Diakoptic Higher-Order FEM-MoM Approach

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Introduction

The principal objective of the work presented here is to demonstrate the proof-of-concept of coupling a higher-order finite element method (FEM) with a higher-order surface integral equation (SIE) method of moments (MoM) using the diakoptic approach. The diakoptic MoM-SIE approach has been successfully used for the electromagnetic (EM) analysis of microwave transmission lines, large scatterers, and antenna arrays [1], [2]. The diakoptic approach in this work uses coefficients calculated by a higher-order FEM code [3] and a higher-order MoM code [4], [5], and combines them to obtain the solution of the original EM problem. We briefly present the theory behind the concept and results for a case where the proposed approach is successfully applied with some diakoptic subsystems being solved using the FEM and the others using the MoM-SIE formulation.

Brief Theory of Diakoptic Approach

The diakoptic approach is based on the idea that the EM system can be split into subsystems that are more efficiently solved, and their solution can be combined together to yield the solution of the original EM problem while maintaining the accuracy. The diakoptic subsystem consists of a part of the original EM structure and the artificial closed surface, called a diakoptic boundary, which enclosed the subsystem domain. The domains of the different diakoptic subsystems do not overlap. The union of all subsystem domains is the domain of the original problem. At the diakoptic boundaries, (new) unknown coefficients are introduced to model the equivalent surface electric and magnetic currents. These coefficients must be the same (although with the different sign) for both inside and outside subsystems, with respect to the boundary. This is due to the surface equivalence principle [6]. Every diakoptic subsystem is represented by matrices that contain the linear relations between the coefficients at the diakoptic boundary and the part of the original EM system within the diakoptic subsystem. (The approach works if the media are piecewise homogeneous and linear.) Combining such matrices for all subsystems, the EM field of the original problem is calculated. In this paper, we show that some diakoptic subsystems can be solved using the higher-order MoM with the SIE formulation, while other subsystems are solved using the higher-order FEM. These partial solutions are combined together using the diakoptic approach to find the solution of the original EM problem.

Numerical Example: Dielectric Cube

In order to demonstrate the diakoptic approach, we consider a dielectric cube. The side of the cube is $a = 1\,\mathrm{m}$. The dielectric is isotropic, homogeneous, and linear with the relative permittivity $\varepsilon_\mathrm{r} = 5$ and permeability μ_0. The cube is illuminated by a TEM plane wave from the direction $(\theta, \phi) = (0, 0)$. The outer domain, which is air-filled, is analyzed using the MoM-SIE, while the interior region, with the dielectric, is analyzed using the FEM. The interface of these two regions is the cube surface with the surface electric and magnetic currents that appear in both the MoM-SIE and FEM equations, and allow the diakoptic decomposition of the problem.

First, we consider the frequency $f = 13\,\mathrm{MHz}$. The normalized radar cross-section, RCS/λ^2, is shown in Fig. 1, calculated with the pure higher-order MoM (MoM-SIE). The RCS is shown as a

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function of the angle $0 \le \theta \le 180^\circ$, while $\phi = 0$. The numbers in the brackets correspond to the order of the surface current approximation (in MoM-SIE case) and to the order of field approximation (in FEM case). There is only the low-order approximation for the pure MoM-SIE solution, as the cube is electrically small. From the Fig. 1, it is observed that the diakoptic low-order MoM-SIE/FEM (1 1/1 1 1) hybrid solution is very accurate. We purposely increased the expansion orders for the diakoptic solution to investigate the influence of the higher-order approximation to the accuracy and the stability of the solution. Fig. 1 shows that all solutions are numerically stable, i.e., they do not diverge significantly from the expected result. However, the solution with the low-order FEM tends to yield the least precise results.

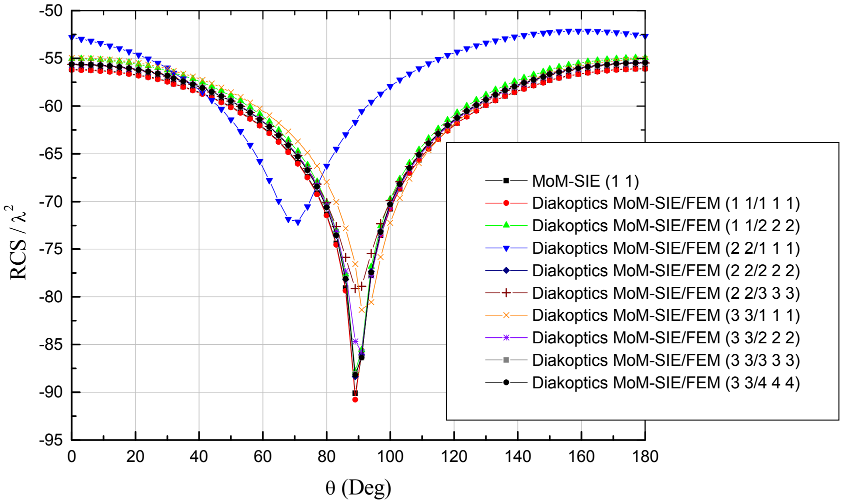

Figure 1. Normalized RCS of the dielectric cube at 13 MHz.

