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CE261 ENGINEERING MECHANICS: DYNAMICS

Spring 2008

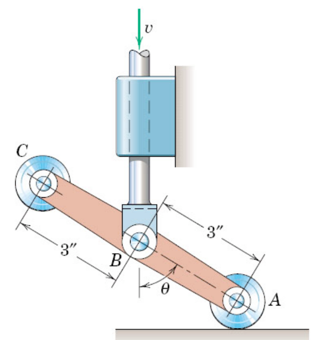
FINAL EXAM

Wednesday, May 14, 2008 – 1:30-3:30 PM Room A204 Clark

You **may not consult** any books, notes, or inanimate references. You **may not consult** with another person. You **may not copy** another student's solutions.

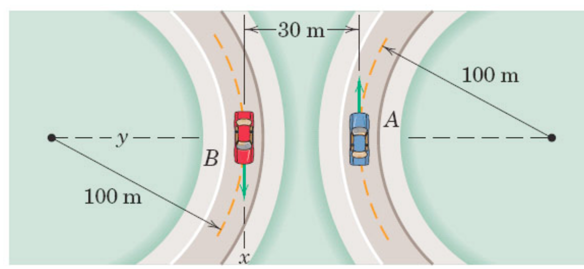
PROBLEM 1

The elements of a switching device are shown. If the vertical control rod has a downward velocity *v* of 3 ft/sec and is decreasing at a rate of 20 *ft/sec*^{2} when $\theta = 60°$ and if roller *A* is in continuous contact with the horizontal surface, determine the magnitudes of the velocity and acceleration of *C* at this instant.



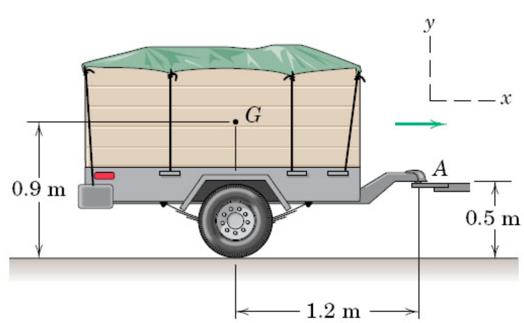
PROBLEM 2

Cars *A* and *B* are rounding the curves with equal speeds of 72 *km/h*. Determine the velocity and acceleration that *A* appears to have to an observer riding in and turning with car *B* for the instant represented.



PROBLEM 3

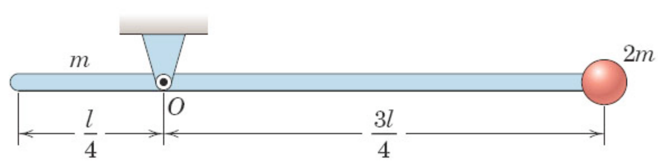
The loaded trailer has a mass of 900 *kg* with center of mass at *G* and is attached at *A* to a rear-bumper hitch. If the car and trailer reach a velocity of 60 *km/h* on a level road in a distance of 30 *m* from rest with constant acceleration, compute the vertical component of the force supported by the hitch at *A*. Neglect the small friction force exerted on the relatively light wheels.



PROBLEM 4

The slender rod of mass *m* and length *l* has a particle (of negligible radius and mass 2*m*) attached to its end. If the body is released from rest when it is in the position shown, determine its angular velocity as it passes the vertical position. The mass moment of inertia about O of the combined rod and particle is:

$$I_O = \frac{61}{48} m l^2$$



Problem 1

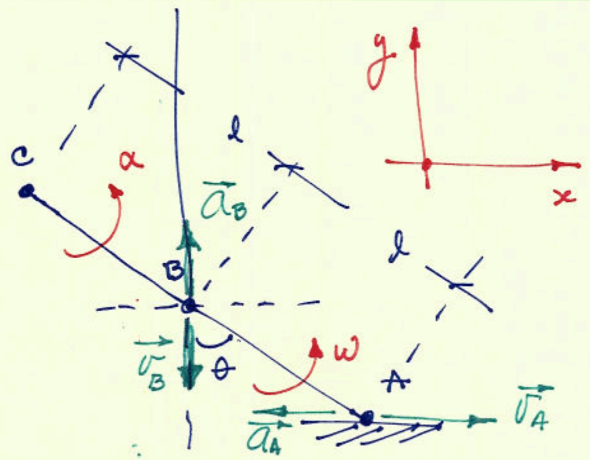
$l = 3'' = 0.25\ ft$

$\vec{v}_B = -3\vec{j}\ ft/s$

$\vec{a}_B = 20\vec{j}\ ft/s^2$

$\theta = 60°$

$\vec{v}_c$ & $\vec{a}_c$



$$* \quad \vec{v}_c = \vec{v}_A + \vec{v}_{C/A} \quad ; \quad \vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$v_A \vec{i} = -v_B j + \omega\vec{k} \times \vec{r}_{A/B} = -v_B \vec{j} + \omega\vec{k} \times l(\sin\theta\,\vec{i} - \cos\theta\,\vec{j})$$

$$v_A \vec{i} = -v_B \vec{j} + \omega l \cos\theta\,\vec{i} + \omega l \sin\theta\,\vec{j}$$

$i: \longrightarrow \quad v_A = \omega l \cos\theta = 1.732\ ft/s$

$j: \longrightarrow \quad v_B = \omega l \sin\theta \longrightarrow \omega = \dfrac{v_B}{l\sin\theta} = 13.856\ rad/s$

$$\vec{v}_c = \vec{v}_B + \vec{v}_{C/B} = \vec{v}_B + \vec{\omega} \times \vec{r}_{C/B} \ ; \ \vec{v}_B = \vec{v}_c + \vec{\omega} \times \vec{r}_{B/C}$$

$$-v_B \vec{j} = \vec{v}_c + \vec{\omega} k \times l(\sin\theta\,\vec{i} - \cos\theta\,\vec{j})$$

$$-v_B \vec{j} = \vec{v}_c + \omega l \cos\theta\,\vec{i} + \omega l \sin\theta\,\vec{j}$$

$$\vec{v}_c = -\omega l \cos\theta\, i - \omega l \sin\theta\, j - v_B \vec{j}$$

$$\vec{v}_c = -\omega l \cos\theta\,\vec{i} - \omega l \sin\theta\,\vec{j} - \omega l \sin\theta\, j$$

$$\vec{v}_c = -\omega l \cos\theta\,\vec{i} - 2\omega l \sin\theta\,\vec{j} = -1.732\vec{i} - 6\vec{j}$$

$$\vec{v}_c = -1.732\vec{i} - 6\vec{j} \longrightarrow |\vec{v}_c| = 6.245\ ft/s$$

$*\quad \vec{a}_A = \vec{a}_B + \vec{\alpha} \times \vec{r}_{A/B} - \omega^2 \vec{r}_{A/B}$

$$-a_A\vec{i} = a_B\vec{j} + \alpha\vec{k} \times l(\sin\theta\vec{i} - \cos\theta\vec{j}) - \omega^2 l(\sin\theta\vec{i} - \cos\theta\vec{j})$$

$$-a_A\vec{i} = a_B\vec{j} + \alpha l\cos\theta\,\vec{i} + \alpha l\sin\theta\,\vec{j} - \omega^2 l\sin\theta\,\vec{i} + \omega^2 l\cos\theta\,\vec{j}$$

$$i: \longrightarrow\quad -a_A = \alpha l\cos\theta - \omega^2 l\sin\theta$$

$$j: \longrightarrow\quad 0 = a_B + \alpha l\sin\theta + \omega^2 l\cos\theta$$

$$\therefore\quad \alpha = -\frac{a_B + \omega^2 l\cos\theta}{l\sin\theta} = -203.227\ \text{rad/s}^2$$

$$\vec{a}_B = \vec{a}_C + \vec{\alpha} \times \vec{r}_{B/C} - \omega^2 \vec{r}_{B/B}$$

$$a_B\vec{j} = \vec{a}_C + \alpha\vec{k} \times l(\sin\theta\vec{i} - \cos\theta\vec{j}) - \omega^2 l(\sin\theta\vec{i} - \cos\theta\vec{j})$$

$$a_B\vec{j} = \vec{a}_C + \alpha l\cos\theta\,\vec{i} + \alpha l\sin\theta\,\vec{j} - \omega^2 l\sin\theta\,\vec{i} + \omega^2 l\cos\theta\,\vec{j}$$

$$i: \longrightarrow\quad (a_C)_x = -\alpha l\cos\theta + \omega^2 l\sin\theta = 66.97\ \text{ft/s}^2$$

