Three Types of Higher-Order MoM Basis Functions Automatically Satisfying Current Continuity Conditions

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1. Introduction

Higher-order basis functions which constitute the large-domain (entire-domain) method of moments (MoM) approach to integral-equation modeling of general 3-D electromagnetic structures have significant computational advantages over traditionally used low-order (subdomain) MoM basis functions [1-2]. Higher-order bases designed for implementing in Galerkin-based MoM techniques automatically satisfy current continuity conditions at the interconnections of elements (divergence-conforming bases) and are generally of either interpolatory or hierarchical form. Interpolatory basis functions have excellent orthogonality properties and produce well-conditioned MoM matrices. Hierarchical basis functions enable using different orders of current approximation in different elements for efficient selective discretization of the solution domain, because each lower-order set of functions is a subset of higher-order sets. This property allows for a whole spectrum of element sizes and the corresponding current approximation orders to be used at the same time in a single simulation model of a complex structure. Additionally, each individual 2-D or 3-D element can have drastically different sizes in different directions, enabling a whole variety of "irregular" element shapes.

This paper presents our investigations aimed at improving the orthogonality properties of polynomial higher-order hierarchical basis functions leading to better conditioned MoM matrices and more stable solutions, in both volume integral equation (VIE) modeling and surface integral equation (SIE) modeling. Three different types of polynomial basis functions are implemented in the large-domain Galerkin SIE method [1], enabling cross-validation of the results and comparison of numerical properties of the three sets of basis functions. We show that by combining the simple 2-D power functions of parametric coordinates in accordance to ultraspherical and Chebyshev polynomials, and modifying them so that the current-continuity condition across the quadrilateral edges is automatically satisfied, higher-order polynomial MoM basis functions are obtained that have much better orthogonality properties.

2. Basis Functions

As a basic building block for SIE modeling of general 3-D EM structures, we use a bilinear quadrilateral parametric surface element in conjunction with higher-order hierarchical polynomial vector current expansions [1-2],

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$$J_{Su}(u,v)=\frac{e_u}{|e_u\times e_v|}\sum_{i=0}^{N_u}\sum_{j=0}^{N_v-1}a_{uij}\,f_{uij}(u,v),\qquad e_u=\frac{\mathrm{d}r}{\mathrm{d}u},\quad e_v=\frac{\mathrm{d}r}{\mathrm{d}v},$$

$$r(u,v)=r_c+r_u u+r_v v+r_{uv}uv,\quad -1\le u,v\le 1 \tag{1}$$

with analogous expression for the v-component of the surface current density vector. The first class of analyzed basis functions is a set of simple 2-D polynomial functions in the $u-v$ coordinate system that automatically satisfy the current-continuity condition for the normal component of J_S along an edge shared by quadrilateral elements:

$$f_{uij}(u,v)=\begin{Bmatrix}1-u, & i=0\\ u+1, & i=1\\ u^i-1, & i\ge 2,\text{even}\\ u^i-u, & i\ge 3,\text{odd}\end{Bmatrix}v^j\qquad\text{(regular polynomials)} \tag{2}$$

Fig.1 shows the first few basis functions in one dimension.

The second class of basis functions is a set obtained by combining the power functions u^iv^j in accordance to a class of ultraspherical (Gegenbauer) polynomials [3], and modifying them so that the current-continuity condition across the quadrilateral edges be automatically satisfied:

$$f_{uij}(u,v)=\begin{Bmatrix}1-u, & i=0\\ u+1, & i=1\\ P_i(u) & i\ge 2\end{Bmatrix}P_j(v)\qquad\text{(ultraspherical polynomials)}$$

$$P_0(x)=1,\quad P_1(x)=-x,\quad nP_n(x)=(2n-3)xP_{n-1}(x)-(n-3)P_{n-2}(x) \tag{3}$$

The modified ultraspherical basis functions are shown in Fig.2 in one dimension.

Finally, the third class of functions incorporates Chebyshev polynomials [3] in the v-coordinate yielding combined 2-D ultraspherical/Chebyshev polynomial basis functions:

$$f_{uij}(u,v)=\begin{Bmatrix}1-u, & i=0\\ u+1, & i=1\\ P_i(u) & i\ge 2\end{Bmatrix}T_j(v),\qquad\text{(ultraspherical/Chebyshev polynomials)}$$

$$T_0(x)=1,\quad T_1(x)=x,\quad T_n(x)=2xT_{n-1}(x)-T_{n-2}(x) \tag{4}$$

The three types of polynomial basis functions in Eqs.(2)-(4), shown in Figs.3-5 in two dimensions, are implemented in the large-domain SIE method, yielding three independent versions of the code, and the MoM matrix conditioning is investigated using the three codes. The same functions are used also for testing (Galerkin method).

