Sediment load calculations from point measurements in sand-bed rivers

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Abstract
Point velocity and suspended sediment concentration measurements are used to calculate the total sediment discharge in sand-bed rivers. Calculations with the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) depend on grain diameter $d_s$, settling velocity $\omega$, flow depth $h$, shear velocity $u_*$, and sampling depth $h_p$. This procedure extends the applicability of the Modified Einstein Procedure (MEP) by using point sediment concentration and velocity measurements. This procedure is tested using the laboratory data from Coleman, and field measurements from the Enoree, Middle Rio Grande and Mississippi Rivers. Based on 801 point measurements over 124 verticals at flow depths ranging from 0.17 m to 33.5 m and sediment concentrations less than 0.1 kg L$^{-1}$, the accuracy of the calculations depends on $u_*/\omega$ and $h_p/d_s$. Point measurement techniques like SEMEPP are well-suited when $u_*/\omega > 5$ where at least 60% of the total sediment load is measured when 90% of the flow depth is sampled. The determination of sediment discharge from point measurements is most accurate in deep rivers when $h_p/d_s > 10,000$, and $u_*/\omega > 10$. Point measurements are not well-suited for shallow rivers and laboratory flumes where $h < 0.5$ m and when $u_*/\omega < 2$.

Key Words: Point measurements, Concentration profiles, Velocity profiles, Total sediment load, Modified Einstein Procedure

1 Introduction
Difficulties in calculating the total sediment load in sand-bed rivers from the combination of suspended and bed loads continue to persist. Suspended sediment concentration profiles in open channels corroborate the theory of turbulent mixing and the logarithmic velocity profile for turbulent flows (Rouse, 1937). The Rouse number $Ro = \omega / u_*$ is function of the fall velocity $\omega$, the shear velocity $u_*$, the momentum correction factor for sediment $\beta$ and the von Kármán constant $\kappa$. The Rouse number (Ro) is a ratio of the sediment property to the hydraulic characteristics of the flow. High values of the Ro (greater than 5) describe bed-load sediment transport, while low values of Ro (less than 0.5) result in near-uniform sediment concentration profiles and sediment transported in suspension (Dade and Friend, 1998; Julien, 2010).

Einstein (1950) developed a method to determine the total sediment load in open channels. The method uses the bed load function from Einstein (1942) and determines the suspended sediment concentration profiles from Rouse (1937). The integral of the product of the logarithmic velocity and the Rouse concentration profiles resulted in the Einstein integrals, which are analytically intractable. Recent developments through series expansions have improved the accuracy of the estimation of the Einstein integrals (Guo and Julien, 2004), which are applied in this study.

When the Einstein procedure was later applied to sand-bed rivers, inaccuracies emerged because this method could not define the wash load, which comprises the transport of the very fine sediment particles not found in the bed. The determination of the wash load concentration can only be accomplished through field measurements of suspended sediment concentrations. The subsequent analysis of the measured suspended sediment concentration led to the determination of the total sediment load using the Modified Einstein Procedure (MEP) developed by Colby and Hembree (1955). The MEP uses a combination of the Einstein bed load function (Einstein, 1942; 1950) and an extension of the velocity and concentration profiles in the unmeasured zone to estimate a total load as sketched in Fig. 1. The original MEP used depth-integrated sediment concentration measurements and the particle size distribution of the bed material to determine the total load. The MEP was first successfully tested with data from the Niobrara River

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Note: The original manuscript of this paper was received in Apr. 2014. The revised version was received in May 2014. Discussion open until Jan. 2016.
However, further complications did arise when there is not enough overlap between sediment particles sizes in suspension and in the bed material. For instance, the MEP could not calculate the total sediment load for gravel-bed streams with sand sizes in suspension. Several modifications have thereafter been proposed to determine the total sediment load transported in sand-bed rivers (Colby and Hubbell, 1961; Lara and Pemberton, 1963; Ackers and White, 1973; Burkham and Dawdy, 1980; Shen and Hung, 1983; O’Brien, 2000).

Fig. 1. Definition sketch for sediment measurements in suspension

Point measurements of sediment concentration and flow velocity are useful in sand-bed rivers. When compared with depth-integrated measurements, greater accuracy can be obtained with point measurements because of the larger sampling volumes for the determination of sediment concentration as well as particle size distribution of suspended sediment. Thus point measurements present definite advantages in the determination of the sediment load in deep sand-bed channels.

