Efficient Modeling of Complex Electromagnetic Structures Based on the Novel Algorithm for Spatial Segmentation Using Hexahedral Finite Elements

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***Abstract* — A new algorithm for spatial segmentation using hexahedral finite elements, combined with the algorithm for spatial segmentation using quadrilateral elements, is presented. Problems with combining the algorithms for segmentation by hexahedral and quadrilateral elements in the hybrid FEM-MOM technique are pointed out. An example of analysis of a scatterer at resonant frequency is given.**

***Keywords* — Electromagnetic analysis, finite element methods, geometrical modeling, hybrid techniques, meshing, method of moments.**

I. Introduction

HYBRIDIZATION of large-domain finite element method (FEM) [1] with large-domain method of moments (MoM) [2] gives a robust large-domain hybrid FEM-MoM technique [3] for numerical modeling of electromagnetic (EM) structures. This technique has great advantages compared to similar small-domain techniques [4]-[8]. Rather than modeling the EM structure using a large number of electrically small elements ($\lambda/10$ on the edge in each direction, where λ stands for the wavelength in the medium), the structure is modeled using a small number of electrically large elements (up to 2λ on the edge in each direction). This significantly reduces the number of unknowns, required for solving of the EM problems, and thus lessens the utilization of the computational resources.

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However, due to the large main building blocks exploited in large-domain modeling, there are difficulties with spatial segmentation, especially in the case of curved structures. Therefore development of algorithms and software for automatic spatial segmentation, in the large-domain modeling, is much more difficult than in the case of small-domain modeling. In small-domain modeling, there exist standard commercially available software packages (meshers), typically based on tetrahedral and triangular elements for modeling of volumes and surfaces, respectively. To the best knowledge of the authors, there are currently no commercially available software packages that perform automatic spatial segmentation using large curved hexahedral elements. This was the main motivation for development of the novel algorithm for spatial segmentation using large hexahedral finite elements.

Section II presents the theoretical background for the large-domain modeling. In section III, the existing algorithm for spatial segmentation using quadrilaterals is described. Problems with combining the algorithms for spatial segmentation using quadrilaterals and hexahedral finite elements are highlighted. Section IV presents the novel algorithm for spatial segmentation using hexahedral finite elements. In section V, the new algorithm is validated in the example of analysis of spherical electromagnetic scatterer, at the resonant frequency.

II. Generalized Curved Parametric Hexahedron

The choice of element-type for geometrical modeling of EM structures involves a trade-off between the flexibility of the element at modeling different geometries on one side, and its mathematical complexity, on the other. Brick, tetrahedron and triangular prism, for instance, are simple to implement and their parameters are fast to compute. On the other hand, geometrical flexibility of these elements is poor and, because of their straight edges and planar sides, modeling of complex curved structures becomes exceedingly cumbersome and requires extremely fine meshes in order to achieve the satisfactory level of geometrical approximation. This, in turn, leads to the necessary reduction of elements' size, i.e., to small-domain technique.

An attractive choice of flexible elements, with (possibly) curved sides, are generalized curved parametric hexahedra of higher (theoretically arbitrary) geometrical orders [1], which are adopted as basic building blocks for geometrical modeling of three-dimensional (3-D) electromagnetic structures of arbitrary shapes and material inhomogeneities. A generalized hexahedron is determined by $M=(K+1)^3$ points (interpolation nodes) arbitrarily positioned in space, K $K \geq 1$ being the geometrical order of the element. It can be described analytically as

$$\boldsymbol{r}(u,v,w)=\sum_{i=1}^{M}\boldsymbol{r}_i \hat{L}_i^K(u,v,w), \qquad -1 \leq u,v,w \leq 1, \tag{1}$$

where $\boldsymbol{r}_1,\boldsymbol{r}_2,\ldots,\boldsymbol{r}_M$ are the position vectors of the interpolation nodes, $\hat{L}_i^K(u,v,w)$ are Lagrange-type interpolation polynomials satisfying the Kronecker delta relation $\hat{L}_i^K(u_j,v_j,w_j)=\delta_{ij}$, with u_j,v_j and w_j representing the parametric coordinates of the j-th node. Equation (1) defines a mapping from a *parent element,* in this case a reference cube occupying a $-1 \leq u,v,w \leq 1$ domain in $u-v-w$ space, into a curved generalized hexahedron in $x-y-z$ space, as shown in Fig. 1.

