Enhanced Higher Order MoM-PO Modeling Using Multiple Reflections in the PO Region

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Introduction

The applicability and practicality of the method of moments (MoM) for electromagnetic modeling of electrically large metallic structures can be substantially extended by hybridization with asymptotic approaches based on the physical optics (PO). One such hybrid MoM-PO technique, using higher order geometrical modeling and higher order current-distribution modeling in both MoM and PO parts of the structure under consideration, is presented in [1]. As in all standard MoM-PO approaches, this technique neglects the mutual interactions between the PO currents, to maximally reduce the complexity and computational cost of the hybrid method as compared to the pure MoM. This simplification is, of course, paid by some loss of accuracy in the solution, which is often noticeable, and can even lead to a complete failure of the standard MoM-PO technique in some applications.

This paper proposes a higher order MoM-PO technique with enhanced accuracy in the PO region based on a multiple-bounce improvement of the solution for PO currents. According to this new technique, the solution for PO current coefficients is computed as the initial solution obtained from the MoM current coefficients within the MoM-PO interaction process plus the contribution from multiple bounces within the PO region using PO-PO interactions only, in a number of iterative steps. The hybrid solution is further improved by nesting a PO-PO multiple reflection procedure within every iteration in an overall iterative process covering the entire MoM-PO computational domain, with multiple reflections between MoM and PO currents.

Numerical results demonstrate a substantial improvement in accuracy of the results using the higher order MoM-PO technique with increasing the number of iterations (reflections). The improvement clearly indicates the advantage of the new technique with multiple PO-PO reflections over the technique in [1], which can be considered as the initial solution (zeroth iteration) of the multiple PO-PO reflection scheme. Although including multiple reflection effects in an iterative fashion in a PO method or the PO part of a MoM-PO method is an old idea implemented by a number of authors [2-4], this paper presents the first higher order implementation of multiple PO-PO reflections.

Theory and Implementation

Consider a metallic antenna or scatterer decomposed into a MoM region and a PO region (Fig.1), with the surface current density vectors denoted by $\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}$ and $\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}$ in the two regions, respectively, and appropriate higher order basis functions [1]. Based on a system

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of coupled surface integral equations, with an electric field integral equation in the MoM region and a magnetic field integral equation in the PO region, the complete hybrid MoM-PO system matrix equation can be written as

$$\begin{bmatrix} Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) & Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \\ P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})-Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) & 1-Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \end{bmatrix} \begin{bmatrix} I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) \\ I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \end{bmatrix} = \begin{bmatrix} V_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}}) \\ V_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}) \end{bmatrix}, \tag{1}$$

where $\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}}$ and $\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}$ are vector testing functions for the MoM and PO regions, respectively, and Z and P are impedance and projection matrices describing different interactions between the MoM and PO currents (as symbolically indicated in Fig.1). We note that the PO-PO projection matrix $P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})$ is represented as an identity matrix in (1), which is enabled by our choice of higher order PO basis and testing functions [1]. Note that in the hybrid MoM-PO technique [1], the PO-PO interaction impedance matrix $Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})$ is eliminated from the matrix equation, thus neglecting the mutual interactions between the PO currents.

