

CE520 PHYSICAL HYDROLOGY

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EVAPOTRANSPIRATION

ENERGY BUDGET METHODS. Energy budget methods are based on the direct application of the energy budget equation, or some approximation thereof. The energy budget equation for a control volume including a transpiring canopy and a thin layer of surface soil can be written as,

$$\frac{\partial W}{\partial t} = R_n + A_h - L_v E - H - G - L_p F_p \quad (2)$$

or

$$Q_n = L_v E + H \quad (3)$$

where $L_v E$ is the latent heat flux or energy flux used in evaporation; H is the sensible heat flux (i.e., diffusive heat exchange between the land surface and the atmosphere); and Q_n is the available energy flux density. Q_n includes contributions from radiative, advective, and diffusive processes, and energy storage changes. Over land surfaces, the available energy flux density can be expressed as,

$$Q_n = R_n - G + A_h - L_p F_p - \frac{dW}{dt} \quad (4)$$

where R_n is the net radiation flux density at the surface; A_h is energy added to the system by advection; dW/dt is the change in energy storage of the system; G is the net diffusive ground heat flux out of the soil layer; and $L_p F_p$ is energy use in photosynthesis. The contributions to the net radiation flux density at the surface are the incident short wave radiation, R_s ; the short wave radiation reflected back into the atmosphere, $\alpha_s R_s$; the net long wave radiation from the atmosphere absorbed by the surface, R_{la} ; and the long wave radiation emitted by the surface, R_{lu} . Thus,

$$R_n = R_s (1 - \alpha_s) + R_{la} - R_{lu} \quad (5)$$

The energy advected A_h is made up of contributions by precipitation, net streamflow, seepage, and evaporation fluxes. The available energy flux density at the surface is

partitioned into energy for evaporation and energy for sensible heating as expressed in equation 3.

In applications of the energy budget method, Q_n is often approximated as $R_n - G$, or as R_n only. However, care must be taken to ensure that such approximations are valid.

Standard Energy Budget Method. Among the many applications of the energy budget methods, one of the most widely used is the Bowen Ratio Method. The Bowen Ratio, B_o , defined as the ratio of sensible heat to latent heat,

$$B_o = \frac{H}{L_v E} \quad (6)$$

can be determined from data on specific humidity and temperature in the dynamic sublayer of the atmospheric boundary layer as,

$$B_o = \frac{pC_p(T_2 - T_1)}{\varepsilon L_v(e_2 - e_1)} = \gamma \frac{(T_2 - T_1)}{(e_2 - e_1)} \quad (7)$$

where C_p is the specific heat of air at constant pressure; p is the surface pressure; L_v is the latent heat of evaporation; ε is the ratio of the gas constant for dry air, R_d to that of water vapor, R_v , and numerically $\varepsilon = 0.622$; e_2 and e_1 are the actual vapor pressures at elevations z_2 and z_1 within the dynamic sublayer; T_2 and T_1 are the corresponding temperatures. The constant γ is

$$\gamma = \frac{pC_p}{\varepsilon L_v} \quad (8)$$

and it is known as the psychrometric constant. Equation 7 above was obtained after invoking similarity arguments for the distributions (*i.e.*, profiles) of specific humidity, velocity, and temperature within the dynamic sublayer of the atmospheric boundary layer.

If the Bowen ratio is known, then the energy budget equation leads to the following equation for the evaporation rate,

$$E = \frac{Q_n}{L_v(1 + B_o)} \quad \text{EBBR and no advection} \quad (9a)$$

In order to account for the energy advected out of the system by the water as it evaporates into the atmosphere, Equations 3, 4, and 6 lead to a modified EBBR expression as follows,

$$E = \frac{Q_n}{L_v(1 + B_o + \frac{C_{pw}T_e}{L_v})} \quad \text{EBBR and advection by evaporated water only} \quad (9b)$$

where C_{pw} is the specific heat of water, and T_e is the temperature of the evaporated water.

Potential Evaporation. Evaporation is a mass transport process that results from gradients in the distribution of water vapor mass concentration (*i.e.*, a gradient of specific humidity). Potential evaporation refers to the rate of evaporation from any large uniform surface which is sufficiently moist or wet, so that air immediately in contact with it is fully saturated, thus leading to the largest possible local gradient of specific humidity, and consequently the maximum rate of evaporation for the given conditions. Some of the simplified methods introduced below are sometimes used as measures of potential evaporation.

