

# Hybrid Numerical–Asymptotic Modeling of Electrically Large EM Structures

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**Abstract:** Efficient numerical-asymptotic electromagnetic simulations of electrically large structures are presented based on an iterative hybridization of the method of moments (MoM) and physical optics (PO) in a higher order computational framework. The iterative MoM-PO technique employs multiple reflections between PO currents nested within an overall iterative MoM-PO multiple-reflection process. The results demonstrate an effective MoM-PO modeling of an automobile with a GSM 1900-MHz antenna and a substantial improvement in accuracy of the analysis of a characteristic reflector antenna geometry using the new technique with multiple PO-PO reflections over the standard higher order technique that neglects the mutual coupling between the currents in the asymptotic region of the structure.

**Key words:** electromagnetic analysis, hybrid methods, method of moments (MoM), physical optics (PO), higher order modeling, vehicles, vehicular antennas, reflector antennas.

## 1. Introduction

One strategy to substantially reduce the computation time and memory requirements in electromagnetic-field simulations for antenna and scattering applications using integral equations at higher frequencies is based on a hybridization of the method of moments (MoM), which is a numerically exact method, with numerically approximate high-frequency techniques for asymptotic analysis of electrically very large smooth parts of the structure. MoM can be hybridized with either ray-based high-frequency asymptotic techniques, such as the uniform geometric theory of diffraction (UTD), or current-based high-frequency asymptotic methods, such as the physical optics (PO).

This paper presents our work in hybrid numerical-asymptotic electromagnetic modeling aimed at developing MoM-PO techniques for accurate and efficient analysis and design of radiating and scattering structures that are not only electrically large but also irregularly shaped, such as airplanes and automobiles. Recently, we have proposed a general hybrid MoM-PO technique using higher order geometrical modeling and higher order current distribution modeling in both MoM and PO parts of the structure [1]. However, when compared with the rigorous pure MoM analysis, the hybrid technique has a number of inherent inaccuracies if regarded as a tool for modeling of arbitrary objects (such as vehicles). These deficiencies, of course, come directly from the PO assumptions introduced to simplify the solution procedure, perhaps the most important one being neglecting mutual coupling between the currents in the asymptotic region of the structure. We present here an effective way to include the PO-PO coupling based on an iterative improvement of the technique in [1] using multiple reflections between PO currents nested within an overall iterative MoM-PO multiple-reflection process. The presented numerical results include:

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(1) an effective modeling of an automobile demonstrating that the higher order MoM-PO technique, even with all of its inherent inaccuracies, can be considered an efficient engineering tool for analysis and design of vehicular antennas in realistic environments and (2) an example of a characteristic reflector geometry clearly indicating the advantage of the new technique with multiple PO-PO reflections over the technique in [1], which can be considered as the initial solution (zeroth iteration) of the multiple PO-PO reflection scheme.

## 2. Iterative Higher Order MoM-PO Technique

Consider a metallic antenna or scatterer decomposed into a MoM region and a PO region, with the surface current density vectors denoted by  $\mathbf{J}_s^{\text{MoM}}$  and  $\mathbf{J}_s^{\text{PO}}$  in the two regions, respectively. The surface currents in both regions are represented as [1]

$$\mathbf{J}_s(u, v) = \frac{1}{\left| \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \right|} \left[ \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v-1} \alpha_{uij} f_{uij}(u, v) \frac{\partial \mathbf{r}}{\partial u} + \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v-1} \alpha_{vij} f_{vij}(u, v) \frac{\partial \mathbf{r}}{\partial v} \right], \quad \mathbf{r}(u, v) = \sum_{k=0}^{K_u} \sum_{l=0}^{K_v} \mathbf{r}_{kl} u^k v^l, \quad (1)$$

where  $\mathbf{r} = \mathbf{r}(u, v)$  describes a generalized curved Lagrange-type parametric quadrilateral element used for geometrical modeling. Basis functions in (1) are higher order divergence-conforming hierarchical polynomials and modified Chebyshev interpolatory polynomials:

$$f_{\text{MoM},uij}(u, v) = \begin{cases} u+1, & i=0 \\ u-1, & i=1 \\ u^i-1, & i \geq 2, \text{ even} \\ u^i-u, & i \geq 3, \text{ odd} \end{cases} v^j, \quad f_{\text{PO},uij}(u, v) = C_{uij} \frac{\mathcal{T}_{N_u+1}^{\text{mod}}(u)}{u-u_i^{\text{mod}}} \frac{\mathcal{T}_{N_v}(v)}{v-v_j}, \quad -1 \leq u, v \leq 1, \quad (2)$$

with analogous expression for  $v$ -components and the Galerkin method and modified point-matching technique used for testing—in the MoM and PO regions, respectively.

