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COMPUTER MODELING OF SOIL EROSION AND SEDIMENT YIELD FROM LARGE WATERSHEDS

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ABSTRACT

Mathematical modeling techniques for predicting soil losses by overland flow and sediment yield from large watersheds are reviewed and three models are presented.

1) A daily simulation model is first discussed. The regression analysis of suspended load and discharge data gives the best results when the analysis of residuals is included.

Mathematical models based on physical characteristics of watersheds figure among the most powerful tools because the rate of sediment transport can be predicted for alternative watershed conditions. Two mathematical models, LAVSED-I and LAVSED-II (LAVal SEDimentological model), were developed for a long-term simulation of sedi-

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ment sources and yields based on topography, precipitation, soil and vegetation.

2) The model LAVSED-I uses the well-known Universal Soil Loss Equation. Maps of the mean annual soil-loss are plotted and a case study is discussed.

3) The model LAVSED-II predicts the wash load in streams as a function of the rate of sediment transport from upland areas. A stochastic component for precipitation and snowmelt runoff is combined with deterministic relationships for overland flow and sediment transport. The average sediment yield is predicted for each month. Validation of the model on several large watersheds in Canada shows excellent agreement between the computed sediment yield and the measured suspended load.

These computer models offer various complementary results which prove to be extremely useful, for instance in management of large watersheds, reservoir sedimentation, long-term changes in land use, soil conservation practices, effect of deforestation and reforestation.

INTRODUCTION

A large fraction of the sediments transported to rivers and reservoirs is eroded from upland areas. Soil erosion is one of the major hazards threatening the productivity of farmlands. The amount of sediment carried in the fluvial system is usually governed by the availability of the upstream supply from watersheds rather than the transport capacity of rivers. Upland erosion pollutes surface waters and often causes serious problems when deposition occurs. The physical processes governing the movement of sediments by rainfall and snowmelt are very complex. In addition to rainfall erosion, sediment transport during snowmelt may significantly increase the sediment load in streams located in mountainous areas or in northern countries. The snow cover protects the frozen ground from soil erosion and usually melts during a relatively short period of time. In some countries, like Canada, the sediment load in rivers peaks during the spring flood while rainfall erosion controls the sediment load in streams only during summer.

This paper reviews the current techniques for predicting soil losses by overland flow and sediment yield from large watersheds. The physical processes are briefly summarized with constitutive relationships and mathematical modeling techniques for overland runoff and sediment transport are presented.

PHYSICAL PROCESSES

The rate of erosion on a particular land area can be determined from the complex interrelations of several factors. These factors include the erosive forces of rainfall and runoff, and the soil resistance to detachment and transport. The detachment and transport of soil particles under the action of raindrop impact and runoff is schematized in Fig. 1. Loose sediment particles are carried downslope by surface runoff and the sediment transport capacity is controlled by the overland flow variables. The snowmelt water percolates through the snow cover until the saturated layer is reached. As the water flows through the snowpack, micro channels develop to gradually yield free surface flow conditions.

Rainfall Characteristics

As suggested by Eagleson (1978) and Todorovic (1968), point rain-

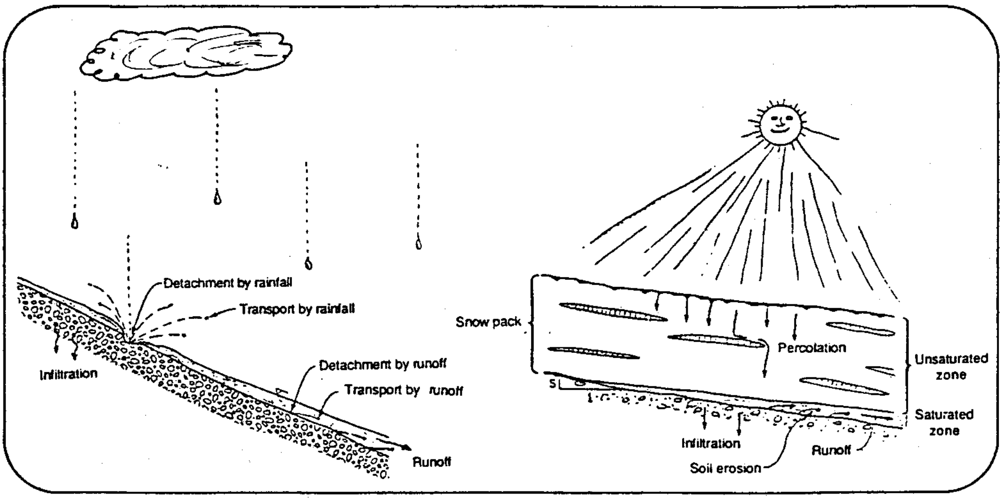


Fig. 1 Soil erosion processes during rainfall and snowmelt

fall can be described as a random series of discrete storm events of finite duration and constant intensity. The two principal variables are the storm duration t_r and intensity i. These two variables have been shown by Julien and Frenette (1985) to be nearly independent and distributed exponentially with probability density functions $p(t_r)$ and $p(i)$ written as:

$$p(t_r) = \lambda_1 e^{-\lambda_1 t_r} \tag{1}$$

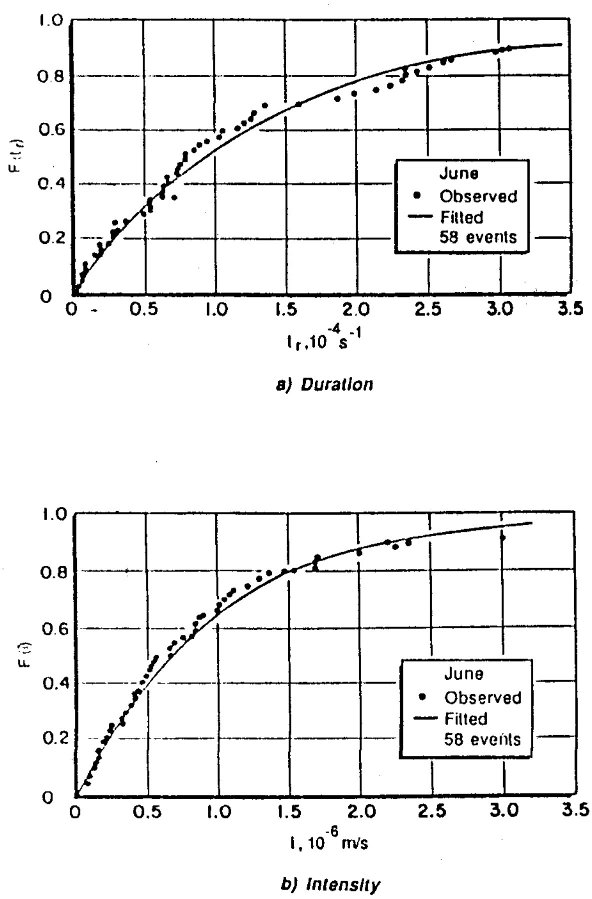
and

$$p(i) = \lambda_2 e^{-\lambda_2 i} \tag{2}$$

The parameters for rainfall duration λ_1 and intensity λ_2 are evaluated for each month. Examples of distribution of rainfall duration and intensity near Québec City, Canada, are shown in Figure 2.

Overland Flow Characteristics

The rainfall-runoff relationship is then considered. After the



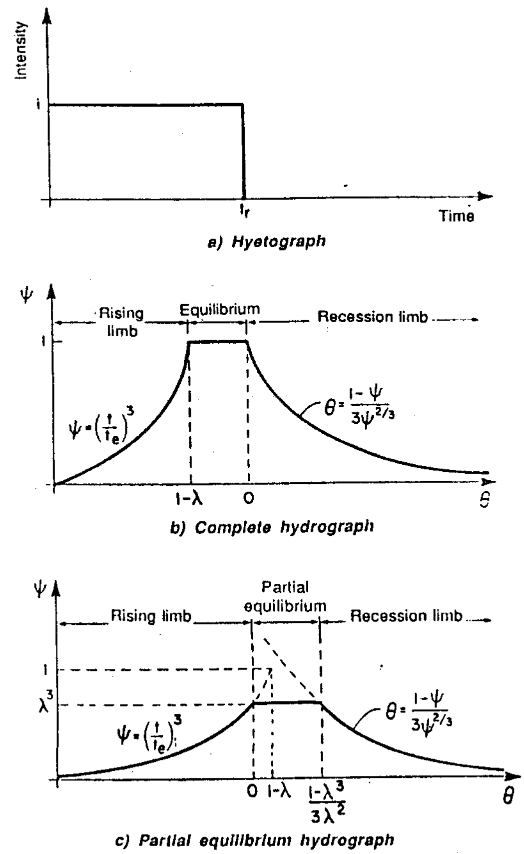


Fig. 2 Exponential distribution function of rainfall characteristics

Fig. 3 Rainfall-runoff relationship

infiltration rate is subtracted from the rainfall intensity, the excess rainfall initiates surface runoff. Though the formation of rills over irregular land surface is generally observed, mathematical modeling techniques are based on the properties of sheet flows. The resulting hydrograph is subdivided into three parts: the rising limb, the equilibrium and the falling limb. A detailed analysis of soil erosion including both complete hydrographs and partial equilibrium hydrographs, schematized in Fig. 3, has been conducted at Laval University and a theoretical solution in terms of series expansion was derived by Julien and Frenette (1985). They concluded that in most cases the first term in the series is equivalent to the complete solution. This first term representing the equilibrium flow conditions has therefore been recommended for practical use. The equilibrium flow conditions in terms of velocity $\bar{u}$, flow depth h and bed shear stress τ_o can be written as power functions of slope s and discharge q:

$$\bar{u} = \left(\frac{8g}{k\nu}\right)^{1/3} s^{1/3} q^{2/3} \qquad (3)$$

$$h = \left(\frac{k\nu}{8g}\right)^{1/3} s^{-1/3} q^{1/3} \qquad (4)$$

$$\tau_o = \gamma \left(\frac{k\nu}{8g}\right)^{1/3} s^{2/3} q^{1/3} \qquad (5)$$

in which γ is the specific weight of the fluid; k is the friction coefficient; ν is the kinematic viscosity of the fluid; and g is the gravitational acceleration.

