

Downstream Hydraulic Geometry of Alluvial Channels

Jong-Seok Lee, A.M.ASCE¹; and Pierre Y. Julien, M.ASCE²

Abstract: This study extends the earlier contribution of Julien and Wargadalam in 1995. A larger database for the downstream hydraulic geometry of alluvial channels is examined through a nonlinear regression analysis. The database consists of a total of 1,485 measurements, 1,125 of which describe field data used for model calibration. The remaining 360 field and laboratory measurements are used for validation. The data used for validation include sand-bed, gravel-bed, and cobble-bed streams with meandering to braided planform geometry. The five parameters describing downstream hydraulic geometry are: channel width W , average flow depth h , mean flow velocity V , Shields parameter τ^* , and channel slope S . The three independent variables are discharge Q , median bed particle diameter d_s , and either channel slope S or Shields parameter τ^* for dominant discharge conditions. The regression equations were tested for channel width ranging from 0.2 to 1,100 m, flow depth from 0.01 to 16 m, flow velocity from 0.02 to 7 m/s, channel slope from 0.0001 to 0.08, and Shields parameter from 0.001 to 35. The exponents of the proposed equations are comparable to those of Julien and Wargadalam (1995), but based on R^2 values of the validation analysis, the proposed regression equations perform slightly better.

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CE Database subject headings: Geometry; Hydraulics; Alluvial channels; Channel design; Channel morphology; Databases; Rivers; River flow.

Introduction

Alluvial rivers continuously change their hydraulic geometry to seek a balance between incoming and outgoing water and sediment discharges. The downstream hydraulic geometry of alluvial channels under bankfull conditions is defined in terms of channel width, average flow depth, mean flow velocity, and channel slope.

Considerable success has been achieved in river engineering after a century of field investigations on the hydraulic geometry of alluvial channels under equilibrium conditions. This analysis expands upon the contribution of Wargadalam (1993) and Julien and Wargadalam (1995). More recent literature on this topic is briefly reviewed. Tu and Graf (1992), Anthony (1992), Karim (1995), Lee (1995), Cheema et al. (1997), and Yen and Lee (1995) studied the velocity distribution, bed-load transport, bed configuration, and bed form changes in alluvial gravel-bed rivers. Cao and Knight (1998), Merigliano (1997), Julien and Anthony (2001), Kassem and Chaudhry (2002), and Olsen (2003) analyzed bed-load motion, numerical modeling of bed evolution and self-forming modeling in meandering streams. Yen (2002), Chiu and Tung (2002), and Smart et al. (2002) analyzed flow resistance and

maximum velocity in open channels. Guo and Jin (2002), Yang and Lim (2003), Abrahams (2003), and Haralampides et al. (2003) studied suspended-load, total-load, and bed-load sediment transport in alluvial streams. Buhman et al. (2002), Simon et al. (2002), and Chitale (2003) studied the fluvial hydraulic geometry and channel stability of the Mississippi, Red, and Missouri rivers. Recent contributions include Moody and Troutman (2002), Singh et al. (2003a,b), Snell et al. (2003), Wohl (2004), Wohl and Wilcox (2005), and Duan and Julien (2005).

With reference to bankfull flow conditions, Fig. 1 shows a typical cross-sectional geometry of a river. The downstream hydraulic geometry of a channel can be described as a function of Q =dominant discharge (m^3/s), W =channel width (m), h =average flow depth (m), V =mean flow velocity (m/s) taken normal to the cross-sectional area, and S =channel slope. Several downstream hydraulic geometry relationships are available in the literature, and usually, channel width, depth, velocity, and slope are described in terms of dominant discharge, bankfull discharge, or mean annual discharge either in International System of Units (SI) or English units. Most of these relationships consist independent variable, i.e., discharge, and fewer relationships include the median grain size of bed material as a second independent variable either as d_{50} , d_{65} , d_{84} , and d_{90} . Very few relationships consider the rate of sediment transport or the Shields parameter as a third independent parameter (e.g., Julien 2002).