The main purpose of this study is to determine the accuracy of total sediment load calculations based on point velocity and sediment concentration measurements. In this article, a new procedure referred to as the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) is developed for the analysis of the total sediment load with point measurements. This article employs the use of the series expansion to solve the Einstein integrals (Guo and Julien, 2004). The study provides an extension of the MEP to point samples by fitting the Rouse suspended sediment concentration profiles and logarithmic velocity profiles to the measured point data. The new procedure is tested with several data sets covering a wide range of flow and sediment conditions ranging from shallow laboratory experiments to deep sand-bed rivers. The analysis aims at providing guidelines on the conditions required for an accurate determination of the total sediment load. The most relevant questions addressed in this article include: (1) under what conditions can total sediment load be accurately determined using point measurements of flow velocity and sediment concentration; and (2) what percentage of the water column should be sampled for an accurate determination of the total sediment load. The analysis of field measurements will also identify the minimum sampling depth needed to obtain accurate total load calculations.

2 Total sediment load from point measurements

2.1 Measured total load

Point velocity and concentration measurements are collected at fixed locations along a vertical water column, as sketched in Fig. 2a. At each point \( i \), flow velocity \( v_i \) and sediment concentration measurement \( C_i \) are taken. The data from the point measurements allow for fitting the sediment concentration profile directly to the data set. In some cases, the particle size distribution of each sample is collected at each point, which allows the determination of the suspended sediment concentration for each size fraction. In general, suspended sediment concentration measurements cannot be obtained at the bed or free surface (Edwards and Glysson, 1999). Sediment concentration is assumed to be zero at the water surface and flow velocity is assumed zero at the bed. On the other hand the concentration at the bed and the velocity at the free surface are assumed to be equal to the nearest measurement.

The sediment flux (sediment discharge per unit area imparted by the velocity current) is determined from the product of flow velocity and sediment concentration as shown in Eq. (1). The unit sediment discharge is obtained from the integral of the sediment flux. With point measurements, this is accomplished by using the trapezoidal rule as sketched in Fig. 2b with \( p = 5 \). The measured total unit sediment discharge corresponds to the summation from 0 to \( p \) from the free surface to the bed, or:
where, $q_{tm}$ is the total measured unit sediment discharge based on $p$ point samples, $C_i$ and $v_i$ are respectively the sediment concentration and flow velocity at a given point $i$, and $h_{i+1} - h_i$ is the vertical layer thickness.

By increasing the number of point measurements, the accuracy of the calculated total measured unit sediment discharge is expected to increase. This occurs because the concentration and velocity profiles have a higher degree of certainty. By including additional discrete points the summation of the measured sediment concentration profile will be closer to the actual sediment discharge. However, when both the Rouse number $R_o$ and the ratio of the lowest point sampling depth to the water column depth ($P = z_p/h$) are large, this approach can lead to highly inaccurate results.

When bed load is dominant (large $R_o$), most of the sediment transport will occur below the lowest sampling point location (Dade and Friend, 1998; Julien, 2010). Thus, guidelines are needed to define the range of applicability of point measurement techniques in terms of $u^*/\omega$ (or $R_o$) and $P$. To quantify the accuracy of sediment transport from point measurements, the concept of partial sediment discharge is introduced. The partial sediment load is the sediment transported in the water column between the free surface and the lowest sampling point $p$. It may be interesting to point out that the partial sediment load is normally called the measured load (Julien, 2010). However, the term “partial load” is preferred in this article to avoid possible confusion between sediment quantities that have been measured and calculated. Therefore, the measured partial unit sediment discharge $q_{pm}$ can be determined from 0 to $p-1$, using the rectangular rule (Fig. 2b):

$$q_{pm} = \sum_{i=1}^{p} 0.5(C_i + C_{i+1})(h_{i+1} - h_i)$$

In Section 3, the analysis of $q_{pm}/q_{tm}$ will determine whether point sampling is appropriate to measure most of the sediment load that is transported. Also, the investigation of $z_p$ (or $P$) will be used to quantify the increase in measured sediment load as the sampling points get closer to the bed.

2.2 SEMEPP procedure

This analysis (SEMEPP) expands upon the Series Expansion of the Modified Einstein Procedure (SEMEP) developed by Shah-Fairbank et al. (2011). While SEMEP was based on depth-integrated samples, the SEMEPP analysis is based on point measurements. SEMEP has been shown to work well in sand-bed rivers (Shah-Fairbank et al., 2011). When the suspended sediment concentration profiles became uniform (low $R_o$) the momentum correction factor for sediment $\beta$ and the von Kármán constant $\kappa$ become irrelevant. One of the main features of SEMEPP has been to simply report the results as a function of $u^*$ and $\omega$. This parametric description does not require any assumption regarding the values of $\kappa$ and $\beta$. The ratio $u^*/\omega$ is favored over the Rouse number as part of the overall improvements of the MEP procedure. For the comparison of $u^*/\omega$ with the Rouse number in this study, it has been assumed that $\beta = 1$ and $\kappa = 0.4$. The value of $\omega$ is calculated as:

$$\omega = \frac{8v_s}{d_s}\left[1 + 0.0139d_s^{1.5}\right]$$

The main features of SEMEPP include the following: (1) fit the concentration profile to the point concentration measurements to determine the reference concentration $C_a$ and $R_o$; (2) fit a logarithmic velocity profile to the point velocity data to determine $u^*$ and depth of zero velocity $z_{o}$; (3) use the series expansion algorithm of Guo and Julien (2004) to solve the Einstein integrals; and (4) calculate the sediment load by integrating the advective flux of sediment $(Cv)$ using parts (1) and (2). SEMEPP is therefore an extension of the MEP to point measurements. The reason we continue to refer to the procedure as the MEP is because the procedure uses field measurements and the Einstein integrals. Improvements beyond the existing MEP are expected because point concentration measurements can define the particle concentration profile.
the sediment concentration profiles, which the depth-integrated samples of the original MEP could not do. The use of the series expansion should improve the accuracy of calculating the Einstein integrals over other methods. Finally, guidelines based on \( u^* / \omega \) will also remove any uncertainly associated with \( \beta \) and \( \kappa \).

2.3 Sediment load calculation using SEMEPP

The SEMEPP procedure requires point measurements to define the velocity distribution and the sediment concentration along a vertical profile (water column). The turbulent velocity profile (law of wall) within a river is based on the logarithmic velocity distribution (Keulegan, 1938).

\[
v = \frac{u^*}{\kappa} \ln \frac{z}{z_o}
\]  

(3)

where \( v \) is the velocity at a distance \( z \) above the bed, \( u^* \) is the shear velocity, \( \kappa \) is the von Kármán constant assumed equal to 0.4, and \( z_o \) is the vertical elevation where the velocity equals zero. In SEMEPP, the values of \( u^* \) and \( z_o \) are determined by fitting Eq. (3) to the entire velocity profile obtained from the point velocity measurements. Alternatively, when the friction slope \( S_f \) of the river is available, the shear velocity \( u^* \) can be determined based on the following relationship \( u^* = \sqrt{ghS_f} \).

The sediment concentration profile is described by the Rouse equation (Rouse, 1937). The suspended sediment distribution described in Eq.(4) provides the concentration at a specified distance \( z \) above the bed.

\[
C = C_b \left( \frac{h - z}{h - z_b} \right)^{1/4}
\]  

(4)

where \( C \) is the concentration at an elevation \( z \) above the bed, \( C_b \) is the concentration at the reference elevation \( z_b = 2d_s \) above the bed and \( h \) is the total depth of flow. In SEMEPP, the values of \( R_o \) and \( C_b \) are determined by fitting the Rouse power function [Eq. (4)] to the concentration measurements. Figure 3 shows a schematic of the calculation procedure from SEMEPP.

Fig. 3 Sketch of the sediment load calculation from SEMEPP

The unit sediment discharge by advection is given by the depth-integration of the product of sediment concentration \( C \) and flow velocity \( v \). SEMEPP provides calculations of the unit total sediment discharge \( q_t \), the unit suspended sediment discharge \( q_s \), and the partial unit suspended sediment discharge \( q_p \) respectively.

\[
q_t = \int_{z_b}^{h} C v dz
\]  

(5a)

\[
q_s = \int_{z_b}^{h} C v dz
\]  

(5b)

\[
q_p = \int_{z_b}^{z_p} C v dz
\]  

(5c)

where \( h \) is the total flow depth, \( z_b \) is the depth of the bed layer defined by Einstein as the reference depth of \( 2d_s \), and \( z_p \) is the elevation above the bed of the lowest point measurement. The values of \( q_p \) and \( q_t \) are sketched in Fig. 3. The ratio of \( q_p / q_t \) defines the fraction of the total sediment load transported in the sampling depth as calculated by SEMEPP.

From SEMEPP, the partial unit sediment discharge is then evaluated from Eq. (5c), after considering Eqs. (3) and (4), and can be expanded as follows:
\[ q_p = \int_{0}^{b} C_b \left( \frac{h}{z} \frac{z_b}{h} \right)^{R_0} u - \ln \left( \frac{z}{z_p} \right) dz \]  

\[ q_p = \frac{C_i h u_c}{b} \left( \frac{B}{1} \right)^{R_0} \left[ \ln \left( \frac{h}{z_p} \right) J_{1p} + J_{2p} \right] \]  

\[ J_{1p} = \int_{0}^{1} \left( \frac{1}{z_b} \right)^{R_0} \ln \left( \frac{z}{z_p} \right) dz, \]  

\[ J_{2p} = \int_{0}^{1} \ln z_b \left( \frac{1}{z_b} \right)^{R_0} dz, \]

