# On the Higher-Order Hexahedral Meshing for FEM in Electromagnetics

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The finite element method (FEM) for discretizing partial differential equations in electromagnetics is an extremely powerful and versatile general numerical methodology for full-wave three-dimensional (3-D) electromagnetic simulations. Traditionally used FEM tools are low-order small-domain techniques – the finite elements are on the order of $\lambda/10$ in each dimension, $\lambda$ being the wavelength in the medium, and the fields within the elements are approximated by low-order (zeroth-order and first-order) basis functions. Only recently FEM techniques based on curved elements for geometrical modeling and/or higher order basis functions for field modeling have been employed, with an objective to significantly reduce the number of unknowns and computational resources for a given (high) accuracy when compared to low-order solutions. However, because these new and emerging elements are much more complex and flexible in shapes and sizes, generation of optimal meshes for practical 3-D electromagnetic problems based on any particular class of higher order finite elements is an enormously difficult and challenging research task, with a great interest to the electromagnetic community.

This paper presents our development of higher-order large-domain FEM meshing techniques based on recently proposed Lagrange-type curved hexahedral finite elements of arbitrary geometrical orders with hierarchical curl-conforming polynomial vector basis functions of arbitrarily high field-approximation orders (M. M. Ilić and B. M. Notaroš, IEEE Transactions on Microwave Theory and Techniques, Vol.51, No.3, 2003, pp.1026-1033). These elements can be electrically quite large (large domains), on the order of $\lambda$ in each dimension, thus fully exploiting the accuracy, efficiency, and convergence properties of the higher order FEM. We propose a semi-automatic algorithm for the higher-order large-domain hexahedral mesh generation that represents a combination of the domain decomposition and mapped meshing techniques. It utilizes the "top-down" approach to mesh generation, where the first step is a creation of a parametric mesh of topologically equivalent elements with correct connections (adjacencies), based on the problem topology and adopted orders of elements. Interpolation nodes are then assigned to the elements, where the number of nodes and their proper ordering depend on the geometrical orders of elements. Next, parts of the structure are mapped from the parametric space to the curved hexahedral elements in real three-dimensional space (geometrical embedding). Finally, these forms (sub-meshes) are connected together appropriately into an optimal large-domain mesh. The trade-off between the shape/quality of the elements and their sizes will be discussed. The importance of adopting the optimal combination of the field approximation orders will also be explained and possible refinement strategies addressed. Several examples illustrating the effectiveness and flexibility of the proposed mesh generation technique will be presented. The results of the electromagnetic analysis of complex 3-D waveguiding structures demonstrate an excellent accuracy while reducing the number of unknowns by approximately 80% when compared with the existing techniques in the literature.