Computation of FEM-Domain Fields in the Higher Order Hybrid FEM-MoM Solution

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Introduction

Hybrid finite element-boundary integral (FE-BI) or finite element method-method of moments (FEM-MoM) techniques [1]-[3] are powerful numerical tools for electromagnetic simulations. A tremendous amount of effort has been invested in the research of FE-BI techniques in the past two decades [4]-[6]. A comprehensive list of works on this subject can be found in [7], where we presented a novel higher order large-domain Galerkin-type hybrid FEM-MoM technique for 3-D electromagnetic analysis of arbitrary antennas and scatterers.

Our novel method has been validated and its performance rigorously evaluated in computation of the far field radiation and scattering patterns of antennas and scatterers, respectively, in the numerous examples given in [7]. Our goal in this work is to analyze possible methods for calculating the near fields in the FEM domain and to report the importance of inclusion of the magnetic currents, obtained by the MoM part of the hybrid method, in the procedure.

Theory and Implementation

Consider an arbitrarily shaped generally inhomogeneous body situated in linear homogeneous medium. Let this system be excited by a time-harmonic electromagnetic (EM) field of complex field intensities $\mathbf{E}^{\mathrm{inc}}$ and $\mathbf{H}^{\mathrm{inc}}$, and angular frequency ω. We decompose the system into a MoM (exterior) region and a FEM (interior) region (which includes the inhomogeneous body). In our hybrid higher order FEM-MoM technique we employ the surface equivalence principle and express the total electric and magnetic field intensity vectors in the exterior region in terms of the equivalent surface electric current, of density $\mathbf{J}_{\mathrm{S}}$, and equivalent surface magnetic current, of density $\mathbf{M}_{\mathrm{S}}$, that are placed on the outer boundary (surface *S*) of the interior region, and the incident or impressed field vectors. After the fields $\mathbf{E}$ and $\mathbf{H}$ in the exterior region are coupled to the corresponding fields in the interior region, through boundary conditions for the tangential field components on the surface *S*, we obtain

$$-[\mathbf{E}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})]_{\tan} - [\mathbf{E}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})]_{\tan} + (\mathbf{E}^{b})_{\tan} = (\mathbf{E}^{\mathrm{inc}})_{\tan}\,, \qquad (1)$$

$$-[\mathbf{H}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})]_{\tan} - [\mathbf{H}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})]_{\tan} + \mathbf{J}_{\mathrm{S}} \times \mathbf{n} = (\mathbf{H}^{\mathrm{inc}})_{\tan}\,, \qquad (2)$$

where $\mathbf{E}_{\mathrm{J}}$ and $\mathbf{H}_{\mathrm{J}}$ stand for the scattered electric and magnetic fields due to current $\mathbf{J}_{\mathrm{S}}$, while $\mathbf{E}_{\mathrm{M}}$ and $\mathbf{H}_{\mathrm{M}}$ are the scattered fields due to $\mathbf{M}_{\mathrm{S}}$. These fields are computed employing Lorentz' potentials and Green's function for the unbounded homogeneous medium. Currents $\mathbf{J}_{\mathrm{S}}$ and $\mathbf{M}_{\mathrm{S}}$ over *S* and field $\mathbf{E}^{b}$, in the FEM region, are unknown. These quantities are represented as

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$$\mathbf{J}_\mathrm{S} = \sum_{j=1}^{N_\mathrm{MoM}} \alpha_j \, \mathbf{j}_{\mathrm{S}j} , \quad \mathbf{M}_\mathrm{S} = \sum_{j=1}^{N_\mathrm{MoM}} \beta_j \, \mathbf{j}_{\mathrm{S}j} , \tag{3}$$

$$\mathbf{E}^b = \sum_{l=1}^{N_\mathrm{FEM}} \gamma_l \, \mathbf{e}_l , \tag{4}$$

where $\mathbf{j}_{\mathrm{S}j}$ are the adopted higher order MoM (divergence-conforming) vector basis functions on Lagrange-type curvilinear quadrilaterals, with unknown current-distribution coefficients α_j and β_j , while $\mathbf{e}_l$ are higher order FEM (curl-conforming) vector basis functions on Lagrange-type curvilinear hexahedra, with unknown field-distribution coefficients γ_l.