Snowmelt Runoff

Snowmelt runoff from an experimental plot under natural conditions was investigated at Laval University. The cumulative snowmelt F computed from hourly runoff data was correlated to three factors: 1) the cumulative number of degree-days D_j in ^oC-d; 2) the cumulative time when temperature is above 0^oC, t_h in h; and 3) the cumulative time of snowmelt t_f in sec. The following empirical relationships shown in Fig. 4 were obtained:

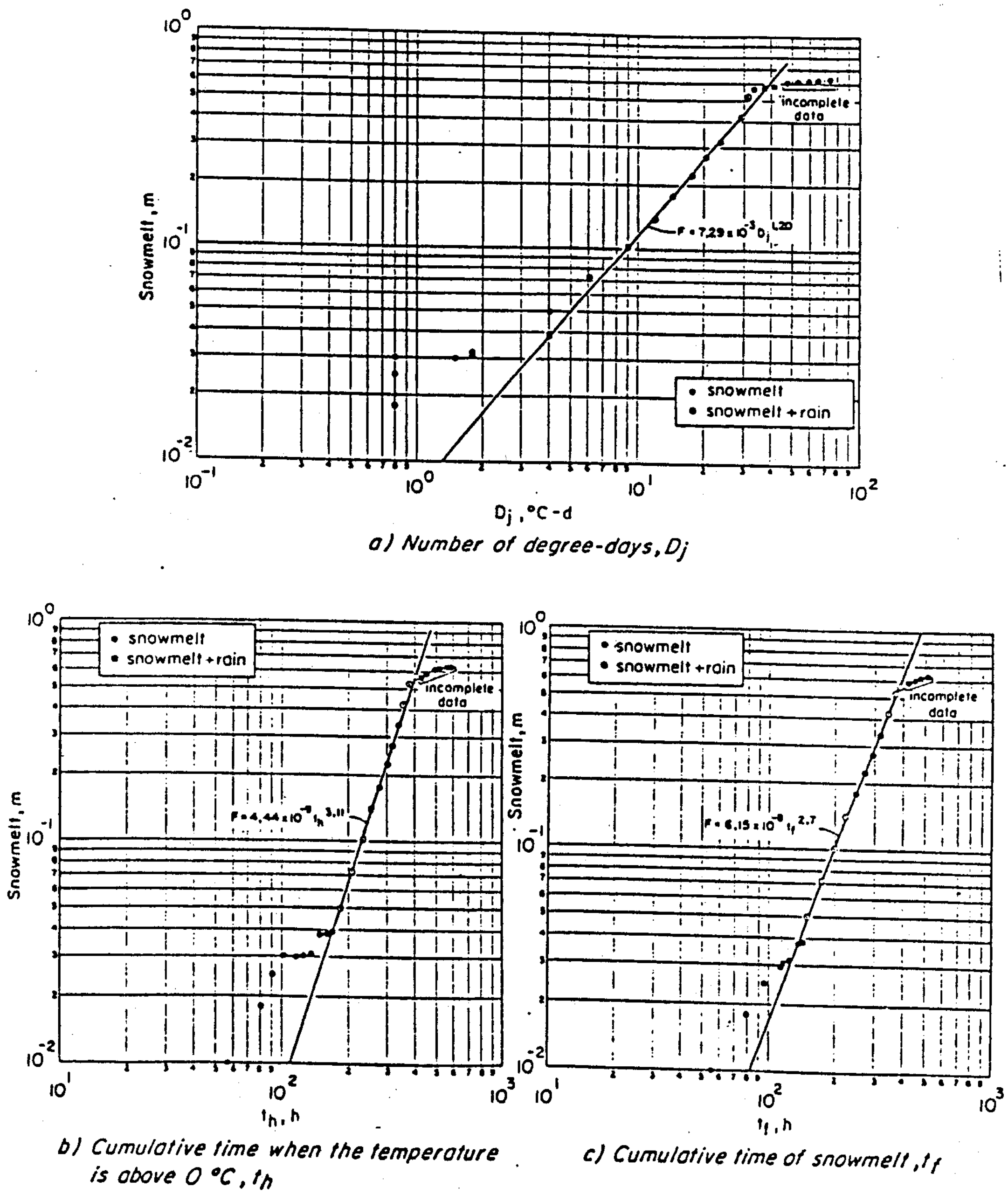


Fig. 4 Cumulative snowmelt as a function of three parameters

$$F = 7.29 \times 10^{-3} D_j^{1.2} \tag{6}$$

$$F = 4.44 \times 10^{-9} t_h^{3.11} \tag{7}$$

$$F = \alpha_f \, t_f^{\beta_f} = 1.54 \times 10^{-17} t_f^{2.7}; \; t_f \text{ in sec} \tag{8}$$

in which α_f and β_f are the snowmelt parameters.

Sediment Transport by Overland Flow

The erosion rate can be estimated from empirical equations such as the well-known USLE,an acronym for the Universal Soil Loss Equation (Wischmeier and Smith, 1978). The USLE gives the mean annual soil loss by rainfall erosion from sheet flow and rill erosion. This equation is a function of several factors and can be written as:

$$E = \alpha \, RKLSCP \tag{9}$$

E = annual soil loss kt/km^2
R = rainfall and runoff factor
K = soil erodibility factor
L,S = topographic factors
C = cover and management factor
p = support practice factor
α = 0.247 coefficient to transform units t/acre into kt/km^2.

These factors are evaluated from nomographs (Wischmeier and Smith, 1978). The USLE has been modified by Williams and Berndt (1975) to include the runoff variables. The modified variable R is a function of the product of the total runoff volume and the peak discharge raised to the 0.56 power.

The sediment transport capacity of overland flow depends on the flow characteristics. The method of dimensional analysis was used to reduce the number of significant variables related to sheet erosion. As a result the sediment discharge written in a dimensional form is:

$$q_s = \alpha\, s^{\beta}\, q^{\gamma}\, i^{\delta}\, \left(1 - \frac{\tau_c}{\tau_o}\right)^{\varepsilon} \qquad (10)$$

in which the coefficients, $\alpha, \beta, \gamma, \delta$ and ε can be determined experimentally. Several sediment transport equations have been transformed into the form of Eq. 10 to define the coefficients shown in Table 1. Julien (1982) and Julien and Simons (1984) found that the coefficient β varies from 1.2 to 1.9 while γ varies from 1.4 to 2.4. In the case of the Universal Soil Loss Equation the approximate values of the coefficient $\beta \cong 1.7$ and $\gamma = 1.5$ compare very well with sediment transport relationships from Table 1. The following coefficients from Kilinc and Richardson (1973) were used in the model LAVSED II for predicting the rate of sediment transport q_s in t/m.s from q in m^2/s; $\alpha = 25500$, $\beta = 1.66$, $\gamma = 2.03$ and $\delta = 0$. In the model LAVSED I, the regression equation from Kilinc and Richardson (1973) has been modified to account for vegetation, soil erodibility and the crop and management factor. The modified relationship can be written as:

$$q_s = e^{-11.65} Re^{2.05} s^{1.46} \frac{K}{0.15} CP \qquad (11)$$

in which Re is the Reynolds number and s is the slope. This relationship combines runoff and topographical factors (Re, s) with three empirical factors from the USLE (K, C, and P).

Sediment-Delivery Ratio

The rate at which eroded material is delivered to river systems and oceans is often much less than the rate at which it is eroded from

Table 1. Transformation of several erosion equations.

