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CIVE261 ENGINEERING MECHANICS: DYNAMICS

Spring 2009

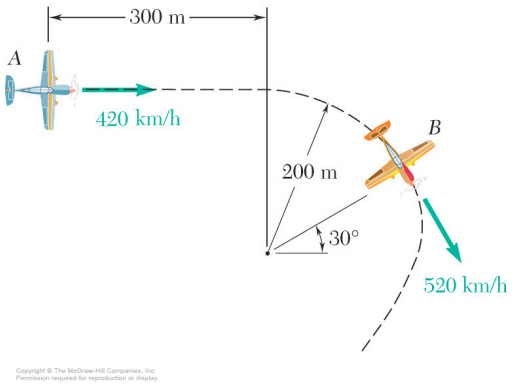
FIRST MIDTERM EXAM

Tuesday, February 17, 2009 – 5:30-7:00 PM Room 120 Engineering

You **may not consult** any books, notes, or inanimate references. You **may not consult** with another person. You **may not copy** another student's solutions.

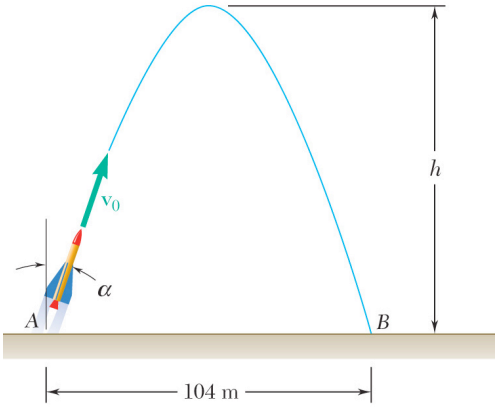
PROBLEM 1 (30 points)

Airplane *A* is flying horizontally in a straight line, and its speed is being increased at a rate of 6 m/s^2. Airplane *B* is flying at the same altitude as airplane *A* and is following a circular path of 200-*m* radius. Knowing that at the given instant the speed of *B* is being decreased at the rate of 2 m/s^2, determine, for the positions shown, (*a*) the velocity of *B* relative to *A*, (*b*) the acceleration of *B* relative to *A*.



PROBLEM 2 (30 points)

A model rocket is launched from point *A* with an initial velocity $\mathbf{v}_0$ and at an angle $\alpha = 3.96°$. If the rocket's descent parachute does not deploy and the rocket lands 104 *m* from *A*, determine (*a*) the magnitude of $\mathbf{v}_0$, (*b*) the maximum height *h* reached by the rocket, (*c*) the duration of the flight.



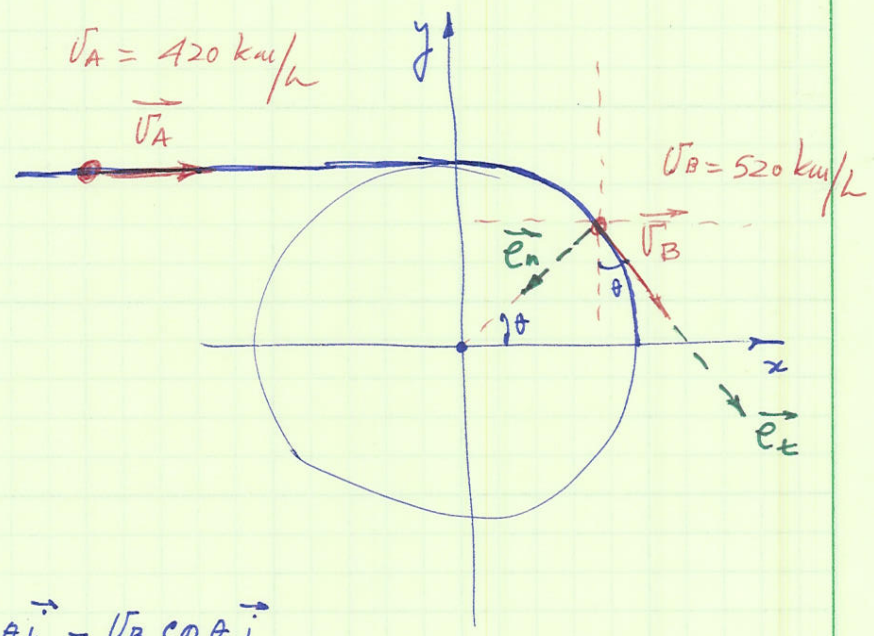
PROBLEM 3 (20 points)

A motorist is traveling on a curved portion of highway of radius 350 *m* at a speed of 72 *km/h*. The brakes are suddenly applied, causing the speed to decrease at a constant rate of 1.25 m/s^2. Determine the magnitude of the total acceleration of the automobile (*a*) immediately after the brakes have been applied, (*b*) 4 *s* later.

PROBLEM 4 (20 points)

A sprinter in a 400-*m* race accelerates uniformly for the first 130 *m* and then runs with constant velocity. If the sprinter's time for the first 130 *m* is 25 *s*, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

Problem 1



Velocity:

$\vec{v}_A = v_A \vec{i}$

$\vec{v}_B = v_B \vec{e}_t$

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$

Thus,

$\vec{v}_{B/A} = -v_A \vec{i} + \underbrace{v_B \sin\theta \vec{i} - v_B \cos\theta \vec{j}}_{\vec{v}_B \text{ in } x\text{-}y \text{ frame}}$

$\vec{v}_{B/A} = (v_B \sin\theta - v_A)\vec{i} - v_B \cos\theta \vec{j} = -160 \text{ km/h}\, \vec{i} - 450.3 \text{ km/h}\, \vec{j}$

$|\vec{v}_{B/A}| = 477.9 \text{ km/h} \swarrow 70.4°$

Acceleration:

$\vec{a}_A = a_A \vec{i} = 6 \text{ m/s}^2 \vec{i}$

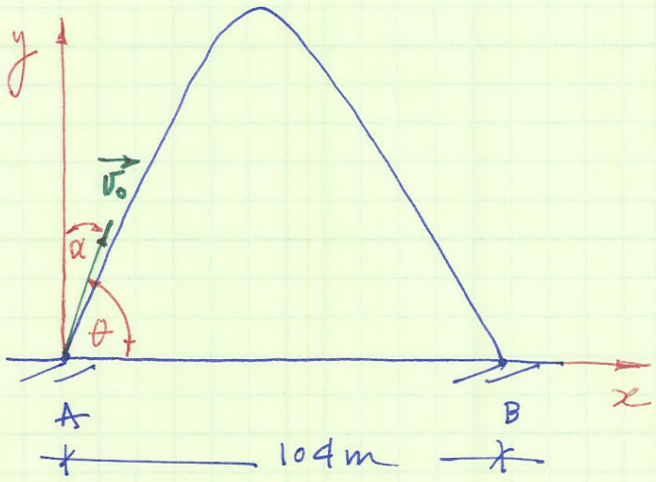
$\vec{a}_B = a_t \vec{e}_t + a_n \vec{e}_n = \frac{dv_B}{dt} \vec{e}_t + \frac{v_B^2}{\rho} \vec{e}_n = -2 \text{ m/s}^2 \vec{e}_t + 104.3 \text{ m/s}^2 \vec{e}_n$

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = \underbrace{-2(\sin\theta \vec{i} - \cos\theta \vec{j}) + 104.3(-\cos\theta \vec{i} - \sin\theta \vec{j})}_{\vec{a}_B \text{ in } x\text{-}y \text{ frame}} - 6\vec{i}$

$\vec{a}_{B/A} = -97.34 \text{ m/s}^2 \vec{i} - 50.42 \text{ m/s}^2 \vec{j}$

$|\vec{a}_{B/A}| = 109.6 \text{ m/s}^2 \swarrow 27.4°$

Problem 2



a)

Trajectory equation:

$$y - y_0 = (x - x_0)\tan\theta - \frac{g(x-x_0)^2}{2v_0^2\cos^2\theta}$$

$y_0 = 0$; $x_0 = 0$; For $x = 104\ m \longrightarrow y = 0$, then

$$0 = 104\tan\theta - \frac{g(104m)^2}{2v_0^2\cos^2\theta} \quad ; \quad \theta = 90° - \alpha$$

Solve for v_0:

$$v_0^2 = \frac{g(104m)}{2\tan\theta\cos^2\theta} = 7404.29\ m^2/s^2$$

$$v_0 = 86.04\ m/s$$

b) $v_y^2 = (v_0)_y^2 - 2g\Delta h \longrightarrow \Delta h = y - y_0$

y_{max} occurs for $v_y = 0 \longrightarrow 0 = (v_0\sin\theta)^2 - 2g\,y_{max}$

$$y_{max} = \frac{(v_0\sin\theta)^2}{2g} = 375.6\ m$$

c) $$t = \frac{x_B}{v_x} = \frac{104\ m}{v_0\cos\theta} = 17.5\ s$$

Problem 3

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n = a_t\vec{e}_t + a_n\vec{e}_n$$

$a_t = \frac{dv}{dt} = -1.25\ m/s^2$; $\rho = 350\ m$

At $t = 0 \longrightarrow v = v_0 = 72\ km/h = 20\ m/s$

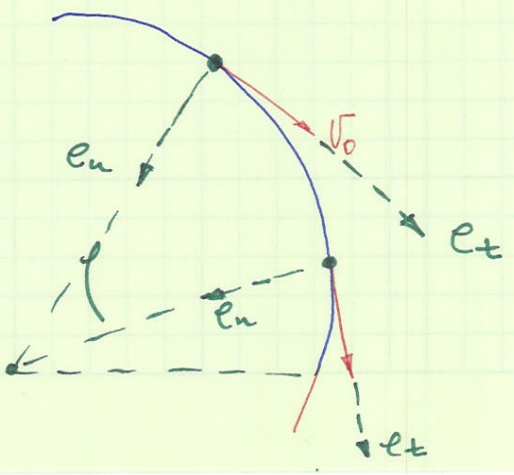
At $t = 4\ s \longrightarrow v = v_0 + a_t t = 20\ m/s + \left(-1.25\ m/s^2\right) t$

$$v = 15\ m/s$$

Thus, magnitude of total acceleration :

At $t = 0$ $\quad a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-1.25)^2 + \left[\frac{(20)^2}{350}\right]^2} = 1.694\ m/s^2$

At $t = 4\ s$ $\quad a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-1.25)^2 + \left[\frac{(15)^2}{350}\right]^2} = 1.406\ m/s^2$



Problem 4

- Rectilinear motion
- $v_0 = 0$; $x_0 = 0$
- Constant acceleration from $x_0 = 0$ to $x = 130\,m$ and $\Delta t = 25\,s$

a) Acceleration:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$130 = \frac{1}{2} a (25)^2 \longrightarrow a = \frac{2 \cdot 130}{(25)^2} = 0.416\ m/s^2$$

b) Final velocity:

$$v = v_0 + at = 0 + 0.416 \times 25 = 10.4\ m/s$$

c) Time to run 400 m:

* Time during acceleration phase: $\Delta t_1 = 25\,s$

* Time during rest of race:

$$\Delta t_2 = \frac{400 - 130}{v} = \frac{270}{10.4} = 25.96\,s$$

$$t = 25 + 25.96 = 50.96\,s$$