In order to demonstrate that the diakoptic MoM-SIE/FEM solution gives accurate results for both far-field and near-field, presented in Fig. 2 is the near-field of the cube. The center of the cube is at $(x, y, z) = (0,0,0)$ in the Cartesian coordinate system. The x-coordinate is within the interval $-1.5\text{ m} \le x \le 1.5\text{ m}$, while $y = 0$ and $z = 0$.

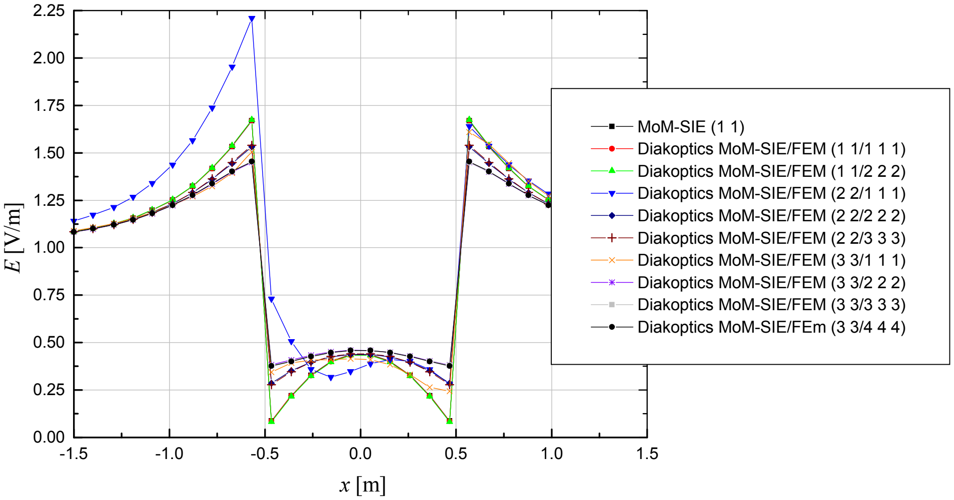

Figure. 2. Near-field of the dielectric cube at 13 MHz for different expansions.

The rms electric field intensity is shown in Fig. 2. The accuracy and convergence trends of the results for the near-field are similar to those for the RCS; the lowest-order FEM when combined with the MoM-SIE using the diakoptic approach yields the least precise results.

Next, we increase the frequency of the incident TEM wave to $f = 40\,\mathrm{MHz}$ in order to explore the accuracy and the stability of the diakoptic solution further. The cube remains the same. The normalized RCS and the near-field are shown in Figs. 3 and 4, respectively.

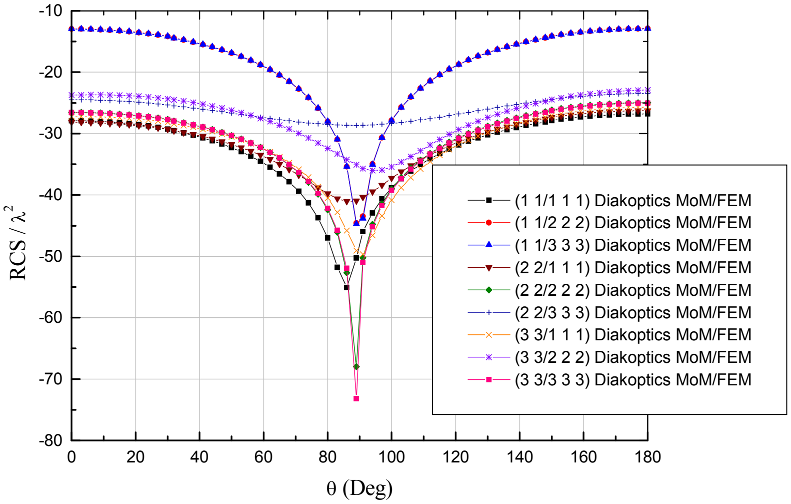

Figure 3. Normalized RCS of the dielectric cube at 40 MHz.