The main objective is to define practical downstream hydraulic geometry relationships from the nonlinear regression analysis of a large data set as a function of channel width, average flow depth, mean flow velocity, channel slope, bed material size, and Shields parameter at bankfull condition.

Downstream Hydraulic Geometry Equations

Julien and Wargadalam (1995) derived semitheoretical hydraulic geometry relationships by combining four fundamental equations

¹Associate Professor, Dept. of Civil Engineering, Hanbat National Univ., Daejeon 305-719, Korea; presently, Visiting Professor, Dept. of Civil Engineering, Engineering Research Center, Colorado State Univ., Fort Collins, CO 80523. E-mail: ljs96@hanbat.ac.kr

²Professor, Dept. of Civil Engineering, Engineering Research Center, Colorado State Univ., Fort Collins, CO 80523. E-mail: pierre@engr.colostate.edu

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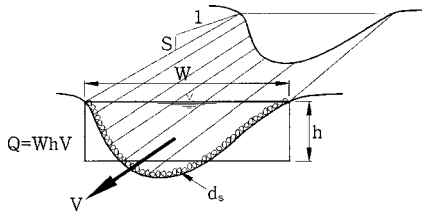


Fig. 1. Definition of alluvial meandering channel

for flow rate, resistance to flow, bed material mobility, and secondary flow in bends. These equations were combined and solved for channel width W , average flow depth h , mean flow velocity V , channel slope S , and Shields parameter τ^* . They are written as a power function of three independent variables to define discharge Q , median size of bed particles d_s , channel friction slope S , or Shields parameter $\tau^* = hS / [(G-1)gd_{50}]$ where G = specific gravity of sediment and g = gravitational acceleration. Hager (1996) later showed that a single dimensionless parameter can be defined, however this is the case only once the independent variables are preselected (Julien and Wargadalam 1996). Huang and Warner (1995) also obtained relationships as a function of Manning n .

Table 1 shows the range of data used for this analysis. The total data set of 1,485 measurements covers a wide range of flow conditions for sand-bed, gravel-bed, and cobble-bed streams with meandering to braided planform geometry. This study determined new regression equations for comparison with the Julien and Wargadalam (1995) (J-W) equations. The calibration is done with a

large database including 1,125 data points from Emmett (1972), Williams (1978), Griffiths (1981), Church and Rood (1983), Hey and Thorne (1986), Higginson and Johnston (1988), Lee (1999), and Kodoatie (2000). From this data set 1,125 field measurements at dominant flow conditions are used for calibration and verification. The validation uses 360 measurements with 309 field and 51 laboratory in order to ensure applications in hydraulic engineering.

Calibration of Regression Equations

A nonlinear analysis has been performed on the 1,125 field measurements of width, depth, velocity, slope, and Shields parameter as a power function of discharge Q , bed particle size d_s , and channel slope S or Shields parameter τ^* . The regression analysis has been conducted with the program SAS (Release 8.02) PROC NLIN. The nonlinear regression equations that were obtained are shown in Table 2, especially Eqs. (1a)–(5a). For comparison, the J-W equations corresponding to the Manning-Strickler resistance equation with $m=1/6$ are given in Table 2 Eqs. (1a')–(5a'). The J-W equations were obtained from 382 field data measurements. In general, the regression Eqs. (1a)–(5a) listed in Table 2 are quite comparable to J-W Eqs. (1a')–(5a'). Figs. 2–6 show the calibration results for the five parameters. Figs. 2–4 show very good agreement within the range from 1 to 1000 m for channel width, 0.05 to 20 m for flow depth, and 0.2 to 5 m/s for mean flow velocity, respectively. The channel slope and Shields parameter are also compared in Figs. 5 and 6 over a range from 0.0001 to 0.1 for slope and 0.001 to 20 for the Shields parameter.