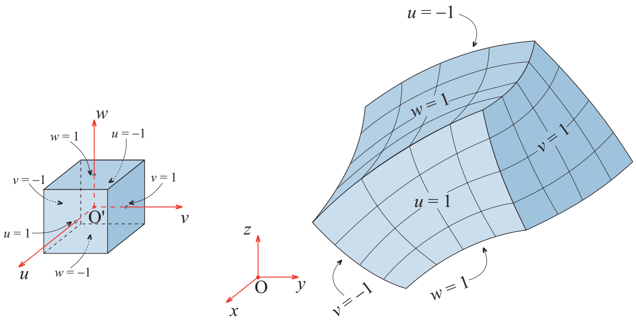

Fig. 1. Cube to hexahedron mapping defined by (1).

The first-order element ($K=1$), called the trilinear hexahedron [9], along with the visible coordinate lines, is shown in Fig. 2(a). It is determined solely by $M=8$ interpolation points – its 8 vertices. Its edges and all coordinate lines are straight, whereas its sides, so-called *bilinear quadrilateral surfaces* [10], are somewhat curved (inflexed). Note that even these hexahedra provide the same or better flexibility for geometrical modeling of general electromagnetic structures as compared to commonly used elements in the form of brick, tetrahedra, and triangular prisms.

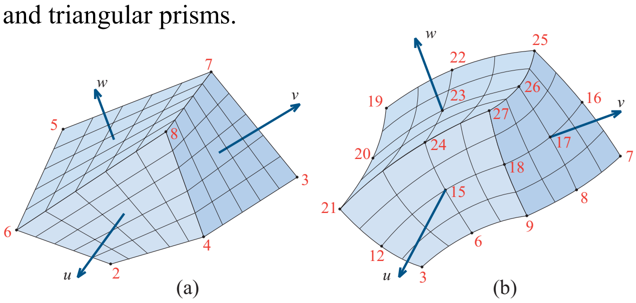

Fig. 2. (a) Trilinear hexahedron ($K=1$, $M=8$), (b) Triquadratic hexahedron ($K=2$, $M=27$).

The second order element ($K=2$), called the triquadratic hexahedron and shown in Fig. 2(b), is determined by $M=27$ interpolation points arbitrarily positioned in space. Its edges and all coordinate lines are second-order polynomial curves. Its sides, generalized curvilinear quadrilaterals, are curved and flexible and can easily approximate different curvatures. Of course, parametric bodies of higher ($K>2$) geometrical orders provide even better flexibility and accuracy at modeling complex curved structures. Drawback is calculation of complex geometrical parameters which can significantly increase the total running time of the EM simulation.

A generalized curvilinear quadrilateral, like the one shown in Fig. 3, has been chosen for surface geometrical modeling in large-domain MoM method [2].

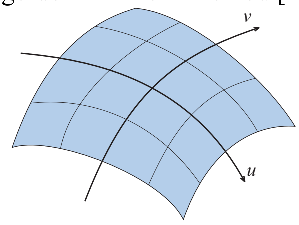

Fig. 3. Lagrange-type curved parametric quadrilateral.

III. Algorithm for Spatial Segmentation Using Quadrilaterals

In hybrid FEM-MoM technique, FEM domain is bounded and closed with MoM quadrilaterals (MoM boundary conditions). The main idea for creating an algorithm for the spatial segmentation using hexahedral finite elements, is to achieve compatibility with the existing algorithm for spatial segmentation using quadrilateral elements. A compatible solution implies that after meshing of an EM structure, utilizing the hybrid FEM-MoM technique with hexahedral and quadrilateral elements, each of the sides of hexahedra, belonging to the FEM domain outer boundary, coincides with only one MoM quadrilateral. Also, one MoM quadrilateral can belong to only one hexahedral side on the boundary.

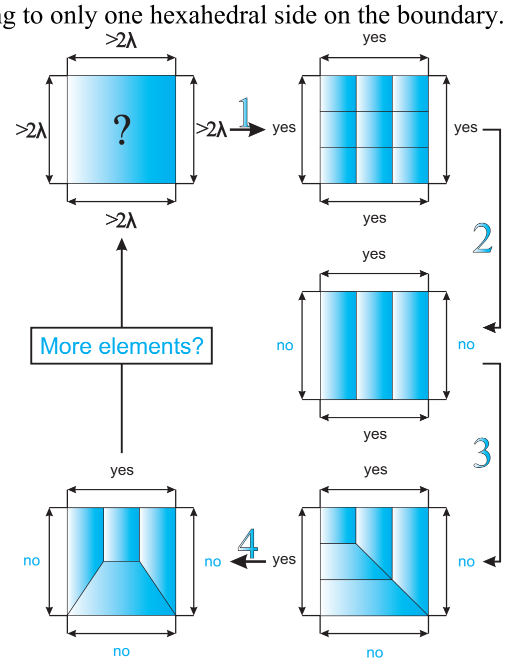

Fig. 4. Existing algorithm for spatial segmentation using quadrilaterals.