Eq. No.	Reference	Equation	α^a	β	γ	δ	ε
22	Musgrave (1947)	$q_s = \alpha' S^m L^n i^P$	α'	m	n	p-n	0
23	Zingg (1940)	$q_s \propto L^{1.66} S^{1.37}$	--	1.37	1.66	-1.66	--
24	Wischmeier and Smith (1978)	$q_s \propto L^{1.5}\ (.00076S^2 + .0053S + .0076)$	--	≅1.7	1.5	-1.5	--
25	Meyer and Monke (1965)	$q_s \propto L^{1.9} S_o^{3.5}$	--	3.5	1.9	-1.9	--
26	Young and Mutchler (1969)	$q_s \propto L^{2.24} S^{0.74}$	--	0.74	2.24	-2.24	--
27	Li et al. (1973)	$q_s = \alpha' \int_o^L \tau_o^2\, dx$	$3\ \alpha'\ \frac{\gamma^2}{5}\left(\frac{K\nu}{8g}\right)^{2/3}$	1.33	1.67	-1	0
28	Komura (1983)	$q_s \propto q^{11/8}\ i^{1/2}\ S^{1.5}$	--	1.5	1.38	0.5	0
29	Kilinc (1972)	$q_s = e^{2.05}(\tau_o - \tau_c)^{2.78}$	$e^{2.05}\ \gamma'^{2.78}\left(\frac{K\nu^2}{8g}\right)^{0.93}$	1.86	0.93	0	2.78
30	Kilinc (1972)	$q_s = e^{0.122}\ ((\tau_o - \tau_c)\bar{u})^{1.67}$	$e^{0.122}\ \gamma^{1.67}$	1.67	1.67	0	1.67
31	Kilinc (1972)	$q_s = e^{-3.17}\ \bar{u}^{-3.625}$	$e^{-3.17}\ (\frac{8g}{K\nu_e})^{1.21}$	1.21	2.42	0	0
32	Kilinc (1972)	$q_s = e^{1.24}\ \bar{u}^{-4.67}\ Re^{-0.878}$	$e^{1.24}\ \nu^{0.878}(\frac{8g}{K\nu})^{1.56}$	1.56	2.24	0	0
33	Kilinc (1972)	$q_s = e^{-11.6}\ Re^{2.05}\ S^{1.46}$	$e^{-11.6}\ \nu^{-2.05}$	1.46	2.05	0	0
34	Kilinc (1972)	$q_s = e^{11.7}\ q^{2.035}\ S^{1.66}$	$e^{11.7}$	1.66	2.03	0	0

[a]Sediment discharge in pounds per ft-sec.

the land surface. The bulk of sediment is deposited at intermediate locations wherever the runoff waters are insufficinet to sustain transport. The percentage of sediment delivered from the erosion source, including gully erosion and streambank erosion, to any specified downslope location defines the sediment-delivery ratio. There are no generalized delivery relationships applicable to every situation. The decrease in sediment-delivery ratio with increasing drainage area was observed for most locations by several researchers.

COMPUTER MODELING TECHNIQUES

To simulate the combined processes of rainfall-runoff and sediment runoff, Negev (1967) developed a sediment model coupled with the Stanford Watershed Model IV. The Negev Model can be considered as an extension of the sediment-rating curve approach. More recently, Simons et al. (1982) developed a computer program based on physical processes to route water and sediments from relatively small watersheds. The model MULTSED simulates single storm events and works best when base flow is relatively small. Sediment is routed by size fractions and several alternatives can be quickly evaluated by altering the appropriate physical parameters. The model CREAMS (Knisel, 1980) has been developed by the Staff of the Science and Education Administration-Agricultural Research to evaluate nonpoint source pollution for field-scale areas.

Stochastic models for predicting sediment yield have been extended by Woolhiser and Todorovic (1974) without extensive testing against field data. The mathematical modeling techniques developed at Laval University can be classified as follows:

a) Regression models based on measured sediment load and discharge on a daily basis,

b) Empirical techniques based on the principal characteristics of watersheds,

c) Modeling techniques combining stochastic rainfall processes and deterministic runoff and sediment transport.

These models have been used extensively on several large watersheds in the province of Quebec, Canada (see Fig. 5).

Models Based on Sediment Load and Discharge Data

Sediment discharge in a stream is commonly evaluated by integration of the product of depth-integrated concentration and unit discharge along the channel width. The daily sediment load is then estimated from the product of the average sediment concentration by the total daily volume of water. If the sediment discharge or the concentration is plotted against the water discharge, a power relationship can be fitted through the data:

$$Q_s = \alpha Q_l^{\beta} \tag{12}$$

in which α and β are obtained by regression analysis. Hysteresis effects between discharge and concentration, seasonal variation, inaccuracies in flow and sediment measurements and variability in the wash load may explain the scatter of points on the sediment transport graph. Better results may sometimes be achieved when sufficient data is available and a regression analysis is made for each month. Rising and falling parts of each hydrograph can also be analyzed separately. For a given discharge, higher sediment discharge rates are generally observed during the rising limb of the hydrograph. Streambank erosion effects can sometimes be separated from upland sediment sources. Snowmelt erosion rates can be compared to rainfall erosion rates. Examples are given in

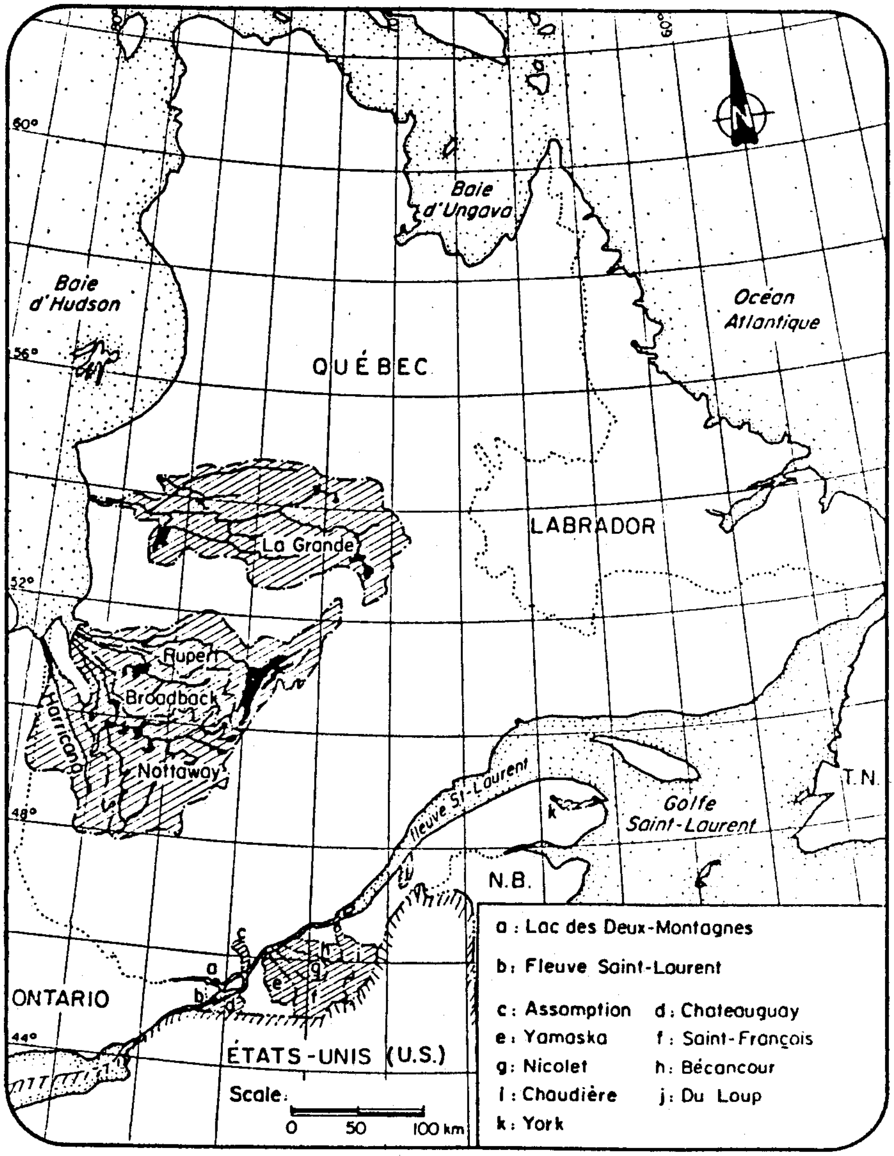


Fig. 5 Location of watersheds

Figures 6, 7, and 8.