where, \( q_p \) is the partial unit sediment discharge calculated from SEMEPP, \( z_b = 2d_i \) is the reference elevation, \( z_p \) is the nozzle height above the bed for the lowest measured point sample (Fig. 3), the bed layer ratio is \( B = z_b/h \), the partial depth ratio is \( P = z_p/h \), the integration variable is the relative depth \( z* = z/h \), and \( J_{1p} \) and \( J_{2p} \) are the modified Einstein integrals for the partial sediment load. In SEMEPP, \( u_c \) and \( z_p \) are determined from the point concentration measurements. The Einstein integrals for partial load \( J_{1p} \) and \( J_{2p} \) in Eqs. (8) and (9) are solved with the series expansion algorithm of Guo and Julien (2004).

The total unit sediment discharge is calculated based on the sum of the bed and suspended sediment discharge. The bed load transport function \( q_b \) in the bed layer of thickness \( z_b = 2d_i \) and the near-bed velocity \( v_b \) are based on Einstein’s bed load analysis (1942, 1950).

\[ q_i = q_b + q_s; \quad \text{where} \quad q_b = C_b v_b z_b; \quad \text{and} \quad v_b = 11.6u, \]

\[ q_i = q_b + \int_{z_p}^{b} C_b \left( \frac{h}{z} \frac{z_b}{h} \right)^{R_0} u - \ln \left( \frac{z}{z_p} \right) dz \]

\[ q_i = C_i v_i z_i + C_i h u_c \left( \frac{B}{1} \right)^{R_0} \left[ \ln \left( \frac{h}{z_p} \right) J_{1s} + J_{2s} \right] \]

\[ J_{1s} = \int_{0}^{1} \left( \frac{1}{z_b} \right)^{R_o} \ln \left( \frac{z}{z_p} \right) dz, \]

\[ J_{2s} = \int_{0}^{1} \ln z_b \left( \frac{1}{z_b} \right)^{R_0} dz, \]

where, \( q_i \) is the total calculated unit sediment discharge, \( q_b \) is the total unit bed load discharge, \( V_i \) is the flow velocity in the bed layer of thickness \( z_b = 2d_i \), \( B = z_b/h = 2 d_i/h \), and \( J_{1s} \) and \( J_{2s} \) are the Einstein integrals for the suspended sediment zone. SEMEPP uses \( u_c, z_p, Ro \) and \( C_i \) from the velocity and concentration profiles, and \( q_i \) is determined from \( C_i, v_b = 11.6u \). The Einstein integrals for the suspended sediment load \( J_{1s} \) and \( J_{2s} \) in Eqs. (13) and (14) are solved with the series expansion algorithm developed by Guo and Julien (2004).

**3 Experimental verification of -SEMEPP**

3.1 Laboratory and field data

To validate SEMEPP, accurate point measurements were analyzed for both laboratory and river conditions. The data sets listed in Table 1 include point measurements (velocity and concentration) from Coleman’s laboratory experiments, the Enoree, Middle Rio Grande and Mississippi Rivers. These data sets were obtained from previously published sources and compiled by Shah-Fairbank (2009). The total data sets include 124 verticals (water columns) and a total of 801 point velocity and point concentration measurements. These data sets were selected to provide a wide range in flow depth (0.017 m < h < 33.5 m), unmeasured depth (0.006 m < z_p < 2.2 m), measured total sediment transport [(0.01 kg m^{-1} \cdot s^{-1}) < q_i < (32 kg m^{-3} \cdot s^{-1})] and relative submergence (1,600 < \( h/d_i \) < 530,000). This also allowed for a continuous range of values for \( u_c/h \) and \( h/d_i \) to identify the domain of applicability of SEMEPP.

Coleman’s laboratory data were measured in a laboratory flume (15 m long by 33 cm wide). The flow velocity and sand concentration profiles were collected for sand particles in suspension ranging in size from 0.088 mm to 0.5 mm (Coleman, 1981, 1986).

The Enoree River data set is based on field measurements made in relatively regular and straight reach (Anderson, 1942). At the sampling location, the sand-bed river was 15 m wide with near vertical banks. Numerous samplers regularly spaced on a sounding cable provided concurrent measurements of flow velocity and suspended sediment concentrations. The particle measured in suspension ranged from 0.07 to 0.99 mm.
Table 1  Main characteristics of the data set

