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CE522 ENGINEERING HYDROLOGY

GAMMA AND RELATED FUNCTIONS

The complete gamma function is defined as,

$$\Gamma(\nu) = \int_0^\infty t^{\nu-1} e^{-t} dt$$

The incomplete gamma function is defined as,

$$\gamma(\nu, x) = \int_0^x t^{\nu-1} e^{-t} dt$$

The complementary gamma function required is defined as,

$$\Gamma(\nu, x) = \int_x^\infty t^{\nu-1} e^{-t} dt$$

The Nash model question in your homework requires that you evaluate the following integral,

$$g(t) = \int_0^t u(\tau) d\tau = \frac{1}{\Gamma(n)} \int_0^t \frac{e^{-\tau/k}}{k} \left(\frac{t}{k}\right)^{n-1} d\tau = \frac{1}{\Gamma(n)} \int_0^{t/k} s^{n-1} e^{-s} ds = \frac{\gamma(n, t/k)}{\Gamma(n)}$$

Therefore, because $\Gamma(\nu) = \gamma(\nu,x) + \Gamma(\nu,x)$,

$$\frac{\gamma(n, t/k)}{\Gamma(n)} = 1 - \frac{\Gamma(n, t/k)}{\Gamma(n)}$$

For integer values of ν (which is the case for this homework), the complementary gamma function can be evaluated using the following expression,

$$\Gamma(n, x) = (n-1)! e^{-x} \sum_{k=1}^{n} \frac{x^{k-1}}{(k-1)!} = (n-1)! e^{-x} \left(1 + x + \frac{x^2}{2!} + \ldots + \frac{x^{a-1}}{(n-1)!}\right)$$