The analysis of measured sediment load also gives the sediment yields of watersheds for consecutive years and extreme sediment loads can be assessed. However, short-term predictions of the

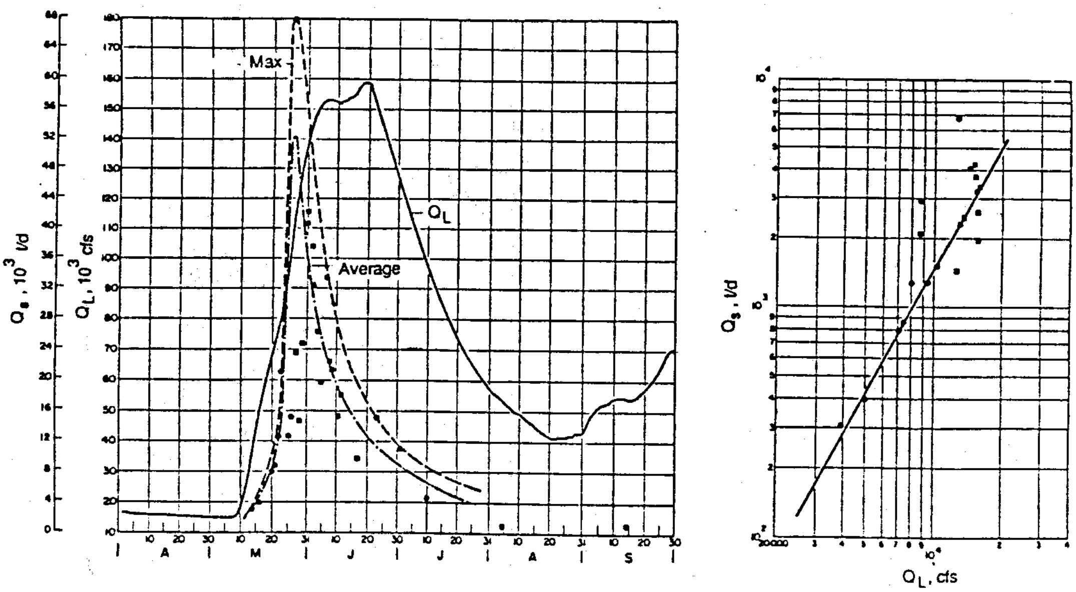


Fig. 6 Suspended load in La Grande River, 1974

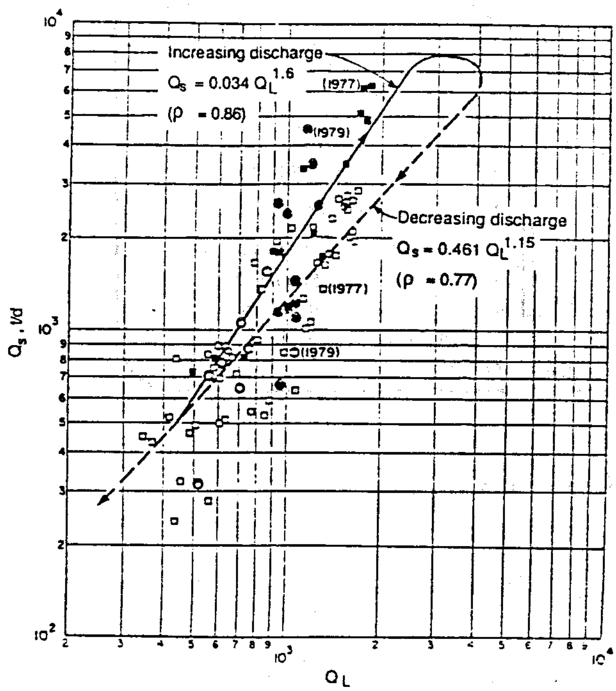


Fig. 7 Sediment rating curves, Bell River, 1977-1979

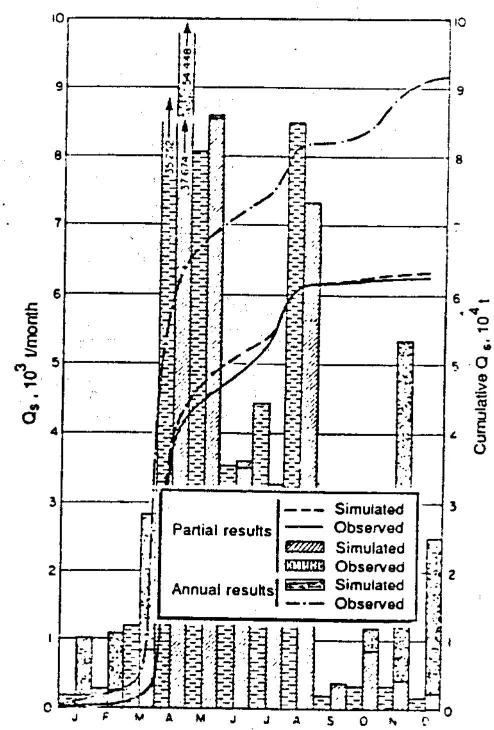


Fig. 8 Sediment loads in the Chateauguay River, 1972

sediment load lack accuracy unless the residuals are considered in the analysis. Daily sediment load can be predicted from one of the following two methods: a linear model and a nonlinear model.

a) Linear modeling of daily sediment load

The sediment load is assumed to have a linear relationship with water discharge. The best relationship (Frenette et al.,1974; Frenette and Nzakimuena, 1976) found for several rivers in Canada is:

$$C_{i+1} = \alpha_0 + \alpha_1 C_i + \alpha_2 Q_{i+1} + \alpha_3 Q_i + \bar{S}_k A^* \tag{13}$$

in which

C_{i+1}	sediment concentration of day i+1 in mg/L
C_i	sediment concentration of day i in mg/L
Q_i	water discharge of day i in ft^3/s
Q_{i+1}	water discharge of day i+1 in ft^3/s
$\alpha_{0,1,2,3}$	multiple linear regression coefficients
A^*	random numbers following a normal distribution N (0,1)
$\bar{S}_k$	standard deviation of residuals in class k
k	class to which the discharge Q_{i+1} belongs

For simulation purposes the variance was modified for different classes of water discharge since for this linear model, the third and fourth moments of the distribution of residuals are significantly high.

b) Nonlinear modeling of daily sediment load

This nonlinear modeling technique can be written in the general form with unknown exponents as:

$$C_{i+1} = C_i^{\alpha_1} Q_{i+1}^{\alpha_2} Q_i^{\alpha_3} e^{\alpha_0 + \bar{S}_k A^* + \bar{E}_k} \qquad (14)$$

in which $\bar{E}_k + \bar{S}_k A^*$ is the logarithm of the residual simulated. The variables $\bar{E}_k$ and $\bar{S}_k$ represent the mean value and the standard variation of the residual for the class k.

The daily sediment load is computed from the following relationship:

$$Q_{s_{i+1}} = 0.0027\, Q_{i+1} C_{i+1} \qquad (15)$$

The agreement between computed and observed sediment load is shown in Figure 9 for two rivers in Canada: Chaudière River, 1969-74 and York River, 1973-74.

These two models reflect the importance of peak discharge in sediment transport. The nonlinear model has been shown to be better suited for predicting suspended load. The main advantage of this model lies in its direct relationship with observed sediment data. Homogeneity of water and sediment data are assumed and the sediment transport rate is not related to the physical characteristics of watersheds. This methodology is therefore not appropriate for watersheds and river systems under significant modifications in land use.

Model LAVSED-I Based on the Characteristics of Watersheds

In this model (Frenette and Julien, 1984) the sediment yield from watersheds is predicted as a function of the sediment sources from upland areas. Such models are applicable when upland erosion is the major source of sediments in the river system. A watershed is subdivided in small homogeneous unit areas of the order of one hectare. Each parameter influencing the rate of soil erosion must be considered and the evaluation of each unit cell is required. The following variables must be determined: climate (rainfall, snow),