Simplified Methods for Wet Surfaces. When the surface is wet, it can be assumed that the surface specific humidity is the saturation specific humidity at the surface temperature. This allows an approximation, first proposed by Penman, that eliminates the need for measurements at two levels (*i.e.*, levels at z_1 and z_2), as required by the standard energy budget method, or the so called profile methods (*i.e.*, those based on similarity profiles in the boundary layer presented later in this section.) Penman equation is,

$$E = \frac{\Delta}{\Delta + \gamma} Q_{ne} + \frac{\gamma}{\Delta + \gamma} E_A$$

$$Q_{ne} = \frac{Q_n}{L_v} = \frac{R_n + A_h - G - \frac{\partial W}{\partial t}}{L_v} \quad (10)$$

where $\gamma = pC_p / (0.622L_v)$ is the psychrometric constant at surface pressure p , and where $\Delta = (de_s/dT)$ is the slope of the saturation water vapor pressure curve. Penman equation assumes that Δ can be approximated as,

$$\Delta \equiv \frac{de_s(T)}{dT} = \frac{L_v e_s}{R_v T^2} \cong \frac{e_s(T_2) - e_s(T_1)}{T_2 - T_1} \quad (11)$$

E_A is sometimes referred to as the drying power of the air. It is of the general form,

$$E_A = f(u_2)(e_{s_2} - e_2) \quad (12)$$

where $f()$ is the so-called wind function and e_{s_2} and e_2 refer to saturation water vapor pressure, and actual water vapor pressure of the air at elevation z_2 , respectively. A particular form of the drying power of the air is the so-called aerodynamic method or Thornthwaite-Holzmann equation,

$$E_A = \frac{\rho k^2 \epsilon u_2}{p \ln^2(z_2/z_o)} (e_{s_2} - e_2) \quad (13)$$

where z_o is the roughness length of the surface, and k is von Karman's constant ($k = 0.4$).

Penman (1948) suggested the following form for E_A

$$E_A = f(u_2)(e_{s_2} - e_2) = 0.26(1 + 0.54u_2)(e_{s_2} - e_2) \quad (14)$$

where u_2 is in m/s and the vapor pressures are in mb yielding E_A in units of mm/day .

Equilibrium Evaporation. When overpassing air has been in contact with a wet surface over a long fetch, it may tend to become vapor saturated (under conditions of no advection). Thus, the drying power of the air should tend to vanish. Therefore, in this limit, Penman equation yields a lower limit to the evaporation rate from moist surfaces, known as the equilibrium evaporation rate, and given by,

$$E_e = \left(\frac{\Delta}{\Delta + \gamma}\right) \frac{Q_n}{L_v} \quad (15)$$

Partial Equilibrium Evaporation. However, equilibrium conditions are encountered only rarely, as air in the boundary layer is continually responding to large-scale weather patterns which tend to maintain a vapor deficit even over the oceans. Thus, there is always some degree of advection. Priestley and Taylor (1972) took the concept of equilibrium evaporation as the basis for an empirical equation giving evaporation from a wet surface under conditions of minimal advection, the so-called Priestley and Taylor equation for partial equilibrium evaporation,

$$E_{pe} = \alpha_e E_e = \alpha_e \left(\frac{\Delta}{\Delta + \gamma}\right) \frac{Q_n}{L_v} \quad (16)$$

Priestley and Taylor concluded that the best estimate of α_e is $\alpha_e = 1.26$.

Empirical Equations. Jensen and Haise (1963) proposed the following empirical equation for the estimation of evapotranspiration for agricultural purposes.