A Galerkin weak-form discretization of the curl-curl electric-field vector wave equation [8] in the interior region yields the FEM matrix equation

$$[FEM_{kl}]\{\gamma_l\} = [C_{kj}]\{\alpha_j\} , \tag{5}$$

where elements of the $[FEM]$ and $[C]$ matrices can be readily evaluated and $\{\gamma\}$ and $\{\alpha\}$ are the unknown vectors of FEM field and MoM electric-current distribution coefficients, in (4) and (3), respectively. Galerkin discretization of (1) and (2), with testing and basis functions in (3) [9], yields the MoM matrix over the FEM-MoM interface

$$\begin{bmatrix} [-Z_{ij}^{ee} + < \mathbf{j}_{\mathrm{S}_i}, \mathbf{E}^b(\mathbf{j}_{\mathrm{S}_j}) >] & [-Z_{ij}^{em}] \\ [-Z_{ij}^{me} + < \mathbf{j}_{\mathrm{S}_i} \times \mathbf{j}_{\mathrm{S}_j}, \mathbf{n}) >] & [-Z_{ij}^{mm}] \end{bmatrix} \begin{bmatrix} \{\alpha_j\} \\ \{\beta_j\} \end{bmatrix} = \begin{bmatrix} \{v^e\}^\mathrm{inc} \\ \{v^m\}^\mathrm{inc} \end{bmatrix} . \tag{6}$$

All terms in (6) can be readily evaluated except $\mathbf{E}^b(\mathbf{j}_{\mathrm{S}_j})$, for which we solve from (4) and (5) as follows:

$$\mathbf{E}^b(\mathbf{j}_{\mathrm{S}_{\hat{j}}}) = \sum_{l=1}^{N_\mathrm{FEM}} \gamma_l^{\hat{j}} \, \mathbf{e}_l = \{\gamma_l^{\hat{j}}\}^\mathrm{T} \{\mathbf{e}_l\} , \tag{7}$$

$$\{\gamma_l^{\hat{j}}\} = [FEM_{kl}]^{-1}\{C_{k\hat{j}}\} , \tag{8}$$

where $\{C_{k\hat{j}}\}$ (with a fixed $j = \hat{j}$) stands for the $\hat{j}$-th column of $[C_{kj}]$, and (7) and (8) are computed for all values of $\hat{j}$ from 1 to N_MoM , i.e., for unit excitations. The system (6) is solved for the unknown current distribution coefficients $\{\alpha_j\}$ and $\{\beta_j\}$, i.e., for the MoM surface currents $\mathbf{J}_\mathrm{S}$ and $\mathbf{M}_\mathrm{S}$ on *S*, which, in turn, are used to obtain the exterior fields.

We now proceed to compute the fields in the interior (FEM) region. Theoretically, FEM field coefficients $\{\gamma_l\}$ can be found from (5),

$$\{\gamma_l\} = [FEM_{kl}]^{-1}[C_{kj}]\{\alpha_j\} , \tag{9}$$

and the interior electric field can be evaluated by means of the expansion (4). We refer to this solution as to one "without magnetic currents" in the following section. Additionally, however, from the boundary condition $\mathbf{M}_\mathrm{S} = -\mathbf{n} \times \mathbf{E}$ on *S*, there exists a direct relation between the magnetic currents and tangential components of the electric field in the FEM domain on *S*. This relation can be formally expressed as

$$\{\gamma_l\} = [D_{lj}]\{\beta_j\} , \tag{10}$$

where only those coefficients $\{\gamma_l\}$ which govern $\mathbf{E}_{\tan}$ on S are involved. After careful examination of the employed basis functions, the coefficients in $[D_{lj}]$ are found to be 0, $1/2$, or $-1/2$, depending on the orders of the basis functions in both FEM and MoM regions and orientations of both FEM and MoM elements. In the following section, we refer to the solution of the interior electric field where (10) is applied in succession to (9) as to one "with magnetic currents".