Table 1. Calibration and Validation Data Range (Total 1,485 Measurements)

Notations	Calibration data (1,125 data)	Validation data (360 data)	
		Field (309 data)	Laboratory (51 data)
W (m)	1.44–1,097	2.32–610	0.22–2.44
h (m)	0.04–16	0.10–13	0.005–0.244
V (m/s)	0.02–7.1	0.04–4.26	0.11–0.86
Q (m ³ /s)	0.048–26,560	0.137–11,546	0.0002–0.227
S (-)	0.00001–0.081	0.000044–0.0508	0.00023–0.017
d_{50} (m)	0.00001–0.945	0.00002–0.343	0.000011–0.0023
τ^* (-)	0.00092–35.5	0.00093–12.7	0.1337–5.89

Table 2. Comparisons of Downstream Hydraulic Geometry Equations

Parameter	Author	Equation	Equation number	Calibration R^2	Validation R^2
W (m)	L-J	$W = 3.004Q^{0.426}d_{50}^{-0.002}S^{-0.153}$	(1a)	0.83	0.87
	J-W	$W = 1.33Q^{0.44}d_s^{-0.11}S^{-0.22}$	(1a')		0.77
h (m)	L-J	$h = 0.201Q^{0.336}d_{50}^{-0.025}S^{-0.060}$	(2a)	0.86	0.93
	J-W	$h = 0.2Q^{0.33}d_s^{0.17}S^{-0.17}$	(2a')		0.81
V (m/s)	L-J	$V = 2.996Q^{0.198}d_{50}^{0.007}S^{0.242}$	(3a)	0.54	0.89
	J-W	$V = 3.76Q^{0.22}d_s^{-0.05}S^{0.39}$	(3a')		0.87
S (-)	L-J	$S = 4.981Q^{-0.346}d_{50}^{0.955}\tau^{*0.966}$	(4a)	0.92	0.93
	J-W	$S = 12.4Q^{-0.4}d_s\tau^{*1.2}$	(4a')		0.86
τ^* (-)	L-J	$\tau^* = 0.090Q^{0.423}d_{50}^{-0.965}S^{0.912}$	(5a)	0.96	0.96
	J-W	$\tau^* = 0.121Q^{0.33}d_s^{-0.83}S^{0.83}$	(5a')		0.93

Note: The L-J equations refer to the regression equations presented in this paper by Lee and Julien. The J-W equations had variable exponents depending on meters. The value $m=1/6$ were used in this analysis. Also, the units for Q are in m³/s and d_s or d_{50} is in meters, while the slope is in m/m.

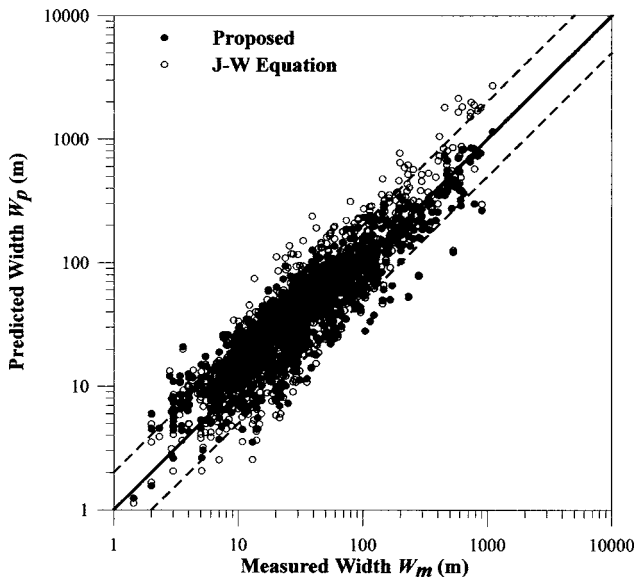


Fig. 2. Calibration of channel width

Solid dots refer to the regression equations in comparison to open dots for the Julien and Wargadalam equations. Results are comparable for width and depth in Figs. 2 and 3, but Eq. (1a) does not overestimate the width of large channels. Except for a few outliers at flow velocities $V < 0.1$ m/s, the results in Fig. 4 are slightly higher for Eq. (3a) than Eq. (3a'). The slope values in Fig. 5 are well predicted in both cases. In Fig. 6, the values of the Shields parameter from Eq. (5a) are better than those of Eq. (5a') at high values of τ^* .