The existing algorithm for spatial segmentation using quadrilaterals (similar to the one used in the software package WIPL-D [11]) is symbolically shown in Fig. 4. The algorithm will be executed only for those quadrilaterals with at least one edge longer than 2λ. Through four steps, depending on which edge is longer than 2λ, quadrilateral will be divided into smaller quadrilaterals.

In certain cases, it is not possible to directly map the existing algorithm for spatial segmentation using quadrilateral elements onto the algorithm for spatial segmentation using hexahedral elements.

An example of the hexahedron with two edges longer than 2λ, is shown in Fig. 5. Hexahedron edges longer than 2λ have one common vertex (in the bottom plane). If we try to divide the hexahedron in a manner compatible with the existing algorithm for spatial segmentation using quadrilaterals, the domain for which it would not be possible is shown in dark color in Fig. 5.

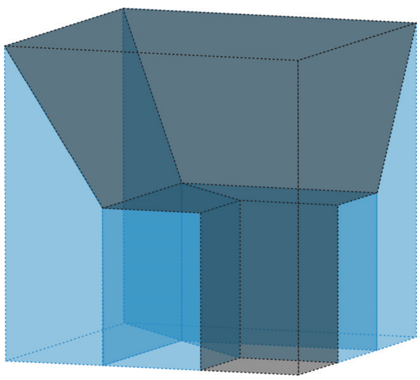

Fig. 5. Two edges of the hexahedron with common vertex are longer than 2λ. Compatibility of algorithms for spatial segmentation using quadrilaterals and hexahedra is not possible.

IV. Novel Algorithm for Spatial Segmentation Using Hexahedral Finite Elements

Since it is impossible to make a compatible solution for spatial segmentation of the hexahedron with the existing algorithm for spatial segmentation using quadrilaterals (shown in section III), we developed a new semi-automatic algorithm for spatial segmentation using hexahedral finite elements, together with a compatible algorithm for spatial segmentation using quadrilaterals.

For each hexahedron, user specifies how it would be divided (meshed). For a selected hexahedron, for each direction (i.e. for each local coordinate *u*, *v* and *w*), user specifies the number of divisions. Appropriate quadrilaterals are automatically meshed in a compatible way. Algorithm is capable of working with hexahedra and quadrilaterals of arbitrary geometrical orders. An example of the mesh, is shown if Fig. 6. The corresponding MoM quadrilaterals are also shown.

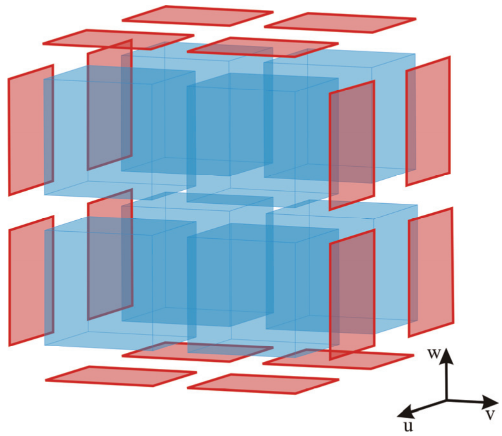

Fig. 6. Hexahedral mesh using the new algorithm for spatial segmentation. Corresponding quadrilaterals are shown in red.

V. Numerical Result

Consider a dielectric coated perfect electric conductor (PEC) spherical scatterer shown in Fig. 7. Relative permittivity and permeability of the dielectric are $\varepsilon_r = 4$ and $\mu_r = 1$, respectively. The radii of the spheres are $a = 0.3423\lambda_0$ for the PEC sphere and $b = 0.4440\lambda_0$ for the dielectric coating, λ_0 being the free-space wavelength. This corresponds to the frequency of the internal resonance of the sphere.

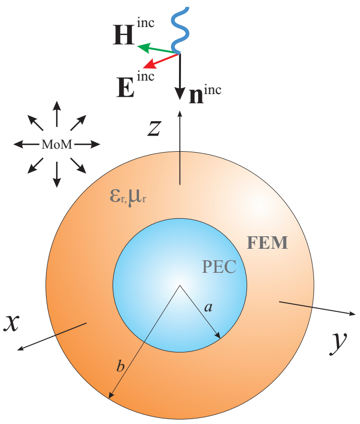

Fig. 7. Scattering by a dielectric coated PEC spherical scatterer ($a = 0.3423\lambda_0$, $b = 0.4440\lambda_0$, $\varepsilon_r = 4$, and $\mu_r = 1$).

