Hierarchical and Interpolatory Higher-Order Vector Basis Functions for Finite Element Method and Method of Moments

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This paper presents our investigations in utilizing higher-order vector polynomial basis functions for approximating fields and currents in computational electromagnetics. We demonstrate significant computational advantages of higher-order (large-domain) basis functions over traditionally used low-order (subdomain) basis functions, as implemented in 3-D computational techniques based on the finite element method (FEM) and the method of moments (MoM). In addition to FEM discretization of partial differential equations, we present the MoM discretization of surface integral equations (SIE) and volume integral equations (VIE), as well as thin-wire modeling. As a basic building block for FEM and VIE geometrical modeling of 3-D structures, we use a generalized curved parametric hexahedron. Generalized curved parametric quadrilaterals are used for SIE geometrical modeling.

We propose and discuss several types of higher-order bases of both hierarchical and interpolatory forms defined in generalized hexahedrons and quadrilaterals. The functions are implemented in the Galerkin-based FEM and MoM techniques. The FEM functions are curl-conforming, whereas the MoM functions are divergence-conforming. Interpolatory basis functions have excellent orthogonality properties and produce well-conditioned FEM and MoM matrices. Hierarchical basis functions enable using different orders of field and current approximation in different elements for efficient selective discretization of the solution domain, because each lower-order set of functions is a subset of higher-order sets. This property allows for a whole spectrum of element sizes (e.g., from a very small fraction of the wavelength to a couple of wavelengths) and the corresponding field and current approximation orders (e.g., from 1 to 10) to be used at the same time in a single simulation model of a complex structure. Additionally, each individual 2-D or 3-D element can have drastically different sizes in different directions, enabling strip-like, trapezoid-like, and triangle-like quadrilaterals, as well as a whole variety of "irregular" hexahedral shapes. We show that hierarchical basis functions based on using ultraspherical (Gegenbauer) and Chebyshev polynomials have much better orthogonality properties, leading to better conditioned matrices and more stable solutions.

Numerous examples of diverse canonical and practical 3-D structures are presented, which are analyzed by the FEM and MoM utilizing various higher-order discretizations. The results are cross-validated, compared with measurements, and against the numerical results obtained by different low-order techniques. Numerical properties of the presented higher-order basis functions are discussed in detail.