow per unit length of channel in the direction of flow is $q = 0.0001$ ft$^2$/s, and the initial flow in the channel reach is $Q(x, t = 0) = 500$ ft$^3$/s.

Use the finite difference expressions developed in class to solve the kinematic wave equations$^1$,

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q ; \quad S_o = S_f ; \quad A = \alpha Q^\beta ; \quad \alpha = \left(\frac{nP^{2/3}}{1.49S_f^{1/2}}\right)^{3/5} \quad \text{and} \quad \beta = 3/5;$$

a. Use the linear solution procedure:

$$Q_{i+1}^{j+1} = \frac{\Delta t}{\Delta x} Q_{i+1}^j + \alpha \beta Q_{i+1}^j \left(\frac{Q_{i+1}^j + Q_{i+1}^{j+1}}{2}\right)^{\beta-1} + \Delta \left(\frac{q_{i+1}^j + q_{i+1}^{j+1}}{2}\right)$$

$^1$ For the given conditions, this channel may be assumed “wide”, that is, $P = B$. 
b. Use the non-linear solution procedure, that is, use Newton’s method to find the solution to the following equation,

\[ f(Q_{i+1}^{j+1}) = \frac{\Delta t}{\Delta x} Q_{i+1}^{j+1} + \alpha (Q_{i+1}^{j+1})^\beta - C = 0 \]

where

\[ C = \frac{\Delta t}{\Delta x} Q_{i}^{j+1} + \alpha (Q_{i+1}^{j+1})^\beta + \Delta t \left( \frac{q_{i+1}^{j} + q_{i+1}^{j+1}}{2} \right) \]

For each \((x,t)\), assume a trial solution and iterate such that the new approximation to the solution is

\[ (Q_{i+1}^{j+1})_{new} = (Q_{i+1}^{j+1})_{old} - \frac{f(Q_{i+1}^{j+1})_{old}}{f'(Q_{i+1}^{j+1})_{old}} \]

where,

\[ f'(Q_{i+1}^{j+1}) = \frac{df(Q_{i+1}^{j+1})}{dQ_{i+1}^{j+1}} = \frac{\Delta t}{\Delta x} + \alpha \beta (Q_{i+1}^{j+1})^{\beta-1} \]

Stop the iteration when either of the following two convergence criteria is met,

\[ \left| (Q_{i+1}^{j+1})_{new} - (Q_{i+1}^{j+1})_{old} \right| < \varepsilon \]

\[ \left| f(Q_{i+1}^{j+1})_{new} \right| < \varepsilon \]

For both part a and part b, use \(\Delta t = 50\) s, \(\Delta x = 100\) ft, and simulate the process for 10000 s. Plot the flow along the entire channel every 2000 s. Compare your results.