

CIVE520 PHYSICAL HYDROLOGY

Homework No. 2

Assigned on: September 23, 2011
Due on: October 10, 2011

1. Use the precipitation model discussed in class in order to make predictions of precipitation rates at the ground for the conditions given below. The data given correspond to a decaying cloud system, and were taken at intervals of 450 seconds. Atmospheric soundings have indicated an ambient lapse rate of 6.9 °K/km. The ambient mixing ratio varies with pressure, p , according to

$$w(p) = w_o \exp(-(p_o - p)^2 / \mathbf{P}^2)$$

where \mathbf{P} equals 200. hPa.

Initial inverse mean diameter at cloud base	$c_i=953 \text{ m}^{-1}$
Parameter of raindrop size distribution	$N_0=8.e6 \text{ m}^{-4}$
Updraft velocity	$v=3.0 \text{ m/s}$
Ratio of inverse diameters at top and bottom of cloud	$c_u/c_b = \gamma=5$
Parameter c^* for critical diameter	$c^* = 7.0 \cdot 10^5 \text{ Kg/m}^3/\text{s}$
Parameter of terminal velocity parameterization	$\alpha=1000. \text{ s}^{-1}$
Height to cloud base	$Z_b=420. \text{ m}$
Ambient Lapse Rate	$\Gamma_{\text{amb}}=6.9 \text{ K/km}$
Initial cloud depth	$Z_c=6000. \text{ m}$
Surface pressure	$p_o=1000.0 \text{ hPa}$

Following are the forecasts for the climate parameters:

Climate Variable	t=0. s	t=450. s	t=900. s	t=1350. s
θ_e (°K)	350.	348.	346.	344.
θ (°K)	300.	300.	300.	300.
Cloud top Pressure (hPa)	460.	500.	550.	600.

In addition, determine the level of free convection and compute the energy generated per unit mass of parcel in going from LFC to the pressure level of the cloud top, that is, CAPE. In the absence of environmental mixing and other frictional losses, compute the upward velocity per unit mass attained by the parcel after ascending to cloud top. Plot the ambient and parcel temperatures as a function of pressure. Also, plot the ambient mixing ratio and the parcel mixing ratio as a function of pressure.

CIVE520 PHYSICAL HYDROLOGY

The complementary gamma function can be evaluated for n integer and χ real using the following relationship,

$$\Gamma(n, \chi) = (n-1)! e^{-\chi} \left(1 + \chi + \frac{\chi^2}{2!} + \dots + \frac{\chi^{n-1}}{(n-1)!} \right)$$

The following equations may be useful in making your predictions.

$$I = [w_o - w_s(T_i, p_i)] \rho_m v = \Delta w \rho_m v$$

$$O_b = \frac{\pi \rho_w N_o}{6 c_i^4} \left[\frac{\alpha}{c_i} \Gamma\left(5, \frac{c_i v}{\alpha}\right) - v \Gamma\left(4, \frac{c_i v}{\alpha}\right) \right]$$

$$O_t = \frac{\pi \rho_w N_o \alpha}{(c_i \gamma)^5} \left(\frac{1}{6} \left[\Gamma\left(5, \gamma \frac{c_i v}{\alpha}\right) - \gamma \frac{c_i v}{\alpha} \Gamma\left(4, \gamma \frac{c_i v}{\alpha}\right) \right] + \gamma \frac{c_i v}{\alpha} - 4 \right)$$

$$P = \frac{\pi \rho_w N_o}{6 c_i^4} \left[\frac{\alpha}{c_i} \Gamma(5, c_i D_l) - v \Gamma(4, c_i D_l) - D_c^3 \alpha c_i^2 \Gamma(2, c_i D_l) + D_c^3 v c_i^3 e^{-c_i D_l} \right]$$

$$D_l = \max(D_c, v / \alpha)$$

$$D_c = \left(\frac{4 D^*}{c^* R_v} Z_b \left(\frac{e_s(T_w)}{T_w} - \frac{e_s(T_d)}{T_o} \right) \right)^{1/3}$$

$$D^* = 2.11 \times 10^{-5} \left(\frac{T_o}{273.15} \right)^{1.94} \left(\frac{1013.25}{p_o} \right) \text{ in } m^2 / s$$

$$X = (\pi \rho_w N_o Z_c \delta / c_i^4)$$

$$\delta = (1/\gamma + 1/\gamma^2 + 1/\gamma^3) / 3$$