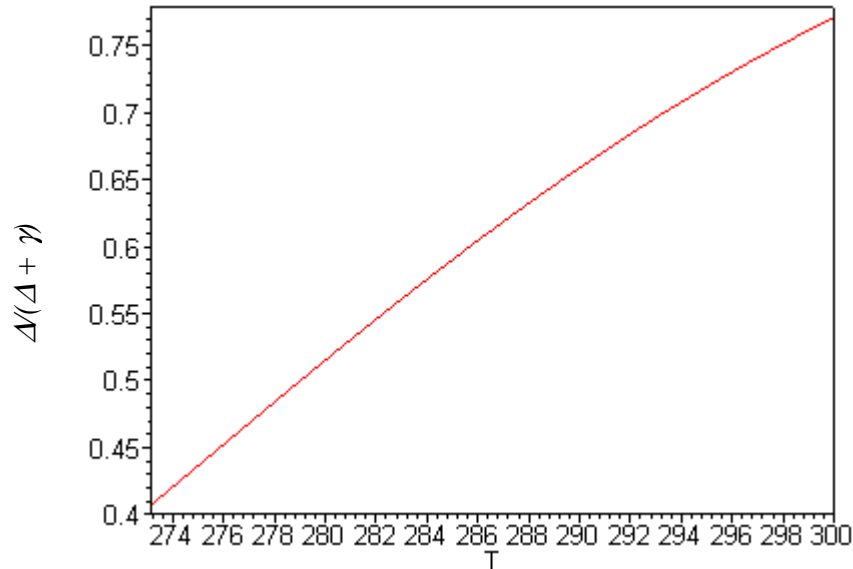
$$E = (aT_a + b)R_{se} \quad (17)$$

where a and b are calibration constants, T_a is the air temperature in degrees Celcius, and R_{se} is the incident solar radiation expressed in equivalent units of evaporated water. That is,

$$R_{se} = \frac{R_s}{L_v} \quad (18)$$

Observe that this equation has a form similar to the equilibrium evaporation equation above. The linear function of temperature of the equation proposed by Jensen and Haise

can be justified by the quasi-linear dependence on temperature of the factor $\Delta/(\Delta + \gamma)$ as shown in the figure below.



In addition, the net short wave radiation is strongly correlated with the net radiation R_n , which is the most important component of the available energy, Q_n , over daily periods or longer. However, the equation needs to be calibrated to the specific region. Based on more than 1000 measurements of consumptive use in the western U.S. representing means over periods longer than 5 days, Jensen and Haise obtained the following values for the constants, $a = 0.025 \text{ }^\circ\text{C}^{-1}$ and $b = 0.078$, with a correlation coefficient $r = 0.86$.

Modified Penman Equation for Vegetated Surfaces. Vegetated surfaces, even when the vegetation is well supplied with water, can not be considered wet, except after rainfall or dew formation. Thus, the specific humidity at the surface of the foliage is likely to be smaller than the saturation value at the corresponding temperature. Therefore, Penman equation is no longer applicable. Introducing a resistance formulation for evaporation such that,

$$E = \frac{\rho}{r_{1,2}}(q_1 - q_2) \quad (19)$$

where q_1, q_2 are specific humidity at elevation z_1 and z_2 , and $r_{1,2}$ is a resistance parameter characterizing the transfer between points at elevation z_1 and z_2 , a modified Penman equation, the so called Penman-Monteith equation, can be derived. In order to do so, two resistance parameters must be defined, one characterizing the transfer between the vapor saturated stomatal cavities and the surface of the leaf, and another characterizing the vapor transfer between the leaf surface and the ambient air within the dynamic sublayer.

The former resistance is known as the stomatal resistance, r_{st} , and the latter as the aerodynamic resistance, r_{av} .

The aerodynamic resistance parameter has the following form,

$$r_{av} = [\ln(z_2 / z_o) / k]^2 / u_2 \quad (20)$$

The modified Penman equation is,

$$E = \frac{\Delta Q_n / L_v + \rho C_p (e_{s2} - e_2) / (L_v r_{av})}{[\Delta + \gamma(1 + r_{st} / r_{av})]} \quad (21)$$

MASS TRANSFER METHODS. Based on arguments about similarity profiles within the fully turbulent dynamic sublayer of the ABL, the following expression was obtained for the water vapor transfer as a result of gradients of specific humidity in the dynamic sublayer,

$$E = - \frac{\rho k u^* (q_2 - q_1)}{\ln(z_2 / z_1)} \quad (22)$$

where u^* is the so-called friction velocity,

$$u^* = (\tau_o / \rho)^{1/2} \quad (23)$$

In the expression for friction velocity, τ_o is the surface shear stress.

From similarity arguments in the ABL, the vertical profile of horizontal velocity in the dynamic sublayer is given by,

$$u_2 - u_1 = \frac{u^*}{k} \ln(z_2 / z_1) \quad (24)$$

where k is von Karman's constant, $k = 0.4$.

Using analogous arguments of similarity profiles in the dynamic sublayer, the vertical transfer of sensible heat is given by,

$$H = - \frac{\rho k u^* C_p (T_2 - T_1)}{\ln(z_2 / z_1)} \quad (25)$$