$$j: \longrightarrow\quad (a_C)_y = a_B - \alpha l\sin\theta - \omega^2 l\cos\theta = 40\ \text{ft/s}^2$$

$$\vec{a}_C = 66.97\vec{i} + 40\vec{j}\ \text{ft/s}^2$$

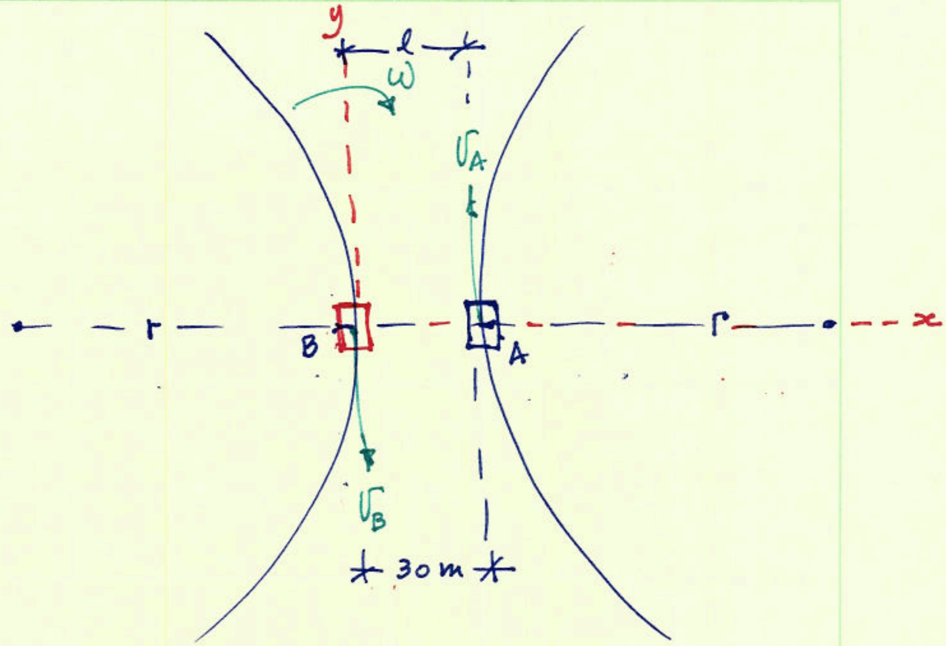
$$|\vec{a}_C| = 78\ \text{ft/s}^2$$

Problem 2

$r = 100\,m$

$v_A = v_B = 20\ m/s$

$l = 30\,m$



* $$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r}_{A/B} + \vec{v}_{A/B_{rel}}$$

$$\vec{v}_{A/B_{rel}} = \vec{v}_A - \vec{v}_B - \vec{\omega} \times \vec{r}_{A/B_{rel}} \quad ; \quad \vec{r}_{A/B_{rel}} = l\vec{i}$$

$$\vec{\omega} = -\omega\vec{k} = -\frac{v_B}{r}\vec{k} = -0.2\vec{k}\ rad/s \quad ; \quad \vec{\omega} \times \vec{r}_{A/B_{rel}} = -6j$$

$$\vec{v}_{A/B_{rel}} = v_A\vec{j} - (-v_B\vec{j}) - (-0.2\vec{k}) \times l\vec{i}$$

$$\vec{v}_{A/B_{rel}} = 20j + 20j + 6j = \underline{\underline{46\vec{j}\ m/s}}$$

* $$\vec{a}_A = \vec{a}_B + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B_{rel}}) + \vec{\alpha} \times \vec{r}_{A/B_{rel}} + 2\vec{\omega} \times \vec{v}_{A/B_{rel}} + \vec{a}_{A/B_{rel}}$$

$$\vec{\alpha} = 0 \quad ; \quad \vec{a}_A = \frac{v_A^2}{r}\vec{i} \quad ; \quad \vec{a}_B = -\frac{v_B^2}{r}\vec{i}$$

$$\vec{\omega} \times (\vec{\omega} \times \vec{r}_{A/B_{rel}}) = -\omega^2 \vec{r}_{A/B_{rel}} = -0.04l\vec{i}$$

$$2\vec{\omega} \times \vec{v}_{A/B_{rel}} = -0.4\vec{k} \times 46\vec{j} = 18.4\vec{i}$$

$$\vec{a}_{A/B_{rel}} = 4\vec{i} + 4\vec{i} + 1.2\vec{i} - 18.4\vec{i} = \underline{\underline{-9.2\vec{i}\ m/s^2}}$$

