Snowmelt Runoff Example

1. Consider a homogeneous snowpack of 0.6 m depth and density 0.5 g/cm^3 and which is in thermodynamic equilibrium at a temperature of 271.15 K.

a) Compute the water equivalent depth of the snowpack, the cold content in units of energy per unit area, and the thermal quality of the snowpack.

Assume that rain having a temperature of 275.15 K begins to fall at a constant rate of 0.3 cm/hr.

b) How long will it be before snow melt begins?
c) What is the water equivalent of the snowpack at the beginning of snowmelt?
d) How long will it be before runoff from snowmelt begins?
e) How long will it be before runoff from rainfall begins?

The specific heat of snow, c_{ps}, is 0.5 cal/g/K, that of liquid water c_p, is 1.0 cal/g/K and the latent heat of fusion of ice is L_f = 80 cal/g. The maximum water holding capacity, Wmax, is 0.035 for the conditions given. Assume a seepage velocity, v_s, of 18 cm/h. Also, assume that the only important heat fluxes are those associated with advection and latent heats.

```
> restart;with(student);
```

[D, *Diff*, *Doubleint*, *Int*, *Limit*, *Lineint*, *Product*, *Sum*, *Tripleint*, *changevar*, *completesquare*, *distance*, *equate*, *integrand*, *intercept*, *intparts*, *leftbox*, *leftsum*, *makeproc*, *middlebox*, *middlesum*, *midpoint*, *powsubs*, *rightbox*, *rightsum*, *showtangent*, *simpson*, *slope*, *summand*, *trapezoid*] **(1)**

Define cold content function, *Hcc*:

$$H_{cc} := \rho_s C_{ps} d_s (273.15 - Ts)$$

```
> H[cc]:=Ts->rho[s]*C[ps]*d[s]*(273.15-Ts);
```

$$H_{cc} := Ts \to \rho_s C_{ps} d_s (273.15 - Ts) \quad \textbf{(2)}$$

Define cold content in units of equivalent depth of melt, *dcc*

$$d_{cc} := \frac{H_{cc}}{L_f \rho_w}$$

```
> d[cc]:=Ts->H[cc](Ts)/L[f]/rho[w];
```

$$d_{cc} := Ts \to \frac{H_{cc}(Ts)}{L_f \rho_w} \quad \textbf{(3)}$$

Define latent heat of fusion of snow, *Lfs*

$$L_{fs} := L_f (1 - W_{\max} + W_{def})$$

```
> L[fs]:=L[f]*(1-(W[max]-W[def]));
```

$$L_{fs} := L_f (1 - W_{\max} + W_{def}) \quad \textbf{(4)}$$

Define the total heat deficit of the snowpack, *Hdef*

$$H_{def} := H_{cc} + \rho_s d_s L_{fs}$$

```
> H[def]:=Ts->H[cc](Ts)+rho[s]*d[s]*L[fs];
```

$$H_{def} := Ts \rightarrow H_{cc}(Ts) + \rho_s d_s L_{fs} \quad \textbf{(5)}$$

Define Thermal Quality function

$$\theta := \frac{L_{fs}}{L_f} + \frac{C_{ps}\,(273.15 - Ts)}{L_f}$$

```
> theta:=Ts->L[fs]/L[f]+C[ps]*(273.15-Ts)/L[f];
```

$$\theta := Ts \rightarrow \frac{L_{fs}}{L_f} + \frac{C_{ps}\,(273.15 - Ts)}{L_f} \quad \textbf{(6)}$$

Define Advection Heat (i.e. heat added by the precipitation), *Hp*. The temperature of precipitation is *Tr* and the total depth of precipitation is *dr*.

$$H_p := \rho_w C_{pw} d_r\,(Tr - Ts)$$

where *dr* is the total precipitation depth over the given time interval.

```
> H[p]:=(Tr,Ts)->rho[w]*C[pw]*d[r]*(Tr-Ts);
```

$$H_p := (Tr, Ts) \rightarrow \rho_w C_{pw} d_r\,(Tr - Ts) \quad \textbf{(7)}$$

Define Latent Heat (resulting from the freezing of the rain water):

$$H_e := \rho_w L_f d_r$$

where *dr* is the total precipitation depth over the given time interval.

```
> H[e]:=rho[w]*L[f]*d[r];
```

$$H_e := \rho_w L_f d_r \quad \textbf{(8)}$$

Define depth of water required to supply the Cold Content. In this case, energy will be supplied from the advection of energy by rain and from the release of latent heat by freezing rain. That is, *Hcc* = *He* + *Hp*, where $H_p := \rho_w C_{pw} d_r\,(Tr - Ts)$ and $H_e := \rho_w L_f d_r$. Use the resulting equation to solve for *dr*.

$$d_r := \frac{\rho_s C_{ps} d_s\,(273.15 - Ts)}{\rho_w C_{pw}\,(Tr - Ts) + \rho_w L_f}$$

dr will be equal to time delay multiplied by the rain rate.

```
> d[r]:=(Tr,Ts)->(rho[s]*C[ps]*d[s]*(273.15-Ts))/(rho[w]*C[pw]*(Tr-
  Ts)+rho[w]*L[f]);
```

$$d_r := (Tr, Ts) \rightarrow \frac{\rho_s C_{ps} d_s\,(273.15 - Ts)}{\rho_w C_{pw}\,(Tr - Ts) + \rho_w L_f} \quad \textbf{(9)}$$