3. Numerical Results

As the first example, consider a perfectly conducting square plate scatterer 1 m on a side. The frequency was varied from 1.5 GHz to 4 GHz, the number of elements in the geometrical model from 9 to 49, and the number of unknowns from 612 to 4704, respectively. Fig.6 shows the condition number of the MoM matrix for three types of basis functions. We observe that the use of ultraspherical/Chebyshev basis functions yields an improvement in the condition number of several orders of magnitude when compared to the original polynomial basis functions, and 10 times when compared to the pure ultraspherical set of basis functions.

The next example is a perfectly conducting spherical scatterer 1 m in radius, in a frequency range from 10 MHz to 600 MHz (the number of unknowns ranges from 192 to 4368). The sphere was modeled by 384 quadrilaterals and symmetry was used. Shown in Fig.7 is the condition number of the matrix obtained by using three types of basis functions. Again, the ultraspherical/Chebyshev set of basis functions yields the best condition number of the MoM matrix.

Finally, Fig.8 shows convergence properties of the iterative solution of the MoM matrix for the plate scatterer at 4 GHz using an unpreconditioned BiCGSTAB iterative solver. We observe that all three methods rapidly converge to the directly computed (by LU-decomposition) normalized RCS of 55.96 dB. The three sets of basis functions appeared to converge with similar speeds also in the case of the spherical scatterer. We expect, however, to fully exploit better conditioning properties of new basis function sets with enhanced orthogonality in modeling of large-scale EM structures (such as cars and aircraft at microwave frequencies), which is our current and future work in this area.

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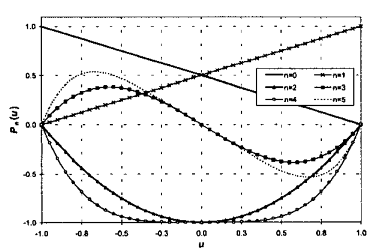

Fig. 1. Regular polynomial basis functions.

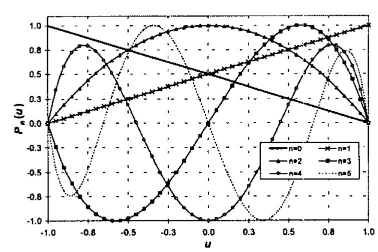

Fig. 2. Modified ultraspherical basis functions.

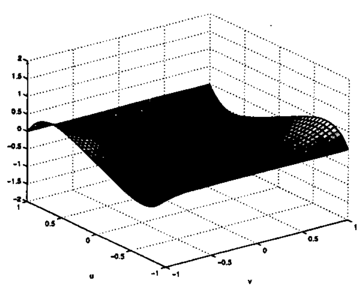

Fig. 3. Regular 2-D polynomial basis function of the (5,3) order

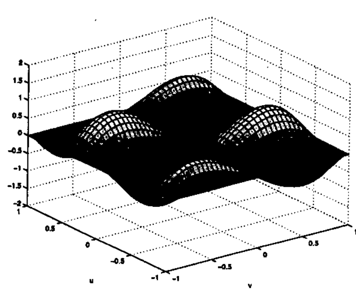

Fig. 4. Ultraspherical 2-D basis function of the (5,3) order

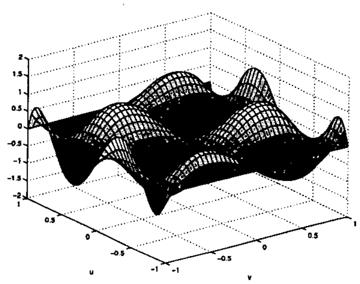

Fig. 5. Ultraspherical/Chebyshev 2-D basis function of the (5,3) order.

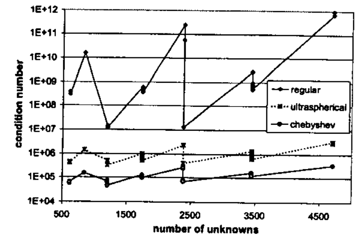

Fig. 6. MoM matrix condition number for a square plate scatterer.

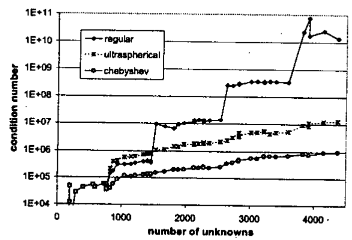

Fig. 7. MoM matrix condition number for a spherical scatterer.

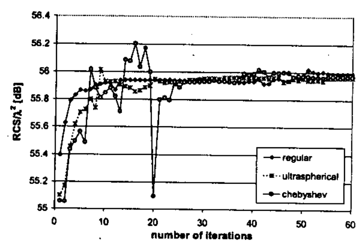

Fig. 8. Convergence curves for the RCS of the square plate at 4.0 GHz