<table>
<thead>
<tr>
<th>Data Set</th>
<th>No. of verticals sampled</th>
<th>No. of points samples</th>
<th>( h ) (m)</th>
<th>( z_p ) (m)</th>
<th>( d_s ) (mm)</th>
<th>( h/d_s \times 1000 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coleman Lab Data (Coleman, 1986)</td>
<td>6</td>
<td>12</td>
<td>0.170 to 0.172</td>
<td>0.006</td>
<td>0.1</td>
<td>1.6</td>
</tr>
<tr>
<td>Enoree River, SC (Anderson, 1942)</td>
<td>6</td>
<td>8</td>
<td>0.9 to 1.6</td>
<td>0.06 to 0.103</td>
<td>0.2 to 0.25</td>
<td>3.2 to 6.2</td>
</tr>
<tr>
<td>Middle Rio Grande at Bernalillo, NM (Nordin and Dempster, 1963)</td>
<td>4</td>
<td>4</td>
<td>0.7 to 0.8</td>
<td>0.27 to 0.37</td>
<td>0.07</td>
<td>11.5 to 12.5</td>
</tr>
<tr>
<td>Mississippi River, MS (Akalin, 2002)</td>
<td>108</td>
<td>6</td>
<td>6.4 to 33.5</td>
<td>0.4 to 2.2</td>
<td>0.4 to 0.6</td>
<td>15 to 530</td>
</tr>
</tbody>
</table>

The United States Bureau of Reclamation took point suspended sediment and velocity measurements on the Middle Rio Grande (Nordin and Dempster, 1963). The velocity measurements were taken using a Price current meter and the suspended sediment measurements were obtained using a US P-46. The particle measured in suspension ranged from less than 0.0625 to 0.5 mm, however the median particle size was always within the sand range.

The United States Army Corps of Engineers collected data near the Old River Control Complex of the Mississippi River in 1998 by the Waterways Experiment Station. The equipment used for the measurements includes: Acoustic Doppler Current Profiler, US P-63, drag bucket, Price current meter and depth sounder. Data at four sites (Union Point, Line 13, Line 6 and Tarbert Landing) were used and compiled by Akalin (2002). The particle measured in suspension ranged from 0.0625 to 2 mm.

3.2 Total sediment load comparison

The measured total sediment load was determined from Eq. (1). The calculated total sediment load was determined from SEMEPP using Eq. (12). A comparison of the calculated and measured total loads is shown in Fig. 4 with each river represented by a distinct symbol. A twenty five percent error band has been placed on the figure. As the measured unit total sediment discharge increases, a higher degree of accuracy is achieved between measured and calculated (SEMEPP) total sediment discharge.

3.3 Applicability of SEMEPP

This section addresses the first issue raised in the introduction: under what conditions can the total sediment load be accurately determined using point measurements. In other words, under what conditions can SEMEPP calculate most of the sediment load transported in a sand-bed channel? The answer to this question is found in the ratio of the partial to the total sediment load \( q_p/q_t \). Point measurements can effectively measure most of the transport rate when the ratio is high. On the other hand, low values of \( q_p/q_t \) describe conditions where most of the sediment is transported as bed load, in which case, point measurement techniques would not be appropriate.

From the measurements, the ratio of the partial to total unit sediment discharge \( q_pm/q_tm \) is determined for each sample based on Eqs. (1) and (2). SEMEPP is also used to determine the calculated values of the ratio of partial to total unit sediment discharge.
sediment discharge $q_p/q_t$ based on Eqs. (7) and (12) for each sample. For the comparison, representative samples covering a wide range of $h/d_s$ were selected and the results are shown in Fig. 5. The measured values of the ratio $q_{pm}/q_{tm}$ are compared to the ratios $q_p/q_t$ calculated using SEMEPP. Measurements are shown as points, while the series of lines represent SEMEPP calculations obtained from varying the value of $u_*$ to generate a continuous line. There is more variability in the percentage of the total load that is measured when $u_*/\omega$ is between 2 to 5. The results of the calculations seem increasingly accurate when $u_*/\omega > 10$. Figure 5 shows that the SEMEPP calculated values of $q_p/q_t$ match the measurements particularly well at higher values of $u_*/\omega$. Overall, point measurement techniques like SEMEPP are considered well-suited when $u_*/\omega > 5$, at least 60% of the total sediment load is measured when 90% of the water column is sampled. As the sampling depth is reduced the variability between the measured and calculated ratio of $q_p/q_t$ increases, however there is less variability between the measured and calculated ratios as $u_*/\omega > 5$.

It is also observed in Fig. 5 that most of the sediment is transported near the bed when the values of $u_*/\omega < 2$. Indeed, only a small fraction (less than 20%) of the total sediment load could be measured by point samplers when $u_*/\omega < 2$, and this even when 90% of the water column is sampled. This means that the unmeasured sediment load is at least five times larger than the measured sediment load. This suggested that the based on SEMEPP method is not acceptable because bed load is the dominant mode of sediment transport.