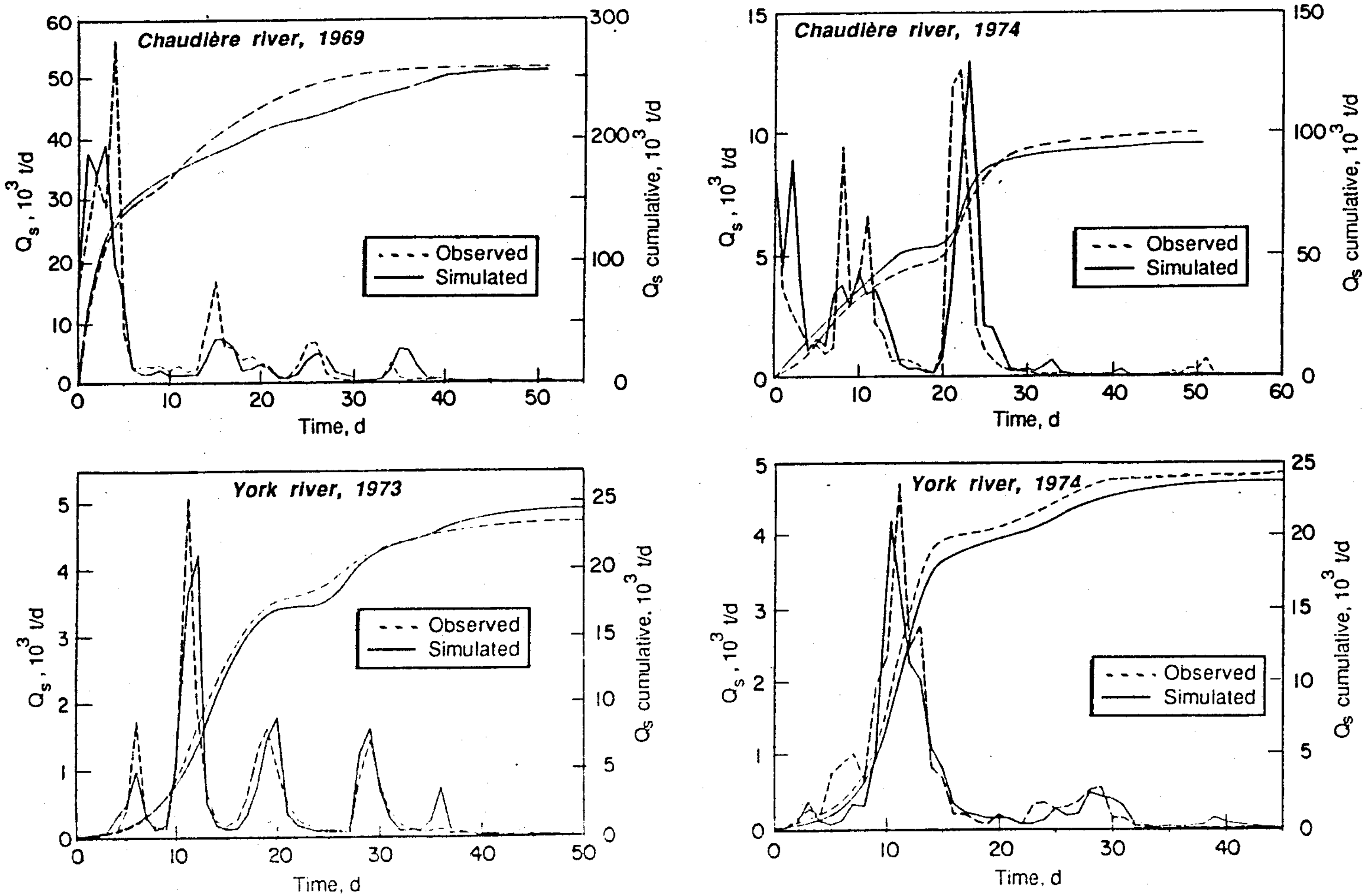


Fig. 9 Daily simulation of the sediment loads in the Chaudière and York Rivers, Canada

topography, soil type, vegetation (trees, shrubs and grass), land use, and runoff characteristics. In many instances these data are available in reports, maps and raw data on meteorology, geology, soil science, topography, forestry, and hydraulics. This information may be completed with LANDSAT, aerial photographs and field trips.

The computation of soil erosion from large watersheds can be handled easily with computers. A fixed square grid system is superimposed on the watershed to define the plan geometry of each unit cell. Appropriate data in terms of meteorology, runoff, topography, soil type, vegetation management practice must be coded for the computerized evaluation of the mean annual soil erosion under natural conditions. Potential erosion rates can be simulated with the sediment yield from the watershed. The basic information on the rate of soil erosion and sensitivity to soil erosion under various hypothetical land use practices are computed with the sediment yield in the river system. The example of the Chaudiere River in Canada (Fig. 10) has been selected to illustrate the use of this model. The total drainage area of the watershed is 6684 km^2, and a sediment gauging station is located near the mouth of the river (A= 5830 km^2). The following four phases are considered for modeling:

a) Analysis of physical and meteorological data for each unit cell of the watershed,
b) Influence of the grid size on soil erosion calculations,
c) Sediment transfer on the watershed,
d) Prediction of actual and potential erosion rates and sediment yield from the watershed.

a) The use of small unit areas of one hectare would require nearly a miliion cells to cover the entire watershed. Obviously, even with

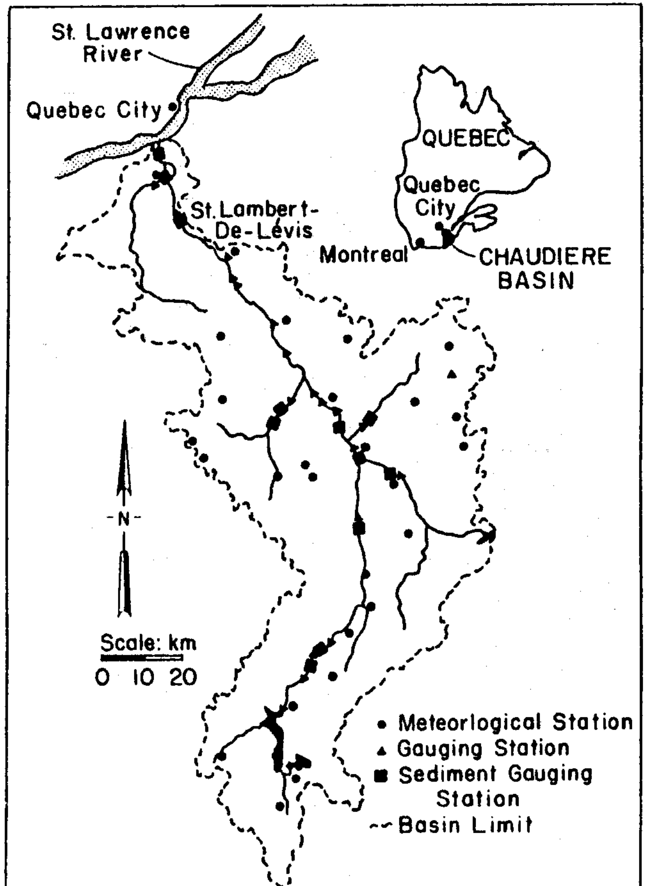


Fig. 10 Chaudière basin

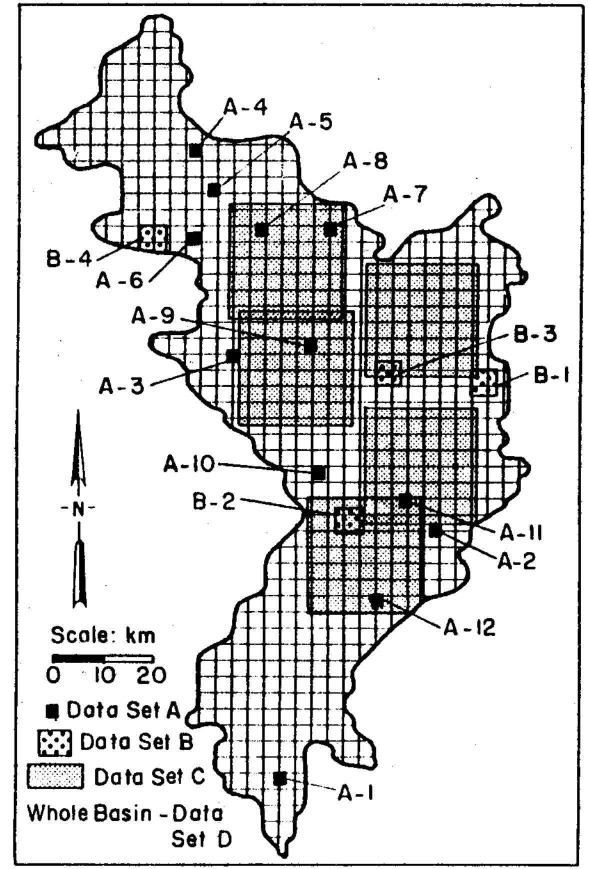


Fig. 11 General scales of the grids

the aid of computers, such a small grid size is not desirable because of the labor involved in collecting the data. The watershed was therefore subdivided in 1671 square cells covering 4 km^2 each as shown in Fig. 11. The factors used in the soil erosion computations were digitized and stored in data files. The relative importance of two variables is illustrated in Fig. 12.