From the boundary conditions for the tangential components of the total (incident plus scattered) electric and magnetic field vectors on the surfaces in the model, a system of coupled surface integral equations is derived, with an electric field integral equation (EFIE) in the MoM region and a magnetic field integral equation (MFIE) in the PO region. In the hybrid MoM-PO method, MFIE is solved using the PO approximation for the surface currents. The complete hybrid MoM-PO system matrix equation can be expressed as

$$\begin{bmatrix} Z_e(\mathbf{T}_s^{\text{MoM}}, \mathbf{J}_s^{\text{MoM}}) & Z_e(\mathbf{T}_s^{\text{MoM}}, \mathbf{J}_s^{\text{PO}}) \\ P(\mathbf{T}_s^{\text{PO}}, \mathbf{J}_s^{\text{MoM}}) - Z_h(\mathbf{T}_s^{\text{PO}}, \mathbf{J}_s^{\text{MoM}}) & 1 - Z_h(\mathbf{T}_s^{\text{PO}}, \mathbf{J}_s^{\text{PO}}) \end{bmatrix} \begin{bmatrix} I(\mathbf{J}_s^{\text{MoM}}) \\ I(\mathbf{J}_s^{\text{PO}}) \end{bmatrix} = \begin{bmatrix} V_e(\mathbf{T}_s^{\text{MoM}}) \\ V_h(\mathbf{T}_s^{\text{PO}}) \end{bmatrix}, \quad (3)$$

where  $\mathbf{T}_s^{\text{MoM}}$  and  $\mathbf{T}_s^{\text{PO}}$  are vector testing functions for the MoM and PO regions, respectively, and  $Z$  and  $P$  are impedance and projection matrices describing different interactions between the MoM and PO currents. We note that the PO-PO projection matrix  $P(\mathbf{T}_s^{\text{PO}}, \mathbf{J}_s^{\text{PO}})$  is represented as an identity matrix in (3), which is enabled by the choice of higher order PO basis functions in (2) [1].

Note that in the hybrid MoM-PO technique [1], the PO-PO interaction impedance matrix  $Z_h(\mathbf{T}_s^{\text{PO}}, \mathbf{J}_s^{\text{PO}})$  is eliminated from the matrix equation, thus neglecting the mutual interactions between the PO currents. This reduces the complexity of the MoM-PO method, as compared to the full MoM, at the expense of some loss of accuracy in the solution. However, the hybrid MoM-PO solution can be improved by including multiple reflections between PO currents into in an overall iterative process

covering the entire MoM-PO computational domain, as follows.

Based on the first equation of the two partitioned matrix equations equivalent to (3), the MoM current coefficients in the  $k$ th iteration are obtained from PO current coefficients in the previous iteration solving the following equation:

$$Z_e(\mathbf{T}_S^{\text{MoM}}, \mathbf{J}_S^{\text{MoM}})I_k(\mathbf{J}_S^{\text{MoM}}) = V_e(\mathbf{T}_S^{\text{MoM}}) - Z_e(\mathbf{T}_S^{\text{MoM}}, \mathbf{J}_S^{\text{PO}})I_{k-1}^{(M)}(\mathbf{J}_S^{\text{PO}}), \quad k = 1, 2, \dots, N, \quad (4)$$

with the initial solution for the PO current coefficients set to zero,  $I_0^{(M)}(\mathbf{J}_S^{\text{PO}}) = 0$ , and  $N$  being the total number of iterations. This MoM current is then used to compute, based on the second partitioned equation of (3), the starting solution for the PO current in the  $k$ th iteration:

$$I_k^{(0)}(\mathbf{J}_S^{\text{PO}}) = V_h(\mathbf{T}_S^{\text{PO}}) - [P(\mathbf{T}_S^{\text{PO}}, \mathbf{J}_S^{\text{MoM}}) - Z_h(\mathbf{T}_S^{\text{PO}}, \mathbf{J}_S^{\text{MoM}})]I_k(\mathbf{J}_S^{\text{MoM}}), \quad (5)$$

which is to be improved in another iterative procedure with the total of  $M$  steps given by

$$I_k^{(n)}(\mathbf{J}_S^{\text{PO}}) = I_k^{(0)}(\mathbf{J}_S^{\text{PO}}) + Z_h(\mathbf{T}_S^{\text{PO}}, \mathbf{J}_S^{\text{PO}})I_k^{(n-1)}(\mathbf{J}_S^{\text{PO}}), \quad n = 1, 2, \dots, M. \quad (6)$$