3.4 Statistical analysis of the accuracy of SEMEPP

The degree of accuracy of SEMEPP is evaluated using the root mean square error (referred to as RMSE). RMSE is a measure of the absolute error (Ott and Longnecker, 2001) and identifying the precision.

\[
RMSE = \sqrt{\frac{\sum (X_i - Y_i)^2}{n}}
\]

where, $X_i$ is the measured total load, $Y_i$ is the calculated total load (SEMEPP) and $n$ is the number of samples. The RMSE is always positive and low numbers are indicative of better calculations.

The mean percent error (referred to as MPE) is a measure of the relative error (Ott and Longnecker, 2001) and is useful in identifying the bias in the calculations.

\[
MPE = \frac{\sum (X_i - Y_i)}{n} * 100
\]

If the value of MPE is less than zero, then SEMEPP has a bias towards over predicting the total sediment load.

Table 2 contains a summary of the statistical parameters. The data show that the magnitude of both the sediment load and error (RMSE) increases with $h/d_s$, which suggest less precision. However, this is associated with high sediment loads and concentrations and the relative value of the error or bias (MPE) clearly decreases as the relative submergence $h/d_s$ increases. In the cases of the Middle Rio Grande and Mississippi Rivers, the measured and calculated total sediment loads are within 25% of each other. It is thus concluded that point measurements have a tendency to provide an accurate total load calculation in deep sand-bed rivers even though the results may be less precise.
3.5 Analysis of the effect of sampling depth \( h_p \)

This section addresses the second issue raised in the introduction: what percentage of the water column should be sampled for an accurate determination of the total sediment load. Both the channel bed material and flow depth will affect the results. Figure 5 readily shows an increase fraction of \( q_p/q_t \) as the sampling depth increases. The accuracy in the calculations of sediment load increases with the value of \( u_s/\omega \). Thus when \( u_s/\omega = 10 \), only 50% of the water column of the Mississippi River needs to be sampled for measured and calculated total load to be within 35%. On the other hand, 90% of the water column of the Enoree River will need to be sampled.

To probe deeper into the effect of sampling depth, a sensitivity analysis on the accuracy of partial sampling is performed by varying the number of sediment concentration measurements used to fit the Rouse equation. The purpose of varying the number of point sediment samples is to determine what partial sampling depth could yield reasonable results in deep rivers. The total suspended sediment load using SEMEPP was re-calculated for the different sampling depths and the RMSE and MPE were determined for the entire data set containing 668 data points. The total sediment load calculated from SEMEPP was compared to the measured total load. Table 3 provides a representative summary of the RMSE and MPE results for various sampling depths. In general, the RMSE and MPE tend to increase as the sampling depth decreases. Under certain circumstances the statistical parameters decrease with reduced sampling depth. This occurs due to lower surface concentrations. Shallow rivers with \( h_p/h_s > 10,000 \) have a tendency to over predict total sediment load, thus resulting in a negative MPE. The Enoree River has a high negative MPE value due to the ratio of \( u_s/\omega \) and the high amounts of washload.

The absolute percent error was determined for each of the 668 samples from:

\[
\text{%Error} = \frac{100 \times (X - Y)}{X} \quad (17)
\]

Figure 6 provides a graphical display of all the results. Table 4 also summarizes the analysis of the average values of the absolute error percentage as a function of \( h_p/d_s \) at each site. The analysis clearly shows that a higher percentage (70%) of points have an error less than 25% when \( h_p/d_s \) increases. This analysis shows that sediment load predictions in deep rivers are more accurate (lower error percentage) than shallow rivers and laboratory flumes, as shown in Table 4. It is concluded that a value of \( h_p/d_s > 10,000 \) (Table 3 and Fig. 6) would result in sufficiently accurate sediment load calculations from SEMEPP because approximately 60% of the samples have an absolute percent error less than 25%. This leads to results of practical significance. This implies that for a grain diameter of 0.1 mm, accurate sediment load calculations from point measurements require a minimum sampling depth in excess of 1 m.

4 Applicability of SEMEPP in sand-bed rivers

The domain of applicability of SEMEPP in shallow and deep sand-bed rivers is determined from a representative application example. The particle size is kept constant at 0.2 mm (fine sand), and the values of \( q_p/q_t \) are calculated for typical values of sampling depth. For instance, the value for \( z_p = 10 \) cm corresponds to the nozzle height of standard point samplers (e.g. P-61 and P-63). Based on the findings in Table 3, it is considered that point measurements would be useful when that \( h_p/d_s > 10,000 \) which corresponds to a minimum flow depth of 2 m.