Validation

The validation data set consists of 360 data points, 309 field data, and 51 laboratory data shown in Table 1. The validation data includes Brownlie (1981), Andrews (1984), Bathurst (1985), Collazzi (1985, 1989), Collazzi and Ireland (1985), Colosimo et

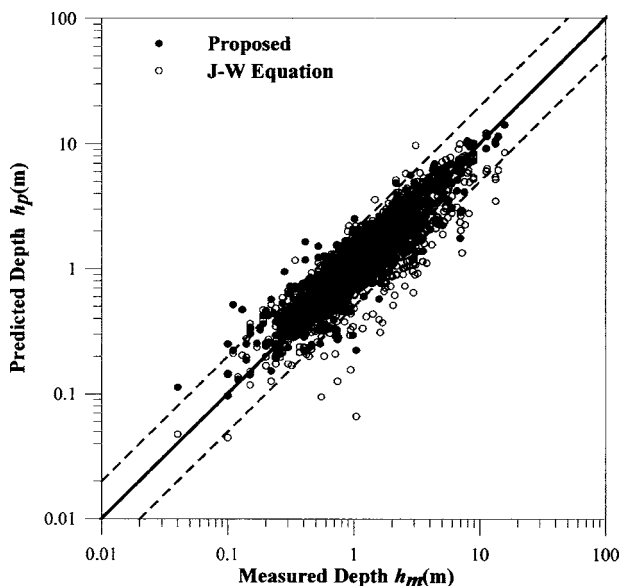


Fig. 3. Calibration of average flow depth

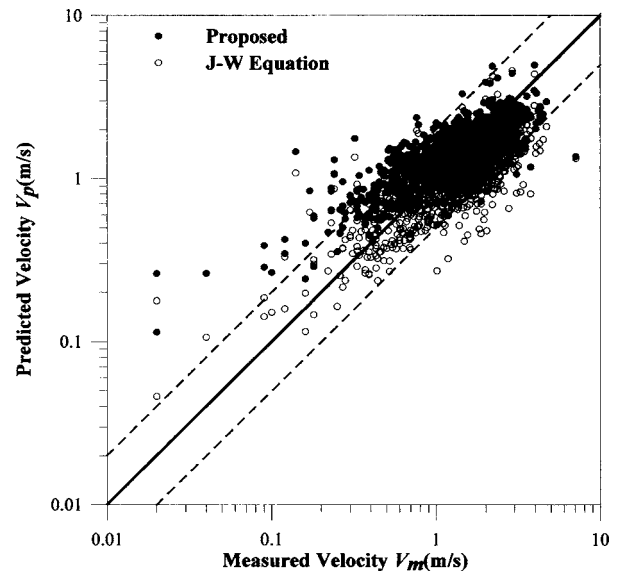


Fig. 4. Calibration of mean flow velocity

al. (1988), Lee (2001), Peter et al. (1999), Castro and Jackson (2001), Kodoatie (2000), and Julien (2002). The field validation data shown in Table 3 are classified into three bed-material types with median diameter d_{50} of sand (2.0 mm or less); gravel ($2 < d_{50} < 64$ mm); and cobble bed ($d_{50} > 64$ mm). The data set includes meandering to braiding planform geometry. In terms of validation, Figs. 7–9 show channel width, depth, and velocity results for sand, gravel, and cobble bed channel and laboratory data. Very good agreement is obtained except at low Reynolds numbers with flow channel depth less than 5 cm and flow velocity less than 0.2 m/s. The channel slope and Shields parameter are shown in Figs. 10 and 11 with excellent agreement.