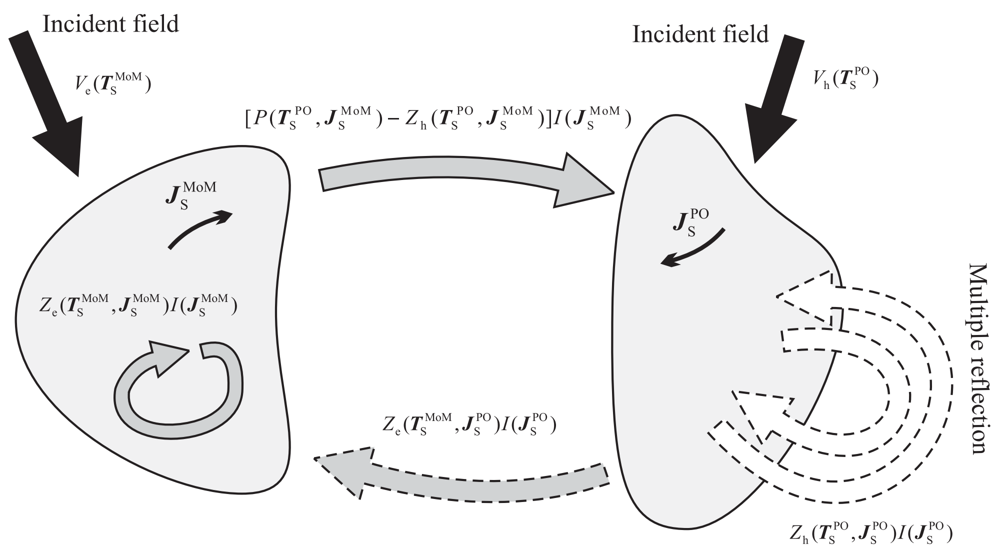

Fig.1 Schematic of the enhanced MoM-PO technique using multiple PO-PO reflections.

From the second equation of the two partitioned matrix equations equivalent to (1), the unknown PO current coefficients $I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})$ can be expressed as

$$I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})=V_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}})+Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})-[P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})-Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})]I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}). \tag{2}$$

Substituting this expression in the first partitioned equation of (1) then leads to the following matrix equation:

$$\begin{aligned}&\left(Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})-Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})(P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})-Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}))\right)I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})\\ &\quad=V_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}})-Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}},\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})V_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}})\end{aligned}, \tag{3}$$

which can be solved for the unknown MoM current coefficients $I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})$.

Once the MoM currents are known, we solve for the PO current coefficients using (2) in the following manner. We note that (2) can be written as

$$I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) + Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \ , \tag{4}$$

where $I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})$ stands for

$$I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = V_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}) - [P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) - Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})] I(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) \ . \tag{5}$$

Note that thus obtained $I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}})$ is actually the solution for PO currents assuming no mutual interactions between currents in the PO region. While this represents the final solution for PO currents in the technique [1], in this present paper it is used as an initial current distribution in the PO region, to be improved in an iterative procedure based on (4),

$$I^{(n)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) + Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) I^{(n-1)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \ , \quad n = 1,2,..., N_{\mathrm{PO}} \ , \tag{6}$$

which leads to

$$I^{(n)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = I^{(0)}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \sum_{k=0}^{n} \left[Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \right]^{k} \ , \quad n = 1,2,.., N_{\mathrm{PO}} \tag{7}$$

(see the symbolic representation in Fig.1), where N_{PO} is the total number of iterations. Essentially, the improved solution for the PO current (in the context of the PO approximation) is found as the initial solution obtained from the MoM current (for $n = 0$) plus the contribution from multiple bounces within the PO region using PO-PO interactions only (for $n = 1,2,3...$).

The hybrid solution can be further improved by reflecting back the enhanced accuracy of the PO current solution to the MoM current solution and vice versa, in an overall iterative process covering the entire MoM-PO computational domain. These overall iterations represent multiple reflections between the MoM and PO regions described as [1]

$$\begin{aligned} &Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) I_{k}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) = V_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}}) - Z_{\mathrm{e}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{MoM}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) I_{k-1}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) \ , \\ &I_{k}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = V_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}) - [P(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) - Z_{\mathrm{h}}(\boldsymbol{T}_{\mathrm{S}}^{\mathrm{PO}}, \boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}})] I_{k}(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{MoM}}) \ , \\ &k = 1,2,3,..., N_{\mathrm{MoM\text{-}PO}} \ , \end{aligned} \tag{8}$$

with the initial solution for the PO current coefficients set to zero, $I_0(\boldsymbol{J}_{\mathrm{S}}^{\mathrm{PO}}) = 0$, and $N_{\mathrm{MoM\text{-}PO}}$ being the total number of multiple reflections. In this technique, a number of multiple reflection iterations between PO currents described by (7) are nested within every iteration in the multiple reflection process between MoM and PO currents described by (8), where the solution for MoM current coefficients in the kth step obtained from the first equation in (8) serves as the known MoM current distribution to compute, using (5), the initial solution for PO current coefficients for the procedure in (7).