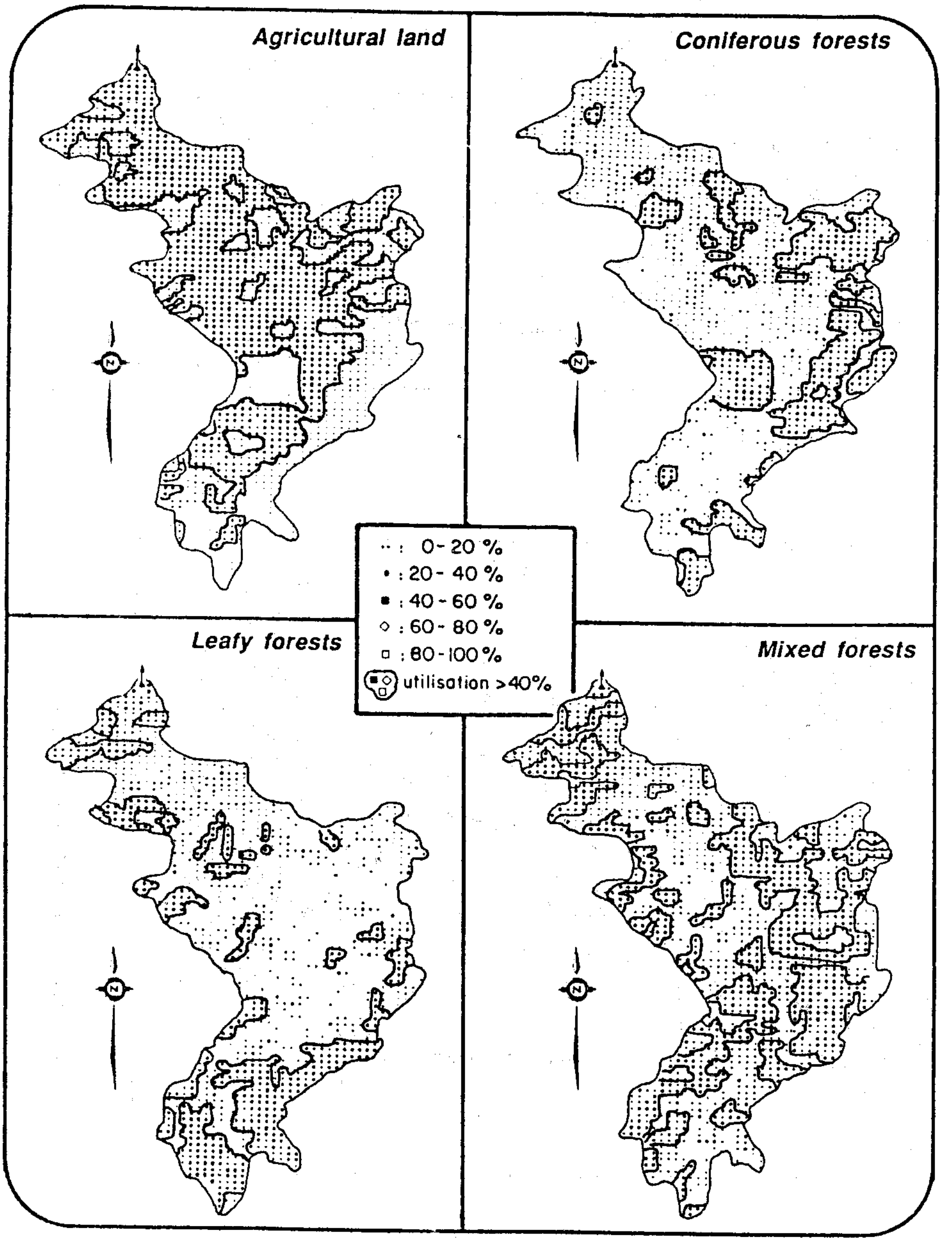


Fig. 12 Chaudière basin, percentage of land uses

b) A correction factor Q_e taking into consideration the influence of the grid size on the evaluation of soil erosion was defined from various grid sizes as shown in Fig. 11. Soil erosion was evaluated on surface areas ranging from 0.03 to 3000 km^2. The mean value of the correction factor Q_e and the confidence intervals at 95% were found to decrease with the size of the cell as illustrated in Fig.13.

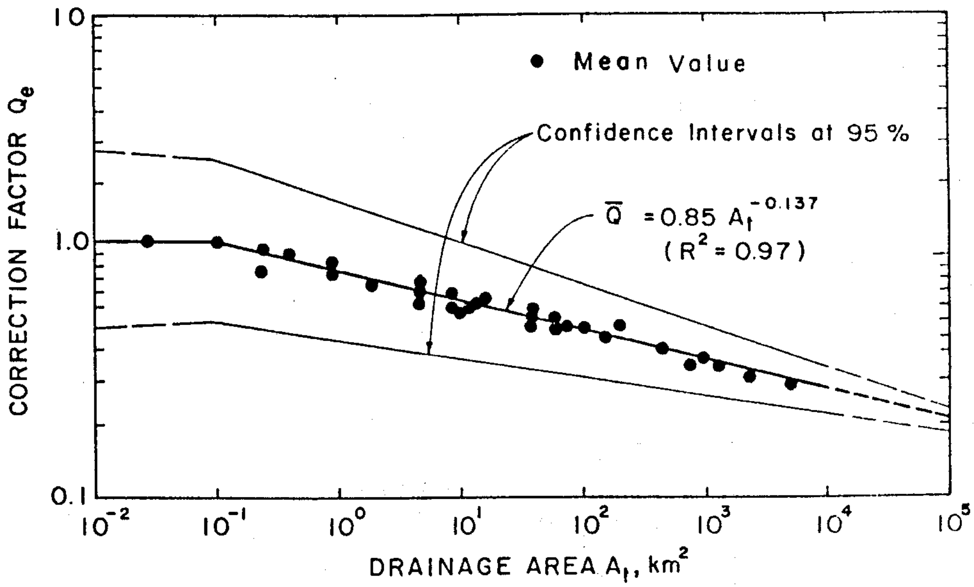


Fig. 13 Correction factor Q_e versus drainage area

c) The sediment yield is estimated from the product of the total soil erosion on the watershed and the mean value of the sediment delivery ratio C_s plotted in Fig. 14. The variability of the delivery ratio ranges between 0.4 and 2.5 times the mean value C_s.

d) The estimated sediment yield simulated by the model LAVSED-I is compared in Fig.15 with the observed sediment load measured in the river. Maps of the mean annual soil losses on the watershed are presented in Fig. 16. It is concluded from this analysis that soil losses and sediment yield computed from the USLE are only valid during rainfall. On this particular watershed, Fig. 15 shows that

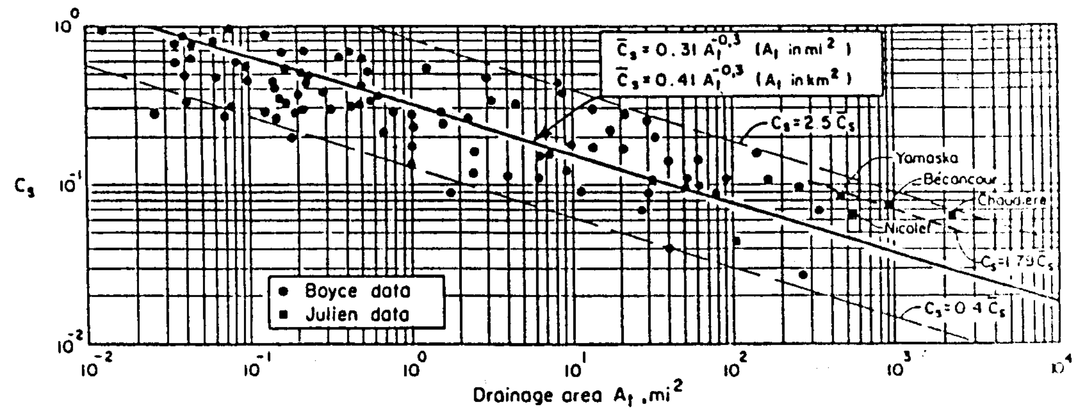


Fig. 14 Sediment-delivery ratio versus drainage area

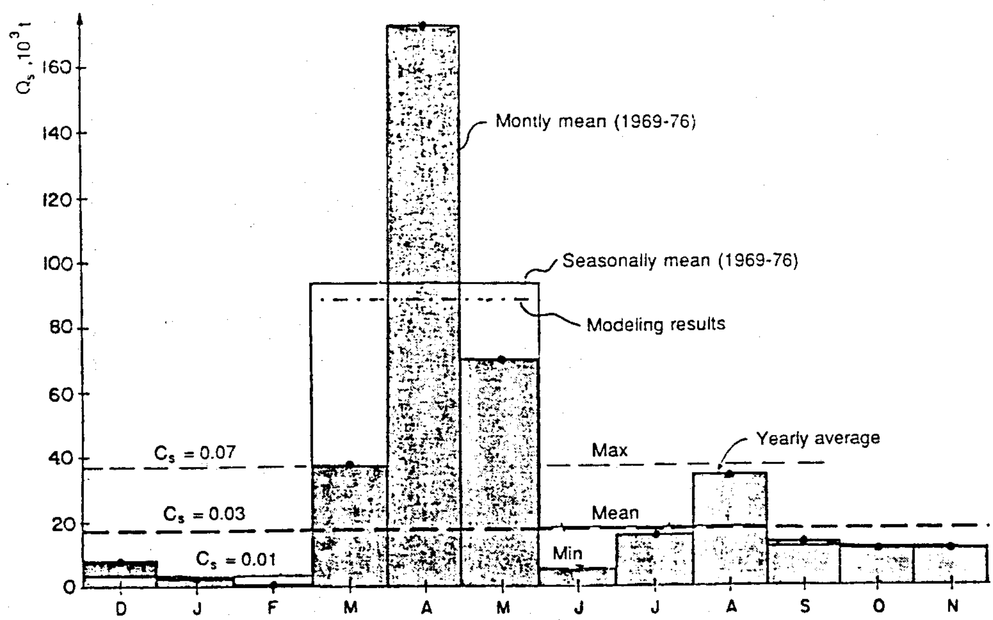


Fig. 15 Observed and computed sediment load of the Chaudière River

nearly 70 percent of the annual sediment load is observed during spring (March, April and May) when snowmelt occurs.