The values of the determination coefficients R^2 for the proposed regression equations in Table 2 are slightly higher than the results obtained from the J-W equations. It is found that the proposed Eqs. (1a)–(5a) show a slightly better agreement with observed values as shown in Figs. 7–11.

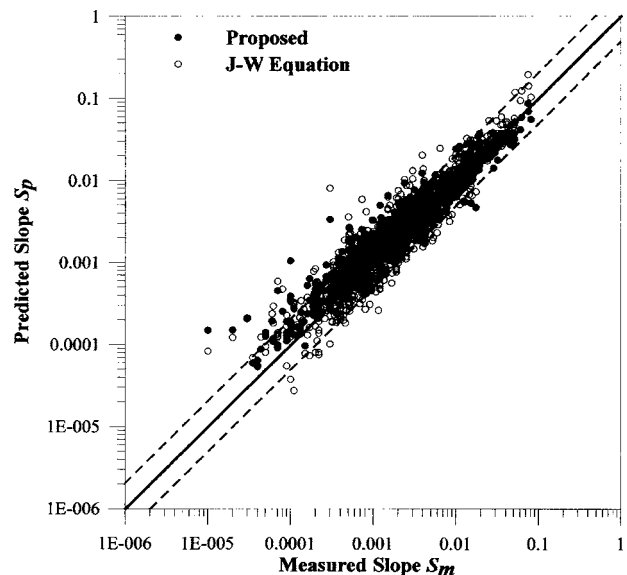


Fig. 5. Calibration of channel slope

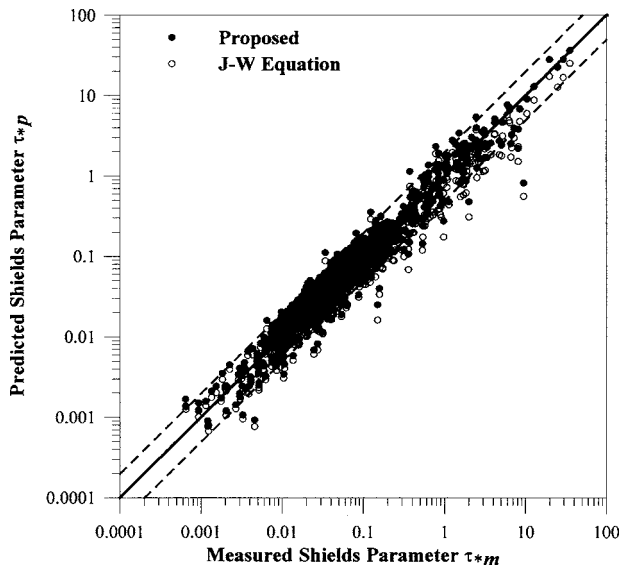


Fig. 6. Calibration of Shields parameter

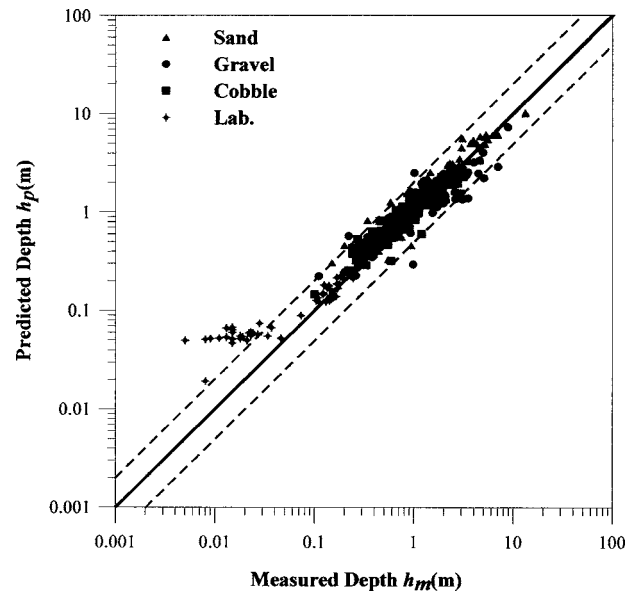


Fig. 8. Validation of flow depth

Table 3. Validation Range of Field Data

Notations	Field data (309 measurements)		
	Sand ($d_{50} < 2$ mm; 45 data)	Gravel ($2 \leq d_{50} < 64$ mm; 138 data)	Cobble ($64 \text{ mm} \leq d_{50}$; 126 data)
W (m)	3.6–610	2.3–594	4–104
h (m)	0.15–13.28	0.11–8.84	0.10–3.13
V (m/s)	0.1–2.93	0.04–3.32	0.26–4.26
Q (m ³ /s)	0.5377–10,222	0.40–11,546	0.137–650
S (-)	0.000044–0.00275	0.0001–0.026	0.0003–0.0508
d_{50} (m)	0.00002–0.00144	0.002–0.063	0.064–0.343
τ (-)	0.042–12.75	0.00093–2.08	0.00176–0.15

Conclusions

This analysis extends the earlier contribution of Julien and Wargadalam (1995). Five equations summarized in Table 2, Eqs. (1a)–(5a), were obtained from a regression analysis of a large data set on downstream hydraulic geometry of alluvial rivers. The equations for channel width, average flow depth, mean flow velocity, Shields parameter, and channel slope contain three independent variables in terms of a power function of discharge, median particle diameter, and either the channel slope or the Shields parameter.

The dataset used in deriving these equations consists of a total of 1,485 data points including 1,125 field measurements at bank-full condition for calibration, and 360 field and laboratory data for validation. The data set covers a wide range of conditions from

