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# CIVE261 ENGINEERING MECHANICS: DYNAMICS

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## 11. KINEMATICS OF PARTICLES

### RECTILINEAR MOTION

$$v = \frac{ds}{dt} = \dot{s}$$

$$a = \frac{dv}{dt} = \dot{v} = \frac{d^2 s}{dt^2} = \ddot{s}$$

$$v\,dv = a\,ds$$

$$\dot{s}\,d\dot{s} = \ddot{s}\,ds$$

where $s$ is displacement, $v$ is velocity, $a$ is acceleration, all of which are, in general, functions of time, $t$.

a) Constant acceleration

$$v = v_o + \int_0^t a\,d\tau = v_o + at$$

$$v^2 = v_o^2 + 2\int_{s_o}^{s} a\,ds = v_o^2 + 2a(s - s_o)$$

$$s = s_o + \int_0^t v(\tau)\,d\tau = s_o + v_o t + \frac{1}{2}at^2$$

where the initial conditions at time $t_o = 0$ are $s_o$, and $v_o$.

b) Variable acceleration as a function of time, $a(t)$:

$$v = v_o + \int_0^t a(\tau)\,d\tau$$

$$s = s_o + \int_0^t v(\tau)\,d\tau$$

c) Variable acceleration as a function of velocity, $a(v)$:

$$t = t_o + \int_{v_o}^{v} \frac{dv}{a(v)}$$

$$s = s_o + \int_{v_o}^{v} \frac{v\,dv}{a(v)}$$

d) Variable acceleration as a function of displacement, $a(s)$:

$$v^2 = v_o^2 + 2\int_{s_o}^{s} a(s)\,ds$$

### PLANE CURVILINEAR MOTION

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}}$$

$$v = |\mathbf{v}| = \frac{ds}{dt} = \dot{s}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}}$$

### RECTANGULAR COORDINATES

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$$v^2 = v_x^2 + v_y^2$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$\tan\theta = \frac{v_y}{v_x}$$

$$a^2 = a_x^2 + a_y^2$$
$$a = \sqrt{a_x^2 + a_y^2}$$

Projectile motion

$$v_x = (v_x)_o$$
$$x = x_o + (v_x)_o t$$
$$v_y = (v_y)_o - gt$$
$$y = y_o + (v_y)_o t - \frac{1}{2}gt^2$$
$$v_y^2 = (v_y)_o^2 - 2g(y - y_0)$$

Trajectory equation:

$$y = x\tan\theta - \frac{gx^2}{2v_o^2\cos^2\theta}$$

corresponding to an initial position $x_o$ = 0; $y_o$ = 0; initial velocity of magnitude $v_o$ and making an angle $\theta$ with the horizontal. Or, alternatively,

$$y - y_o = (x - x_o)\tan\theta - \frac{g(x - x_o)^2}{2v_o^2\cos^2\theta}$$

corresponding to an initial position $x_o$, $y_o$; initial velocity of magnitude $v_o$ and making an angle $\theta$ with the horizontal.

## NORMAL AND TANGENTIAL COORDINATES

$$\mathbf{v} = v\mathbf{e_t} = \rho\dot{\beta}\mathbf{e_t}$$

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e_t} + \frac{v^2}{\rho}\mathbf{e_n}$$
$$a_n = \frac{v^2}{\rho}$$
$$a_t = \frac{dv}{dt} = \dot{v} = \ddot{s}$$
$$a = \sqrt{a_n^2 + a_t^2}$$

## RADIAL AND TRANSVERSE COORDINATES

$$\mathbf{r} = r\mathbf{e_r}$$
$$\dot{\mathbf{e}}_\mathbf{r} = \dot{\theta}\mathbf{e}_\theta$$
$$\dot{\mathbf{e}}_\theta = -\dot{\theta}\mathbf{e_r}$$
$$\mathbf{v} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e}_\theta$$
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$\mathbf{v} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e}_\theta$$
$$v_r = \dot{r}$$
$$v_\theta = r\dot{\theta}$$
$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$a = \sqrt{a_r^2 + a_\theta^2}$$

Circular motion

$$v = r\dot{\theta}$$
$$a_n = v^2 / r = r\dot{\theta}^2 = v\dot{\theta}$$
$$a_t = \dot{v} = r\ddot{\theta}$$

## RELATIVE MOTION (Translating Axes)

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$
$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$
$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

## Law of Sines

$$\frac{\sin\alpha_a}{a} = \frac{\sin\alpha_b}{b} = \frac{\sin\alpha_c}{c}$$

## Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha_a$$