Results and Discussion

To illustrate the numerical effectiveness of the new multiple-reflection hybrid MoM-PO technique, we consider a tri-plate reflector shown in the inset in Fig.2. The width of the reflector equals 4 wavelengths, and its length is broken into 3.6, 1, and 3.6 wavelengths for the three plates, respectively. The angle of the reflector measured between the two lateral plates is 67.4 degrees. The reflector is excited by a short 0.1-wavelength dipole antenna centered at 1.25 wavelengths above the central (horizontal) plate. The dipole is modeled using two wire segments and the reflector is represented by a total of 55 quadrilaterals, as shown in Fig.2. Without the use of symmetry, the total number of unknowns is one in the MoM region (antenna) and 2229 in the PO region (reflector).

Shown in Fig.2 is the computed radiation pattern of the reflector antenna in the plane perpendicular to the dipole, across the full angle around the antenna. The structure is analyzed using the pure MoM and the hybrid MoM-PO technique with six different total numbers of multiple reflections in the PO region (N_{PO} = 0,1,2,3,10, and 100), nested within the MoM-PO multiple reflection process. Note that the multiple PO-PO reflection solution with $N_{PO} = 0$ represents the solution using the MoM-PO technique [1]. Taking the results obtained by the pure MoM technique as a reference (exact) solution, we observe a substantial improvement in accuracy of the results using the hybrid MoM-PO technique with increasing the number of iterations (reflections). The improvement clearly indicates the advantage of the new higher order technique with multiple PO-PO reflections over the technique in [1], where no mutual interactions between the PO currents are included (0-refl. curve in Fig.2). We note a complete failure in predicting the gain in the main beam of the antenna (for $\theta = 0°$) with the standard MoM-PO technique.

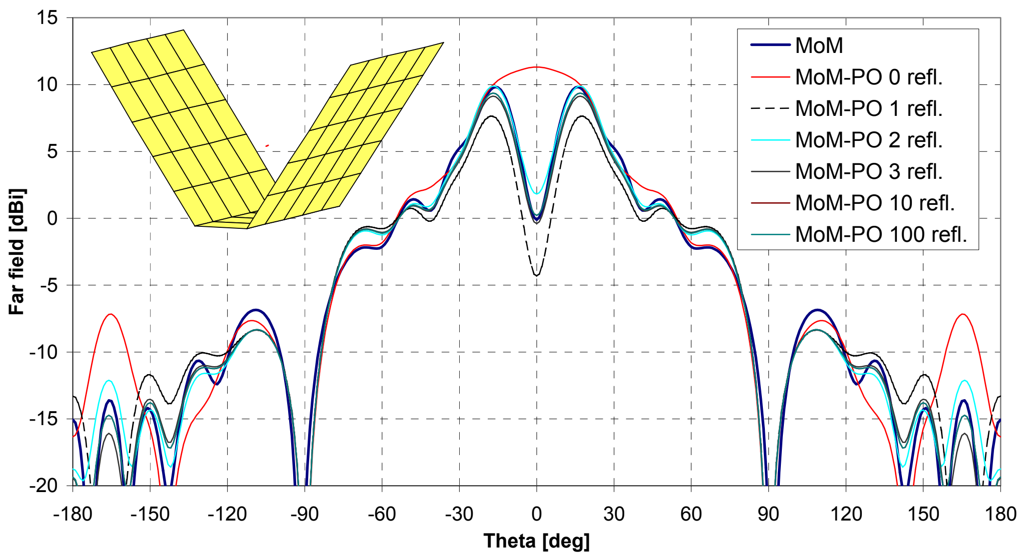

Fig.2 Computed far field around the tri-plate reflector excited by a short dipole antenna above the reflector using the pure MoM and the hybrid MoM-PO technique with different numbers of multiple reflections in the PO region.

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