Model LAVSED-II Based on Runoff Characteristics

The model LAVSED-II(Julien and Frenette, 1984) has been developed to to

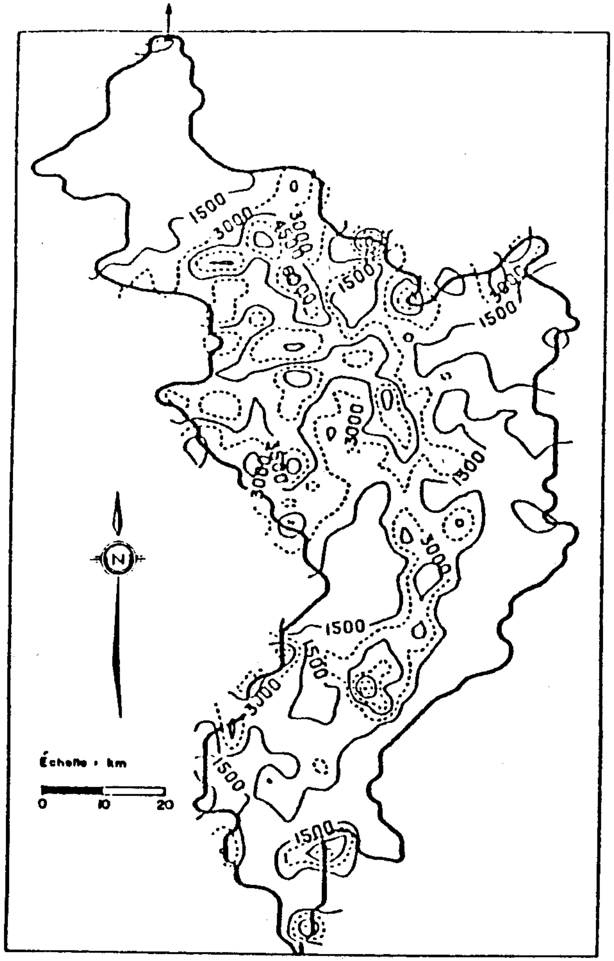


Fig. 16a   Soil erosion map (t/km^2.year) of the Chaudière River using Kilinc and Richardson equation

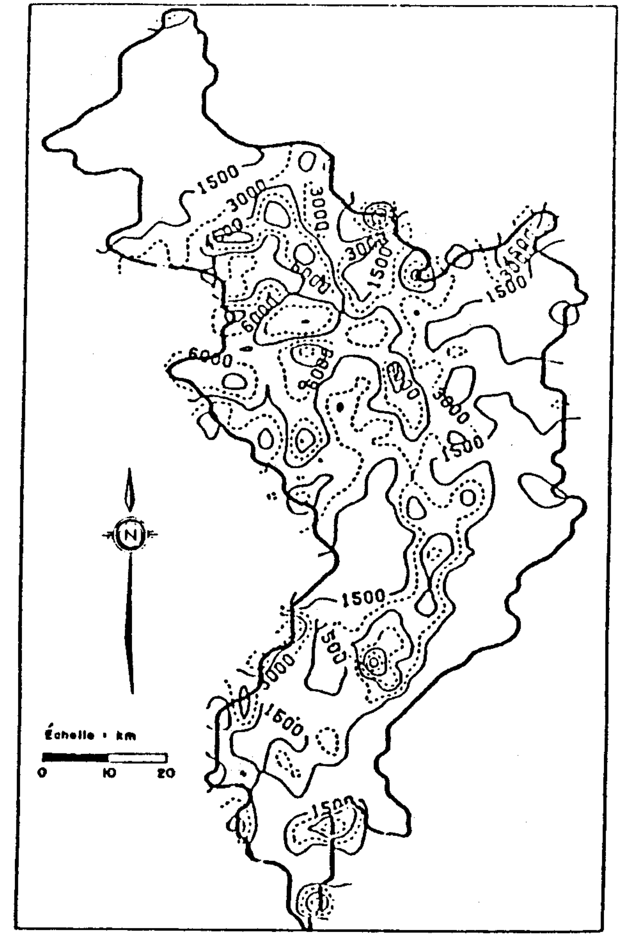


Fig. 16b   Soil erosion map (t/km^2.a) of the Chaudière River using the Universal Soil Loss Equation

better describe the changes in sediment yield associated with rainfall and snowmelt. This model is principally based on the runoff characteristics and is a function of the physical characteristics of the watershed. On upland areas, rainfall-runoff-sediment relationship is treated as deterministic as shown in Fig. 17 while the rainfall duration and intensity are treated as random variables (Fig. 18). The expected value of soil loss on upland areas is obtained by integrating the sediment discharge over the complete range of rainfall duration and intensities shown in Fig. 18 as described by their respective probability density functions from Eqs. 1 and 2. The influence of vegetation and water losses due to infiltration are accounted for during the rainfall period. During snow-melt, the probability density function of runoff follows an exponential function and the mean runoff rate increases gradually during the season. After consideration of the probability density functions of rainfall duration, intensity and snowmelt runoff, the sediment yield from watersheds can be computed from the total erosion and the sediment-delivery ratio. The monthly sediment yield during rainfall Q_{sp} and snowmelt Q_{sf} in kt are respectively:

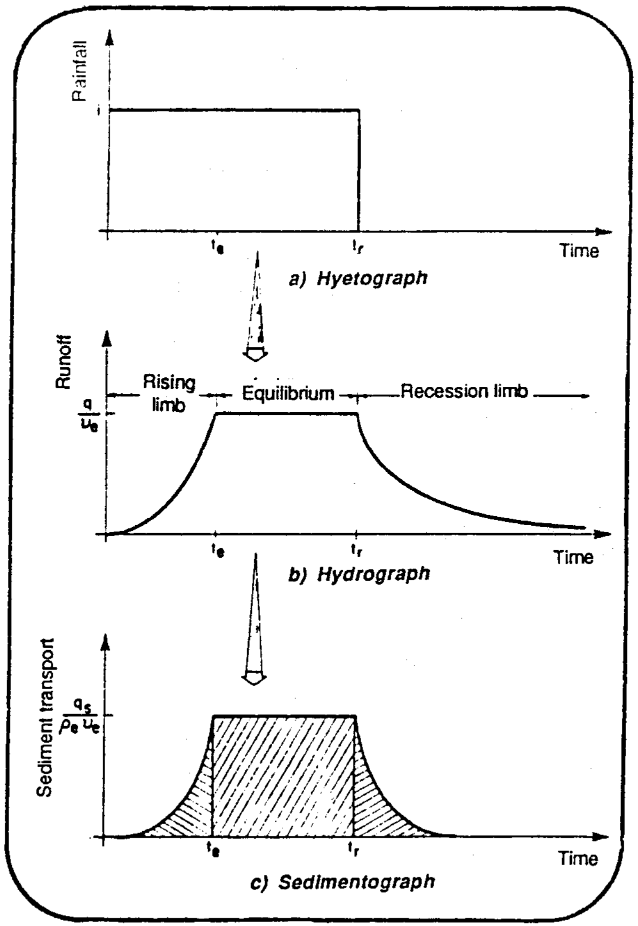
$$Q_{sp} \simeq 2417\, C_s\, \bar{C}\, C_r\, A_t^{1.137}\, \frac{\bar{\nu}\, a\, \bar{S}^{1.66}\, \bar{L}^{1.035}}{\lambda_1\, \lambda_2^{2.035}} \qquad (16)$$

and

$$Q_{sf} \simeq 30.7\, C_s\, \bar{C}\, A_t^{1.137}\, \bar{S}^{1.66}\, \bar{L}^{1.035}\, F^{1.65} \left(\frac{F}{F_T}\right)^{0.37} \qquad (17)$$

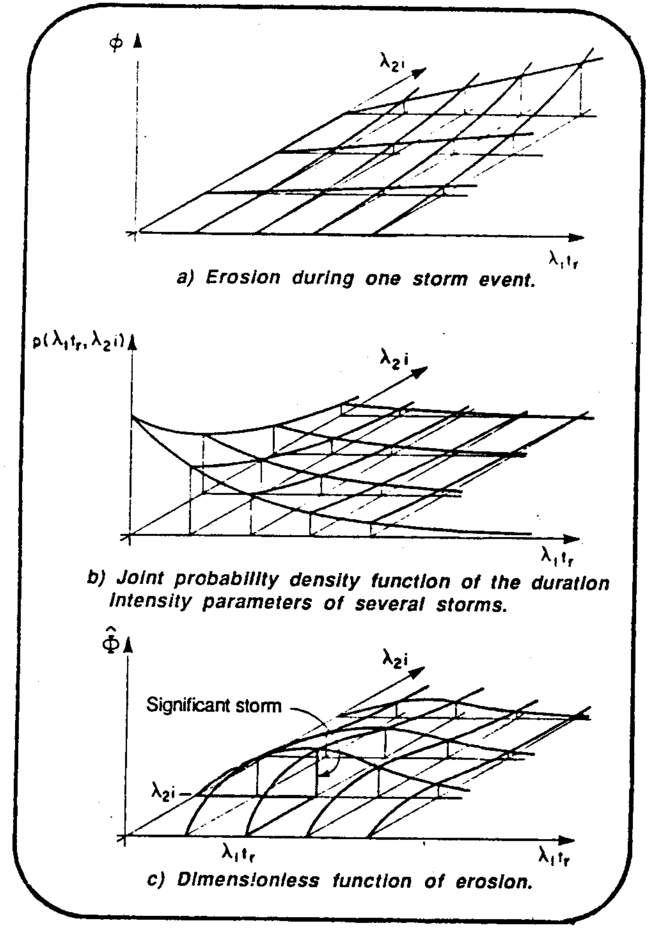
in which

C_s = sediment-delivery ratio
$\bar{C}$ = mean cropping fator
C_r = runoff coefficient
A_t = drainage area in km^2
$\bar{\nu}$ = number of rainstorms



a) Hyetograph
b) Hydrograph
c) Sedimentograph

Fig. 17 Soil erosion for single event



a) Erosion during one storm event.
b) Joint probability density function of the duration intensity parameters of several storms.
c) Dimensionless function of erosion.