The results in Fig. 7 show that regardless of the river size, once the value of \( u_s/\omega > 5 \) it becomes possible to measure most \( (q_p/q_t > 50\%) \) of the total sediment load from a standard sampler, \( z_p = 10 \) cm. For instance in Fig. 7e at a flow depth of 20 m (measuring approximately 99.5% of the water column), over 90% of the total sediment load \( (q_p/q_t) \) would be measured using point samples from 10 cm above the bed to the free surface when \( u_s/\omega = 5 \). In comparison, Fig. 7a shows a shallow stream where a maximum of 80% of the water column is measured and only 53% of the total sediment load being measured. This means that accurate sampling over a partial flow depth is best suited for deep rivers carrying fine sediment while measurements become less accurate in shallow and small streams. The main contribution of this article is that the percentage of the sediment load that is measured in shallow and deep sand-bed rivers can be determined as a function of the sampling depth \( z_p \) and \( u_s/\omega \).
### Table 3: Statistical results for $q_t$ as a function of sampling depth $h_p$

<table>
<thead>
<tr>
<th>Data Set</th>
<th>$h/d_s$</th>
<th>% Flow Measured</th>
<th>$h_p/d_s$</th>
<th>Measured vs SEMEP - Total Load</th>
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<td>-0.036</td>
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<tr>
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<td>90%</td>
<td>49,500</td>
<td>0.013</td>
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<tr>
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<td>70%</td>
<td>38,500</td>
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<td>0.557</td>
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<td>50%</td>
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<td>0.859</td>
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<tr>
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<td>16,500</td>
<td>0.033</td>
<td>1.336</td>
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**Fig. 6** Error analysis vs sampling depth $h_p/d_s$
Table 4  Summary of error analysis as a function of relative sampling depth $h_p/d_s$

<table>
<thead>
<tr>
<th>Data set</th>
<th>Number of samples</th>
<th>Number of samples</th>
<th>% of samples</th>
<th>Number of samples</th>
<th>% of samples</th>
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<tbody>
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<td>Coleman lab data</td>
<td>72</td>
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<td>4.2%</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>Enoree river, SC</td>
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<td>53.5%</td>
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</tr>
<tr>
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<td>12</td>
<td>60.0%</td>
<td>3</td>
<td>15%</td>
</tr>
<tr>
<td>Mississippi - Tarbert</td>
<td>145</td>
<td>122</td>
<td>84.1%</td>
<td>104</td>
<td>71.7%</td>
</tr>
<tr>
<td>Mississippi - Union point</td>
<td>140</td>
<td>128</td>
<td>91.4%</td>
<td>116</td>
<td>82.9%</td>
</tr>
<tr>
<td>Mississippi - Line 6</td>
<td>115</td>
<td>99</td>
<td>86.1%</td>
<td>88</td>
<td>76.5%</td>
</tr>
<tr>
<td>Mississippi - Line 13</td>
<td>133</td>
<td>91</td>
<td>68.4%</td>
<td>91</td>
<td>68.4%</td>
</tr>
<tr>
<td>Total</td>
<td>668</td>
<td>478</td>
<td>71.6%</td>
<td>402</td>
<td>60.2%</td>
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</table>

5 Conclusions

This study determines under what conditions point velocity and sediment concentration measurements can be used to calculate total sediment discharge in sand-bed rivers. A new total sediment discharge calculation algorithm based on the Series Expansion of the Modified Einstein Point Procedure (SEMEPP) is presented. This procedure provides uses to use point sediment data with the Modified Einstein approach for a high degree of reliability with evaluation of total sediment load. The procedure is tested with a wide range of laboratory and field data totaling 801 point measurements on 124 verticals at sediment concentrations less than 0.1 kg l$^{-1}$. The ratio of shear to fall velocities $u*/\omega$ is very important to determine when point measurements can be effective in sand-bed rivers. Point measurement techniques like SEMEPP are well-suited when $u*/\omega > 5$, in such case, at least 60% of the total sediment load is measured when 90% of the flow depth is sampled. The determination of sediment discharge from point measurements is most accurate in deep rivers when $u*/\omega > 10$ and $h_p/d_s > 10,000$. The root mean square error (RMSE) increases (less precision) while the relative error (MPE) decreases (less bias) as $h_p/d_s$ increases. The effect of sampling depth has also been investigated. For instance on the Mississippi at $u*/\omega = 10$ from Fig. 5, only 36% of the total sediment load is measured when sampling 50% of the flow depth. In comparison, more than 80% of the total sediment discharge can be measured when sampling 90% of the water column. As illustrated in Fig. 7, the main contribution of this article is to quantify the percentage of the sediment load that is measured in shallow and deep sand-bed rivers as a function of the sampling depth $z_p$ and $u*/\omega$. Finally, it is found that point measurements are not recommended for shallow rivers ($h < 0.5$ meters) or when $u*/\omega < 2$ because they only capture a small proportion of the total sediment load.