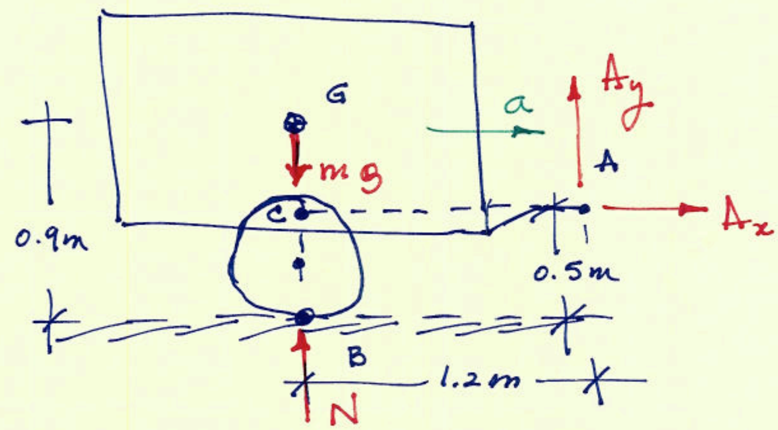
Problem 3

$m = 900\ kg$

$v_0 = 0$; $v_f = 60\ km/h$

$\Delta x = 30m$

$v_f = 16.67\ m/s$



• Constant acceleration : $a = \frac{\Delta v}{\Delta t}$

or $v_f^2 = v_0^2 + 2a(s - s_0) \longrightarrow (16.67\ m/s)^2 = 2a(30m)$

$$a = 4.629\ m/s^2$$

$\Sigma M_c = mad \longrightarrow 1.2$

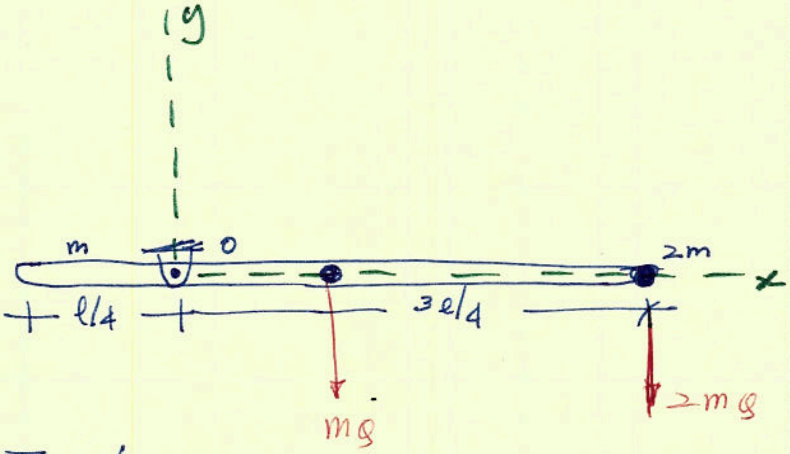
$\Sigma F_x = ma_x \longrightarrow A_x = ma_x = 4166.67$ Newtons

$\Sigma F_y = 0 \longrightarrow N - mg + A_y = 0 \longrightarrow A_y = mg - N$

$\Sigma M_G = 0 \longrightarrow A_x \cdot 0.4 + A_y \cdot 1.2 = 0$

$$A_y = -\frac{0.4\ A_x}{1.2} = -1388.89\ N$$

Problem 4



$$I_0 = \frac{61}{48} m l^2$$

$$T_1 + V_1 + U'_{1-2} = T_2 + V_2$$

$$U'_{1-2} = 0 \quad ; \quad T_1 = 0 \quad ; \quad T_2 = \frac{1}{2} I_0 \omega_2^2$$

$$T_2 = \frac{1}{2}\left(\frac{61}{48} m l^2\right)\omega_2^2 \quad ; \quad r_G = \frac{m(l/4) + 2m\left(\frac{3l}{4}\right)}{3m} = \frac{7}{12} l$$

$$V_1 = 3 m g \cdot \frac{7l}{12}$$

$$0 + 3mg \cdot \frac{7l}{12} = \frac{61}{96} m l^2 \omega_2^2 \longrightarrow \omega_2 = 1.659 \sqrt{\frac{g}{l}}$$