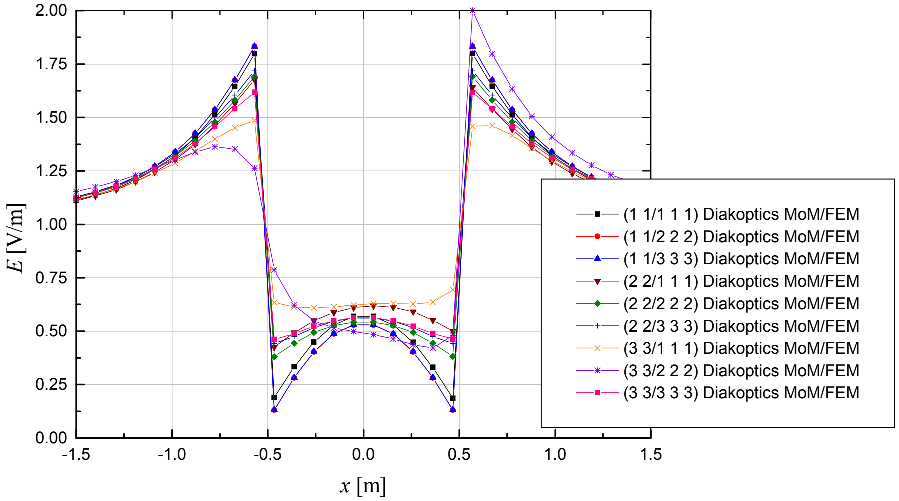

Figure 4. Near-field of the dielectric cube at 40 MHz for different MoM-SIE and FEM expansions.

Results presented in Figs. 3 and 4 show that as the frequency increases, the diakoptic solution (as well as pure MoM-SIE and pure FEM solutions) becomes more dependent on the order of the approximation. The conclusion from the presented results is that in order to pass the electromagnetic information about the diakoptic subsystem through the diakoptic interface, (a) the subsystem should be modeled with an adequately large number of unknown coefficients and (b) there should be an adequately large number of diakoptic unknowns at the diakoptic boundary. (In this example, all MoM-SIE unknown coefficients are the unknown coefficients at the diakoptic boundary too.)

Finally, we present results for the normalized RCS in the direction of the incident wave for frequencies within the range $10\,\mathrm{MHz} \le f \le 100\,\mathrm{MHz}$. The expansion order for MoM-SIE and FEM approximations is varied, and the results are shown in Fig. 5. The results obtained with expansions (1 1/1 1 1) and (4 4/1 1 1) have instabilities at the higher frequencies. These instabilities are due to the internal resonances of the cube (FEM diakoptic subsystem). The internal resonance of the FEM domain (enclosed with PEC) is $f_{\mathrm{r}} = \dfrac{c_0}{2\sqrt{\varepsilon\mathrm{r}}}\sqrt{\dfrac{1}{a^2}+\dfrac{1}{a^2}} \approx 95\ \mathrm{MHz}$. Therefore, the diakoptic solution will have the stability problems with internal resonances if either the FEM subsystem or the MoM-SIE subsystem has that problem.

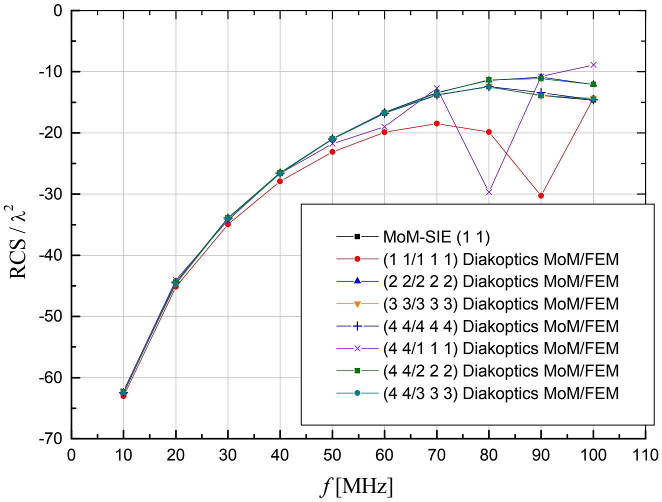

Figure 5. Normalized RCS for various expansion orders as a function of frequency.

Conclusion

We have briefly presented the theory behind the diakoptic higher-order MoM-SIE/FEM approach, as well as numerical results that prove the concept of the diakoptic MoM/FEM hybridization. The work in progress is to apply the diakoptic higher-order MoM/FEM approach to the EM problems that are at the limits of available memory and CPU time consumption of standard computers.

Acknowledgement

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