Fig. 18 Expected value of soil erosion for several events

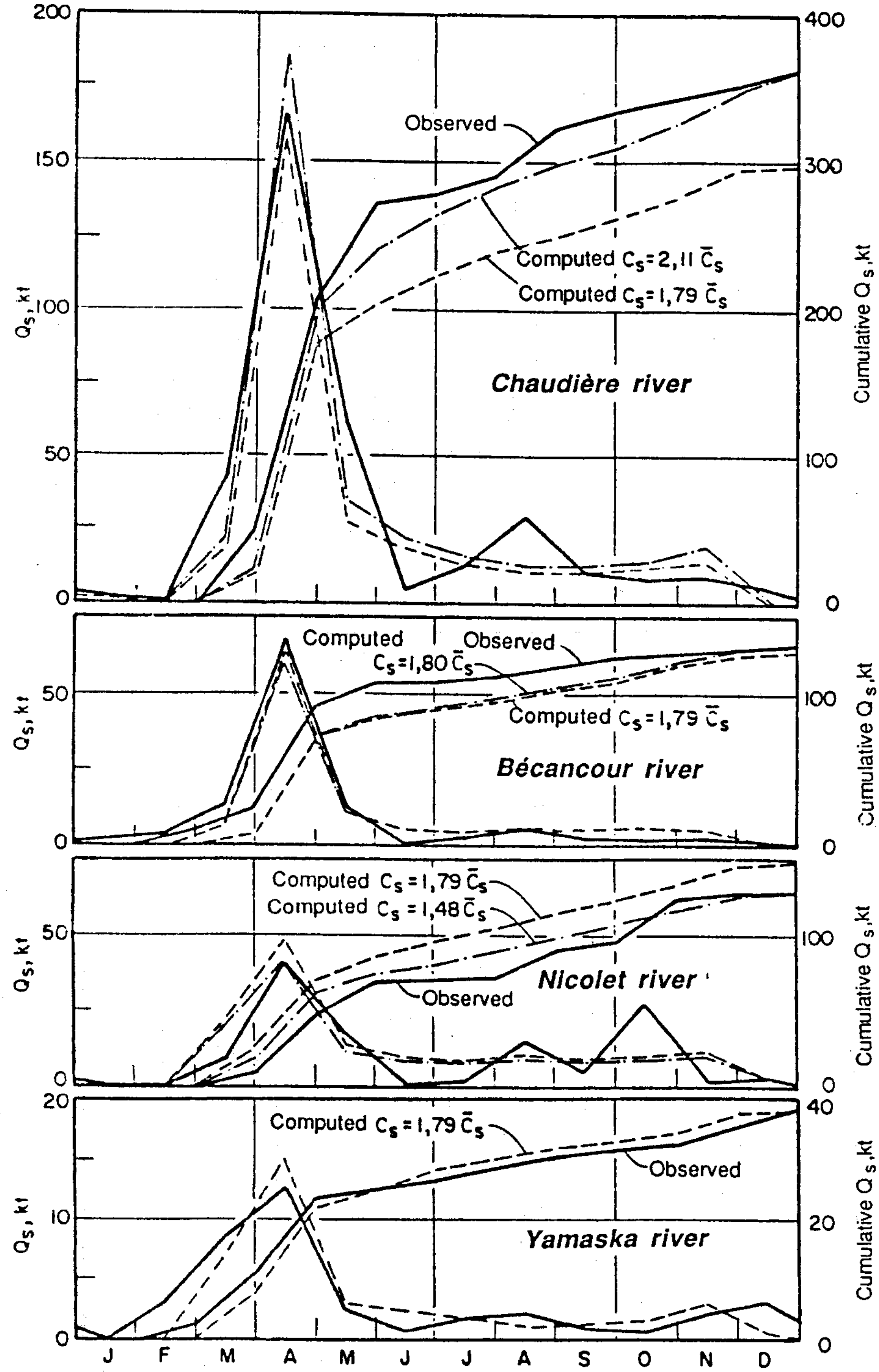


Fig. 19 Observed and calculated sediment loads

α = coefficient of the sediment discharge equation

$\bar{S}$ = average slope of the watershed

$\bar{L}$ = average runoff length

λ_1 = storm duration parameter

λ_2 = storm intensity parameter

F = cumulative snowmelt in m of water

F_T = total snowpack in m of water.

These two equations were used in the model LAVSED-II for predicting the average sediment yield from large watersheds in Canada for each month. The results are shown in Fig. 19 and compared with the measured sediment load in the streams. The model LAVSED-II requires detailed climatological data and several variables defining the overland flow characteristics.

DISCUSSION

Each computer model requires measured data of sediments for validation. These models offer various complementary results which prove to be extremely useful, for instance, in management of large watersheds, reservoir sedimentation, long-term changes in land-use, soil conservation, clear-cutting and reforestation. Research efforts are pursued at Laval University for developing better technologies in the analysis of erosion, transport and sedimentation problems on large watersheds. Though the research developments were primarily oriented toward modeling northern watersheds, the models are flexible so that specific needs in other countries can be met easily.

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REFERENCES

Eagleson, P. S. 1978, Climate, soil and vegetation, Water Resources Research, Vol. 14, No. 5, pp. 705-721.

Frenette, M., Llamas, J. et Larinier, M. 1974, Modèle de simulation du transport solide en suspension des rivières Châteauguay et Chaudière. CENTREAU Report, Laval University, Québec. 34p. (In French)

Frenette, M. et Nzakimuena, T., 1976, Modèle de prédiction des apports solides des rivières Chaudière, York, Châteauguay et Eaton, First Hydrotechnical Conference CSCE-QUEBEC, Montreal. (In French)

Frenette, M. et Julien P., 1984, Etablissement de cartes d'érosion par modélisation mathématique, Proceedings of the Annual Conference of the Canadian Society for Civil Engineering, CSCE, Halifax, May 23-25, pp. 543-564.(In French)

Julien, P., 1982, Prédiction d' apport solide pluvial et nival dans les cours d'eau nordiques a partir du ruissellement superficiel, Ph.D. Dissertation, Department of Civil Engineering, Laval University, Québec, Canada, 240 p. (In French)

Julien, P. and Frenette, M., 1984, A model for predicting suspended load in northern streams, Proceedings of the Annual Conference of the Canadian Society for Civil Engineering, CSCE, pp. 565-585.

Julien, P. and Frenette, M., 1985, Modeling of rainfall erosion, Paper accepted for publication in the Journal of Hydraulic Engineering ASCE.

Julien, P. Y. and D. B. Simons, 1984, Analysis of sediment transport equations for rainfall erosion, Report CER 83-84 PYJ-DBS52, Engineering Research Center, Colorado State University, Colorado, 46 p.

Kilinc, M. and Richardson, E. V., 1973, Mechanics of soil erosion from overland flow generated by simulated rainfall, Hydrology Paper No. 63, Colorado State University, Fort Collins, Colorado, 54 p.

Knisel, W. G., 1980, Editor, A field-scale model for chemical, runoff, and erosion from agricultural management systems, USDA, Conservation Research Report No. 26, 640 p.

Negev, M., 1967, A sediment model on digital computer, Tech. Report No. 76, Department of Civil Engineering, Stanford University, California.

Simons et al., 1982, Engineering analysis of fluvial systems, SLA, P.O. Box 1816, Fort Collins, Colorado.

Todorovic, P., 1968, A mathematical study of precipitation phenomena, Report CER 67-68PT65, Engineering Resrarch Center, Colorado State University, Colorado. 123 p.

Williams, J. R. and H. D. Berndt, 1972, Sediment yield computed with the universal equation, Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY22, Proc. Paper 9426, pp. 2087-2098.

Wischmeier, W. H. and Smith, D. D., 1978, Predicting rainfall-erosion losses - A guide to conservation planning, U. S. Dept. Agriculture, Handbook, No. 537, 48 p.

Woolhiser, D.A. and P. Todorovic, 1974, A stochastic model for sediment yield for ephemeral streams, USDA Misc. Pub. 1275, pp. 295-308.