Results and Discussion

As an example, consider a lossless cubical dielectric ($\varepsilon_r = 2.5$) scatterer, of side length $a = 1$ m, situated in vacuum. The center of the cube is placed in the coordinate system's origin and its vertices are parallel to the Cartesian axes. The cube is excited by a uniform plane TEM wave, of complex intensity (in the plane of origin) $E_z^{\text{inc}} = (-1 + \text{j}0)$ V/m, traveling in the direction opposite to the *x*-axis. The computed *z*-component of the complex electric field, E_z, in the cube along the *x*-axis (middle of the cube) are given in Fig. 1 at two frequencies. The results obtained by the hybrid higher order FEM-MoM are compared with the pure MoM solution, used for reference. The model of the cube in the hybrid higher order FEM-MoM solution is comprised of one FEM hexahedron and 6 MoM quadrilaterals. The FEM-MoM interface is placed exactly on the cube surface. All elements in the model are of the first geometrical order. The field expansion orders in the model are 5 (in all directions) for the FEM hexahedron and current expansion orders are 4 for MoM quadrilaterals. This yields a total of 540 FEM and 384 MoM unknowns. It can be observed from the figures that the field in the cube (FEM domain) is retrieved with significantly higher accuracy when solutions for the surface magnetic currents are included along with those for the electric currents as excitations on the FEM boundary.

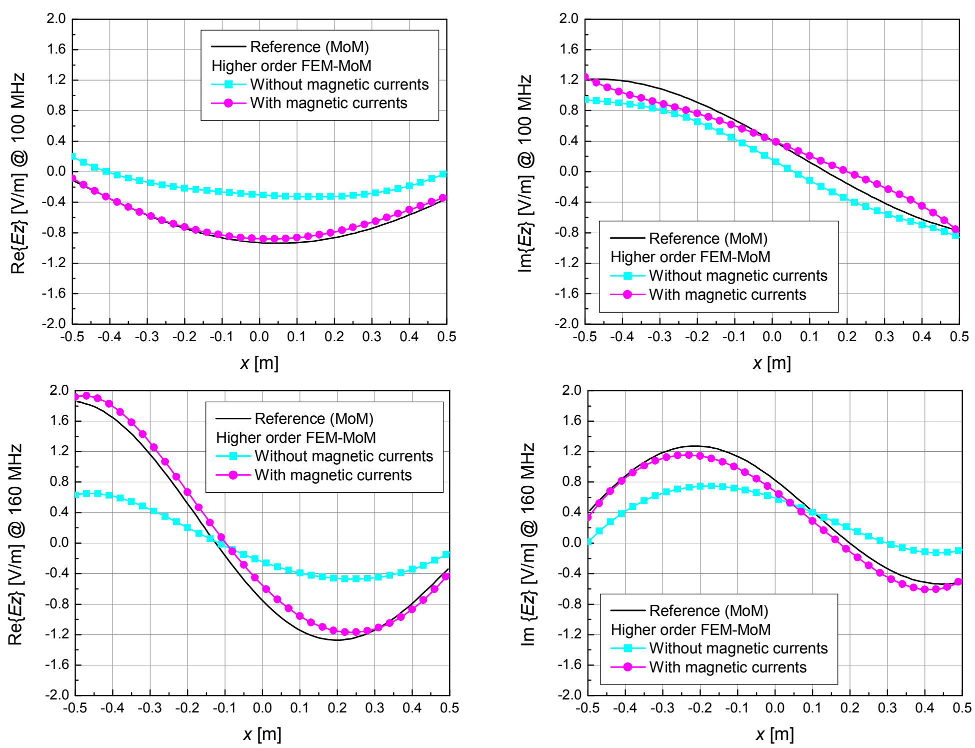

Fig. 1. Real and imaginary parts of E_z in the cubical scatterer along the *x*-axis.

Conclusions

We have developed the means to include magnetic currents in computation of the near field in the FEM domain in our hybrid higher order FEM-MoM technique. Numerical examples have shown significant improvement in accuracy of the electric field solution in the FEM domain when magnetic currents computed by the MoM part of the hybrid method are included in the procedure.

Acknowledgement

This work was supported by the National Science Foundation under Grants ECCS-0647380 and ECCS-0650719 and by the Serbian Ministry of Science and Technological Development.

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