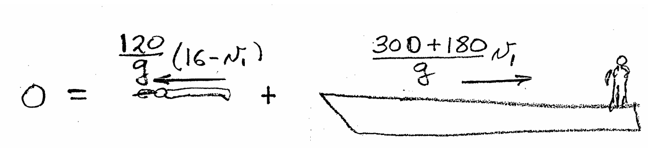
Chapter 14, Solution 8.

(*a*) *Woman dives first.*

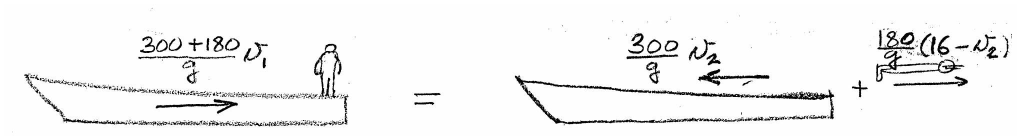


Conservation of momentum:

$$-\frac{120}{g}(16 - v_1) + \frac{300 + 180}{g}v_1 = 0$$

$$v_1 = \frac{(120)(16)}{600} = 3.20 \text{ ft/s} \longrightarrow$$

Man dives next. Conservation of momentum:



$$\frac{300 + 180}{g}v_1 = -\frac{300}{g}v_2 + \frac{180}{g}(16 - v_2)$$

$$v_2 = \frac{-480v_1 + (180)(16)}{480} = 2.80 \text{ ft/s}$$

$$\mathbf{v}_2 = 2.80 \text{ ft/s} \longleftarrow \blacktriangleleft$$

(*b*) *Man dives first.*

Conservation of momentum:

$$\frac{180}{g}(16 - v_1') - \frac{300 + 120}{g}v_1' = 0$$

$$v_1' = \frac{(180)(16)}{600} = 4.80 \text{ ft/s} \longleftarrow$$

Woman dives next. Conservation of momentum:

$$-\frac{300 + 120}{g}v_1' = \frac{300}{g}v_2' + \frac{120}{g}(16 - v_2')$$

$$v_2' = \frac{-420v_1' + (120)(16)}{420} = -0.229 \text{ ft/s}$$

$$\mathbf{v}_2' = 0.229 \text{ ft/s} \longleftarrow \blacktriangleleft$$

Chapter 14, Solution 10.

The masses are $m_A = m_B = m_C = 9$ kg.

Position vectors (m): $\mathbf{r}_A = 0.9\mathbf{k}, \quad \mathbf{r}_B = 0.6\mathbf{i} + 0.6\mathbf{j} + 0.9\mathbf{k}, \quad \mathbf{r}_C = 0.3\mathbf{i} + 1.2\mathbf{j}$

Coordinates of mass center G expressed in m.

$$\overline{\mathbf{r}} = \frac{m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C}{m_A + m_B + m_C}$$

$$= \frac{(9)(0.9\mathbf{k}) + (9)(0.6\mathbf{i} + 0.6\mathbf{j} + 0.9\mathbf{k}) + (9)(0.3\mathbf{i} + 1.2\mathbf{j})}{27}$$

$$= 0.3\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{k}$$

Position vectors relative to the mass center expressed in m.

$$\mathbf{r}'_A = \mathbf{r}_A - \overline{\mathbf{r}} = (0.9\mathbf{k}) - (0.3\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{k}) = -0.3\mathbf{i} - 0.6\mathbf{j} + 0.3\mathbf{k}$$

$$\mathbf{r}'_B = \mathbf{r}_B - \overline{\mathbf{r}} = (0.6\mathbf{i} + 0.6\mathbf{j} + 0.9\mathbf{k}) - (0.3\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{k}) = 0.3\mathbf{i} + 0.3\mathbf{k}$$

$$\mathbf{r}'_C = \mathbf{r}_C - \overline{\mathbf{r}} = (0.3\mathbf{i} + 1.2\mathbf{j}) - (0.3\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{k}) = 0.6\mathbf{j} - 0.6\mathbf{k}$$

Angular momenta.

$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$

$$\mathbf{H}_G = \mathbf{r}'_A \times (m_A\mathbf{v}_A) + \mathbf{r}'_B \times (m_B\mathbf{v}_B) + \mathbf{r}'_C \times (m_C\mathbf{v}_C)$$

Subtracting,

$$\mathbf{H}_O - \mathbf{H}_G = (\mathbf{r}_A - \mathbf{r}'_A) \times (m_A\mathbf{v}_A) + (\mathbf{r}_B - \mathbf{r}'_B) \times m_B\mathbf{v}_B + (\mathbf{r}_C - \mathbf{r}'_C) \times m_C\mathbf{v}_C$$

$$0 = \overline{\mathbf{r}} \times (m_A\mathbf{v}_A) + \overline{\mathbf{r}} \times (m_B\mathbf{v}_B) + \overline{\mathbf{r}} \times (m_C\mathbf{v}_C)$$

$$= \overline{\mathbf{r}} \times (m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C) = \overline{\mathbf{r}} \times \mathbf{L}$$