The basic model, for which the new algorithm for spatial segmentation is applied, is composed of 6 curved, second geometrical order, hexahedra [12]. From the inside, the FEM domain is closed with the PEC boundary condition. From the outside, FEM domain is closed with 6 curved, second geometrical order, quadrilaterals.

Applying the new algorithm for spatial segmentation, each of the 6 FEM elements, in the case 2x2x1 (user-specified number of divisions in two transversal directions and the radial direction, respectively), is divided into 4 equal FEM elements, which gives a total of 24 FEM elements and 24 MoM elements. Each of the 6 FEM elements, in the case 3x3x1, is divided into 9 equal FEM elements, which gives a total of 54 FEM elements and 54 MoM elements. The cross sections of the FEM-MoM models of the spherical scatterer, after applying the new algorithm for spatial segmentation, are shown if Fig. 8.

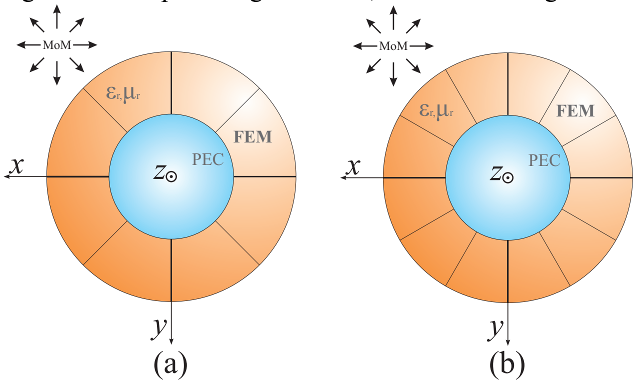

Fig. 8. Cross sections of the FEM-MoM models of the spherical scatterer in Fig. 7 after application of the segmentation algorithm; (a) case 2x2x1, and (b) case 3x3x1.

Shown in Fig. 9 is the normalized bistatic radar cross section RCS/λ_0^2 of the coated spherical scatterer for the plane wave incidence indicated in Fig. 7. The adopted field and current approximation in the FEM-MoM model are second order. The results are compared with the exact Mie's solution and with numerical results obtained by the low-order symmetric FEM-Integral Equation (FEM-IE) [8]. A good agreement of the four sets of results is observed, with the higher order model providing a more accurate RCS prediction at angles around 40 degrees than the low-order FEM-IE model.

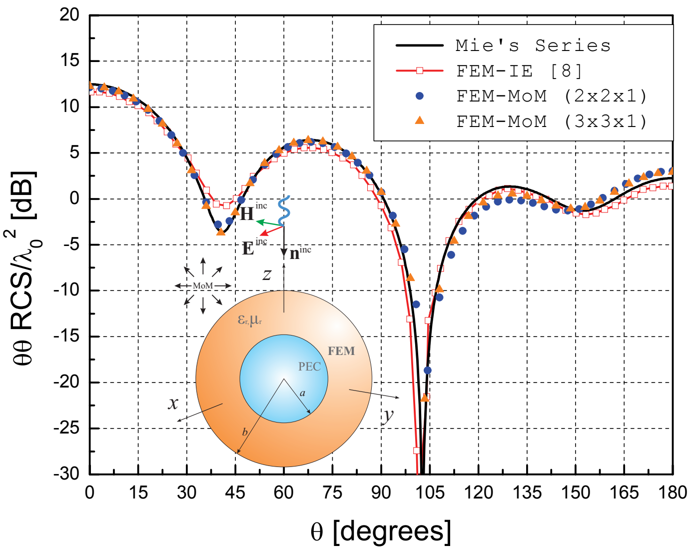

Fig. 9. Normalized bistatic RCS of the coated dielectic sphere from Fig. 7. Comparison of the higher order FEM-MoM solution with the analytic solution and a low-order FEM-IE solution from [8].

VI. CONCLUSION

A new algorithm for spatial segmentation using hexahedral finite elements which is capable of taking all advantages from the large-domain modeling is presented. An example of analysis of the electromagnetic scatterer with pronounced curvature at internal resonance has been given to show the accuracy and robustness of the large-domain modeling, combined with the new algorithm for spatial segmentation. Our future work will include the development of fully automated algorithm for spatial segmentation using hexahedral finite elements, which will enable more efficient way of modeling real world EM structures, and which will be capable of exploiting all advantages of the large-domain modeling.

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