Define time delay before snowmelt begins assuming a rainfall intensity of *ir*.

$$t_c := \frac{d_r}{ir}$$

```
> t[c]:=(Tr,Ts)->d[r](Tr,Ts)/ir;
```

(10)

$$t_c := (Tr, Ts) \rightarrow \frac{d_r(Tr, Ts)}{ir} \quad \textbf{(10)}$$

Define water equivalent at the time when cold content is satisfied. In this case, because additional heat is being supplied by advection, the total depth of rain water is not equal to *dcc* but to *dr*, which is smaller than *dcc*.

$$d_{eq} := \frac{\rho_w d_r + \rho_s d_s}{\rho_w}$$

```
> d[eq]:=(Tr,Ts)->(rho[w]*d[r](Tr,Ts)+rho[s]*d[s])/rho[w];
```

$$d_{eq} := (Tr, Ts) \rightarrow \frac{\rho_w d_r(Tr, Ts) + \rho_s d_s}{\rho_w} \quad \textbf{(11)}$$

Define water deficit at the beginning of snowmelt

$$d_{def} := \frac{W_{\max}\left(\rho_w d_p + \rho_s d_s\right)}{\rho_w}$$

```
> d[def]:=(Tr,Ts)->W[max]*d[eq](Tr,Ts);
```

$$d_{def} := (Tr, Ts) \rightarrow W_{\max}\, d_{eq}(Tr, Ts) \quad \textbf{(12)}$$

Defien time delay between the beginning of snowmelt and fully ripe conditions:

$$t_s := \frac{d_{def}}{i + m}$$

where m is the melt rate. Thus, we need to compute the melt rate. After the pack is at 0 C, the only heat source is the advection heat. Thus, the melt rate is:

$$m := \frac{C_{pw}\, i\, (Tr - 273.15)}{L_f}$$

Since the cold content has already been satisfied, the temperature of the snowpack is 273.15 K.

```
> m:=Tr->rho[w]*C[pw]*ir*(Tr-273.15)/rho[w]/L[f];
```

$$m := Tr \rightarrow \frac{\rho_w C_{pw}\, ir\, (Tr - 273.15)}{\rho_w L_f} \quad \textbf{(13)}$$

```
> t[s]:=(Tr,Ts)->d[def](Tr,Ts)/(ir+m(Tr));
```

$$t_s := (Tr, Ts) \rightarrow \frac{d_{def}(Tr, Ts)}{ir + m(Tr)} \quad \textbf{(14)}$$

Define input data

```
> L[f]:=80;C[ps]:=0.5;d[s]:=60;rho[s]:=0.5;W[max]:=0.035;W[def]:=
0.035;rho[w]:=1;C[pw]:=1;v[s]:=18;ir:=.3;Tss:=271.15;Tpp:=275.15;
```

$$L_f := 80 \quad \textbf{(15)}$$

$$C_{ps} := 0.5$$

$$d_s := 60$$

$$\rho_s := 0.5$$

$$W_{max} := 0.035$$

$$W_{def} := 0.035$$

$$\rho_w := 1$$

$$C_{pw} := 1$$

$$v_s := 18$$

$$ir := 0.3$$

$$Tss := 271.15$$

$$Tpp := 275.15$$

```
> H[cc](Tss);
```

$$30.0000 \qquad (16)$$

```
> d[cc](Tss);
```

$$0.3750000000 \qquad (17)$$

```
> d[eq](273.15,273.15);
```

$$30.0 \qquad (18)$$

```
> theta(Tss);
```

$$1.012500000 \qquad (19)$$

```
> L[fs];
```

$$80.000 \qquad (20)$$

```
> H[def](Tss);
```

$$2430.0000 \qquad (21)$$

```
> H[p](Tpp,Tss);
```

$$4.00\, d_r \qquad (22)$$

```
> H[e];
```

$$80\, d_r \qquad (23)$$

```
> d[r](Tpp,Tss);
```

$$0.3571428571 \qquad (24)$$

```
> t[c](Tpp,Tss);
```

$$1.190476190 \qquad (25)$$

```
> d[eq](Tpp,Tss);
```

$$30.35714286 \qquad (26)$$

```
> d[def](Tpp,Tss);
```

$$1.062500000 \qquad (27)$$

```
> t[s](Tpp,Tss);
```

$$3.455284553 \qquad (28)$$

```
> t[t]:=(Tr,Ts)->(d[s]-m(Tr)*t[s](Tr,Ts))/v[s];
```

$$t_t := (Tr, Ts) \rightarrow \frac{d_s - m(Tr)\, t_s(Tr, Ts)}{v_s} \quad \textbf{(29)}$$

```
> t[t](Tpp,Tss);
```

$$3.331893632 \quad \textbf{(30)}$$