$\mathbf{L}$ is parallel to $\overline{\mathbf{r}}$. $\quad \mathbf{L} = \lambda\overline{\mathbf{r}} \quad \mathbf{L} \cdot \mathbf{L} = \lambda^2 \overline{\mathbf{r}} \cdot \overline{\mathbf{r}}$

$$\lambda^2 = \frac{\mathbf{L} \cdot \mathbf{L}}{\overline{\mathbf{r}} \cdot \overline{\mathbf{r}}} = \frac{(45)^2}{(0.9)^2} = 50^2, \qquad \lambda = \pm 50 \text{ N}\cdot\text{s/m}$$

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = \lambda\overline{\mathbf{r}}$$

$$(9)(v_A\mathbf{j}) + (9)(v_B\mathbf{i}) + (9)(v_C\mathbf{k}) = \pm 50(0.3\mathbf{i} + 0.6\mathbf{j} + 0.6\mathbf{k})$$

(*a*) Resolve into components and solve for v_A, v_B, and v_C.

$v_A = 3.333\text{ m/s}$ $\qquad \mathbf{v}_A = (3.33\text{ m/s})\mathbf{j}$ ◀

$v_B = 1.6667\text{ m/s}$ $\qquad \mathbf{v}_B = (1.667\text{ m/s})\mathbf{i}$ ◀

$v_C = 3.333\text{ m/s}$ $\qquad \mathbf{v}_C = (3.33\text{ m/s})\mathbf{k}$ ◀

(*b*) Angular momentum about O expressed in $\text{kg}\cdot\text{m}^2/\text{s}$.

$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.9 \\ 0 & (9v_A) & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.6 & 0.9 \\ (9v_B) & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 1.2 & 0 \\ 0 & 0 & (9v_C) \end{vmatrix}$$

$$= (-8.1v_A\mathbf{i}) + (8.1v_B\mathbf{j} - 5.4v_B\mathbf{k}) + (10.8v_C\mathbf{i} - 2.7v_C\mathbf{j})$$

$$= (-8.1v_A + 10.8v_C)\mathbf{i} + (8.1v_B - 2.7v_C)\mathbf{j} + (-5.4v_B)\mathbf{k}$$

$$= 9\mathbf{i} + 4.5\mathbf{j} - 9\mathbf{k}$$

$\mathbf{H}_O = (9.00\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (4.50\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} - (9.00\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$ ◀

Chapter 14, Solution 21.

Velocities of pieces C and D after impact and fracture.

$$\left(v'_C\right)_x = \frac{x_C}{t_C} = \frac{2.1}{0.7} = 3 \text{ m/s}, \quad \left(v'_C\right)_y = 3\tan 30° \text{ m/s}$$

$$\left(v'_D\right)_x = \frac{x_D}{t_D} = \frac{2.1}{0.9} = 2.333 \text{ m/s}, \quad \left(v'_D\right)_y = -2.3333\tan\theta \text{ m/s}$$

Assume that during the impact the impulse between spheres A and B is directed along the x axis. Then, the y component of momentum of sphere A is conserved.

$$0 = m\left(v'_A\right)_y$$

Conservation of momentum of system:

$$\xrightarrow{+}: \quad m_A v_0 + m_B(0) = m_A v'_A + m_C\left(v'_C\right)_x + m_D\left(v'_D\right)_x$$

$$m(4.8) + 0 = mv'_A + \frac{m}{2}(3) + \frac{m}{2}(2.3333)$$

(*a*) $\mathbf{v}'_A = 2.13 \text{ m/s} \longrightarrow$ ◀

$$+\uparrow: \quad m_A(0) + m_B(0) = m_A\left(v'_A\right)_y + m_C\left(v'_C\right)_y + m_D\left(v'_D\right)_y$$

$$0 + 0 = 0 + \frac{m}{2}(3\tan 30°) - \frac{m}{2}(2.3333\tan\theta)$$

(*b*)

$$\tan\theta = \frac{3}{2.3333}\tan 30° = 0.7423$$

$\theta = 36.6°$ ◀

$$v_C = \sqrt{\left(v_C\right)_x^2 + \left(v_C\right)_y^2} = \sqrt{(3)^2 + \left(3\tan 30°\right)^2}$$

$v_C = 3.46 \text{ m/s}$ ◀

$$v_D = \sqrt{\left(v_D\right)_x^2 + \left(v_D\right)_y^2} = \sqrt{(2.3333)^2 + \left(2.3333\tan 36.6°\right)^2}$$

$v_D = 2.91 \text{ m/s}$ ◀