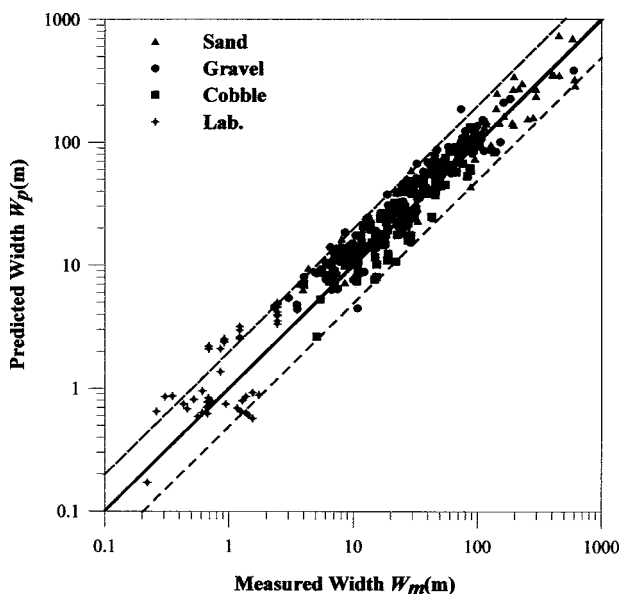


Fig. 7. Validation of channel width

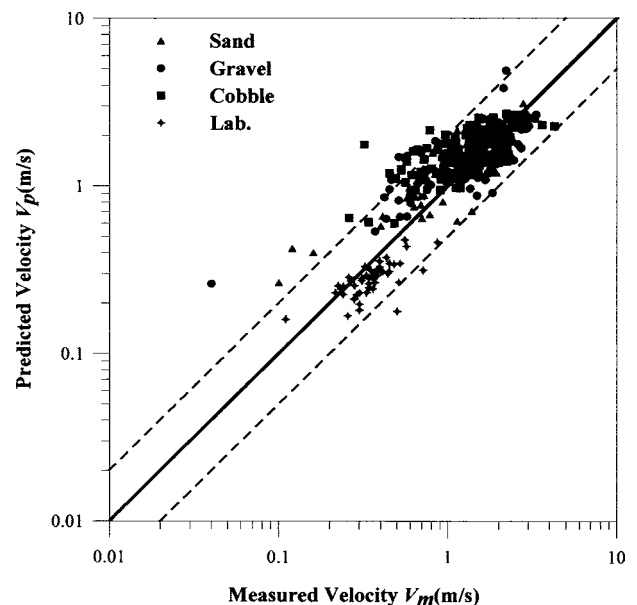


Fig. 9. Validation of flow velocity

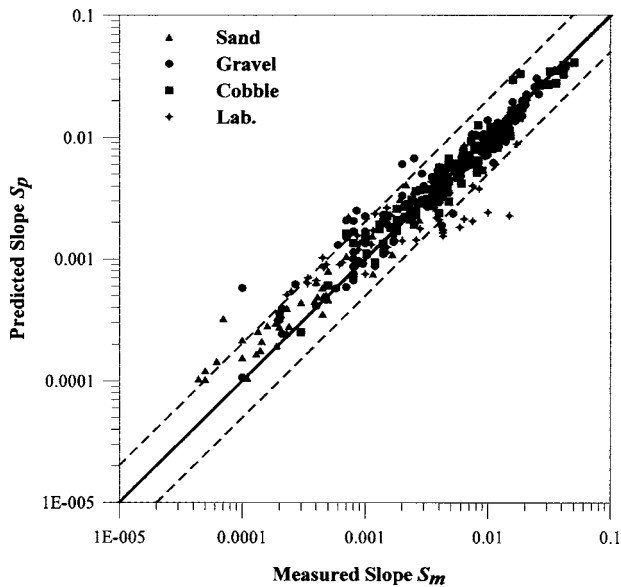


Fig. 10. Validation of channel slope

sand-bed, gravel-bed, and cobble-bed streams with meandering to braided planform geometry. In comparison with the J-W equations, the calculated values by the proposed regression equations are in slightly better agreement with the field and laboratory measurements of channel width, averaged flow depth, mean flow velocity, channel slope, and Shields parameter. The range of applicability of the proposed downstream hydraulic geometry equations is a channel width $1 < W < 1000$ m, average flow depth $0.5 < h < 20$ m, mean flow velocity $0.2 < V < 5$ m/s, channel slope $0.00001 < S < 0.1$, and Shields parameter $0.001 < \tau^* < 20$, respectively.