Acknowledgements

This research has been completed at Colorado State University under primary support from the Department of Defense through the Center for Geosciences/Atmospheric Research at Colorado State University, under Cooperative Agreement
DAAD19-02-2-0005 with the Army Research Laboratory. The writers would also like to thank Dr. J. Guo at the University of Nebraska and D. Baird and R. Padilla at the US Bureau of Reclamation. In addition, this work was supported by the National Science Foundation under Grant No. 0548426 via an ADVANCE Institutional Transformation (IT) Award. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors.

References


Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Ratio of bed layer thickness to flow depth $B = z_b/h = 2d_s/h$</td>
</tr>
<tr>
<td>C</td>
<td>Sediment concentration</td>
</tr>
<tr>
<td>$C_b$</td>
<td>Reference concentration at the reference depth of $2d_s$</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Sediment concentration a location $i$</td>
</tr>
<tr>
<td>$d_s$</td>
<td>Particle diameter</td>
</tr>
<tr>
<td>$h$</td>
<td>Total flow depth</td>
</tr>
<tr>
<td>$h/d_s$</td>
<td>Relative submergence</td>
</tr>
<tr>
<td>$h_f$</td>
<td>Sampling depth from the free surface to the lowest sampling point</td>
</tr>
<tr>
<td>$h_p/d_s$</td>
<td>Ratio of sampling depth to particle size</td>
</tr>
<tr>
<td>$h_p/h$</td>
<td>Relative sampling depth</td>
</tr>
<tr>
<td>$i$</td>
<td>Sampling location with the vertical measured from the water surface</td>
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\[
J_{1S} = \int \left( \frac{1}{z_1} \right)^{Ro} \, dz, \quad \text{first Einstein Integral for the suspended load}
\]

\[
J_{1P} = \int \left( \frac{1}{z_1} \right)^{Ro} \, dz, \quad \text{first Einstein Integral for the partial load}
\]

\[
J_{2S} = \int \ln \left( \frac{1}{z_1} \right)^{Ro} \, dz, \quad \text{second Einstein Integral for the suspended load}
\]

\[
J_{2P} = \int \ln \left( \frac{1}{z_1} \right)^{Ro} \, dz, \quad \text{second Einstein Integral for the partial load}
\]

\text{MPE} \quad \text{Mean Percent Error}

\text{n} \quad \text{Total number of verticals used particular statistical analysis}

\text{p} \quad \text{Number of sampling points along a vertical}

\text{P} \quad \text{Ratio of sampler nozzle height to flow depth} \quad P = \frac{z_p}{h}

\text{q}_b \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Unit bed load discharge by mass in the bed layer} \quad 0 < z < z_b

\text{q}_{pm} \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Partial unit sediment discharge by mass from the point measurements}

\text{q}_i \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Partial unit sediment discharge by mass calculated from SEMEPP}

\text{q}_{pm}/q_{t} \quad \text{Measured ratio of partial to total sediment discharge}

\text{q}_{m} \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Total unit sediment discharge by mass calculated from SEMEPP}

\text{q}_{t} \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Total unit sediment discharge by mass from the point measurements}

\text{RMSE} \quad \text{Root Mean Square Error}

\text{Ro} \quad \text{Rouse number}

\text{u}_* \quad \text{L}^{-1} \quad \text{Shear velocity} \quad u_* = \sqrt{ghS_f}

\text{u}_*/\omega \quad \text{Ratio of shear velocity to fall velocity}

\text{v} \quad \text{L}^{-1} \quad \text{Velocity}

\text{v}_i \quad \text{L}^{-1} \quad \text{Velocity at } i

\text{v}_b \quad \text{L}^{-1} \quad \text{Velocity in the bed layer, } v_b = 0.4u_*

\text{X}_i \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Measured sediment discharge}

\text{Y}_i \quad \text{M}^{-1}\text{L}^{-1}\text{T}^{-1} \quad \text{Calculated sediment discharge}

\text{z} \quad \text{L} \quad \text{Vertical elevation above the bed}

\text{z}_b \quad \text{L} \quad \text{Reference elevation of 2d, or bed layer thickness}

\text{z}_o \quad \text{L} \quad \text{Elevation above the bed where the velocity is zero}

\text{z}_* \quad \text{L} \quad \text{Nozzle elevation of the sampler above the bed}

\text{\beta} \quad \text{Momentum correction factor for sediment, assumed } \beta = 1

\text{\kappa} \quad \text{Von Kármán constant, assumed } \kappa = 0.4 \text{ in this study}

\text{\omega} \quad \text{L}^{-1}\text{T}^{-1} \quad \text{Fall velocity of sediment}