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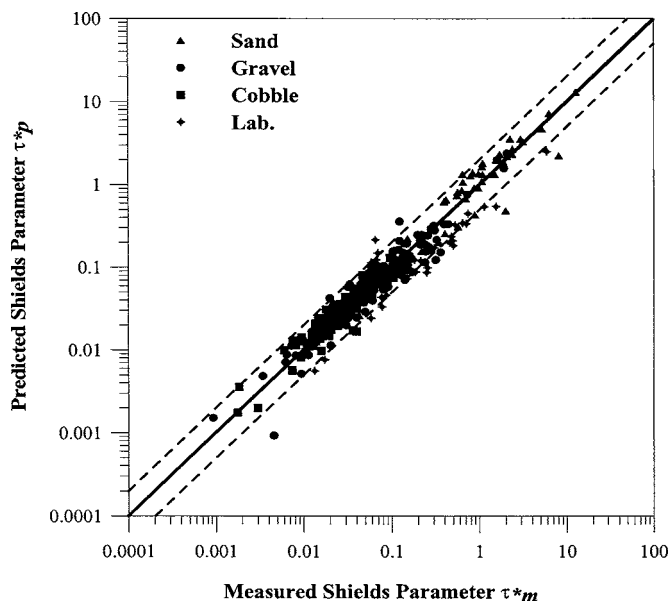


Fig. 11. Validation of Shields parameter

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Notation

The following symbols are used in this technical note:

- d_{50} = median grain size of bed material in m;
- G = specific gravity of sediment;
- g = gravitational acceleration;
- h = mean flow depth in m;
- Q = dominant discharge in m^3/s ;
- R^2 = coefficient of determination;
- S = channel slope;
- V = average flow velocity in m/s;
- W = channel width in m; and
- τ^* = $hS/(G-1)gd_{50}$ =Shields parameter.

Subscripts

- m = measured values; and
- p = predicted values.

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