Here, the improved solution for the PO current (in the context of the PO approximation) is found as the initial solution obtained from the MoM current [for  $n = 0$  – using (5)] plus the contribution from multiple bounces within the PO region through PO-PO interactions only [for  $n = 1, 2, 3, \dots$  – using (6)]. Note that a similar iterative improvement in the PO region in the framework of a low-order MoM-PO technique can be found in [2]. The final ( $M$ th) solution of the PO-PO multiple reflection process,  $I_k^{(M)}(\mathbf{J}_S^{\text{PO}})$ , is now fed back into (4), where it serves as the initial (known) PO solution for finding the MoM current coefficients in the  $(k+1)$ th iteration of the overall MoM-PO multiple reflection process, and so on. Essentially, a number of multiple reflection iterations between PO currents described by (6) are nested within every iteration in the multiple reflection process between MoM and PO currents described by (4) and (5). At the end of the process,  $I_N(\mathbf{J}_S^{\text{MoM}})$  and  $I_N^{(M)}(\mathbf{J}_S^{\text{PO}})$  represent the final solution by the hybrid technique for currents in the MoM and PO regions, respectively.

### 3. Numerical Results

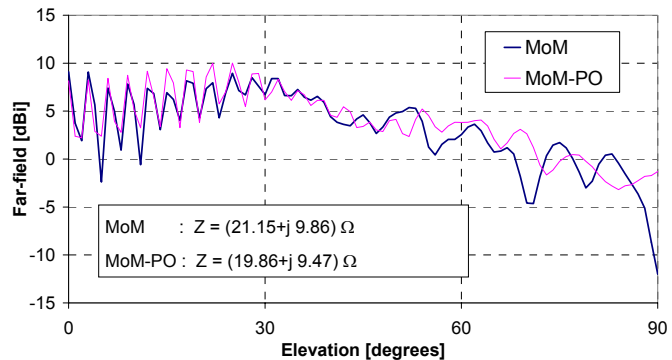
The first example of the hybrid MoM-PO analysis is the simulation of an automobile with a quarter-wave oblique monopole rooftop antenna at an angle of 30 degrees with respect to the vertical axis, at the GSM 1900-MHz band. Fig.1 shows the model of the automobile constructed from 463 quadrilaterals of the second geometrical orders, with the MoM region consisting of the monopole and 6 quadrilaterals in the junction. The ground is approximated by a PEC plane and the total number of unknowns is 4116 using symmetry. Shown in Fig.2 are the results for the radiation pattern of the antenna in the plane perpendicular to the longitudinal axis of the vehicle, along with the antenna impedance. We observe a good agreement of the results obtained by the MoM-PO technique with the pure MoM results, the hybrid technique being about 30 times faster than the numerically rigorous solution.

Consider next a reflector composed of three plates of lengths equal to 3.6, 1, and 3.6 wavelengths, respectively, and the width equal to 4 wavelengths (Fig.3). The full angle of the reflector, measured between the two lateral plates, is 67.4 degrees. The reflector is modeled by a total of 55 quadrilaterals and is excited by a short 0.1-wavelength dipole antenna centered at 1.25 wavelengths above the central plate. Without the use of symmetry, the total number of unknowns is 2230. Fig.4 shows the radiation pattern of the reflector antenna in the plane perpendicular to the dipole computed using the pure MoM and the hybrid MoM-PO technique (only dipole in the MoM region) with a total of  $M = 0, 1$ , and 3 multiple reflections in the PO region, respectively, nested within the MoM-PO multiple reflection process. Taking the results obtained by the pure MoM as a reference (exact) solution, we observe a substantial

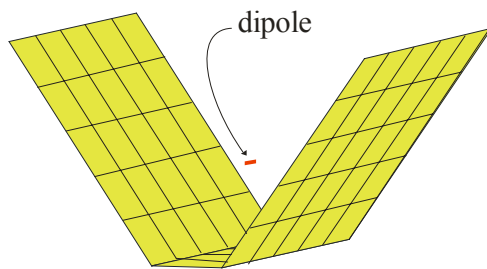
improvement in accuracy of the results using the hybrid MoM-PO technique with increasing the number of reflections. The improvement clearly indicates the advantage of the new higher order technique with multiple PO-PO reflections over the technique in [1], where no mutual interactions between the PO currents are included (zero reflections curve in Fig.4).



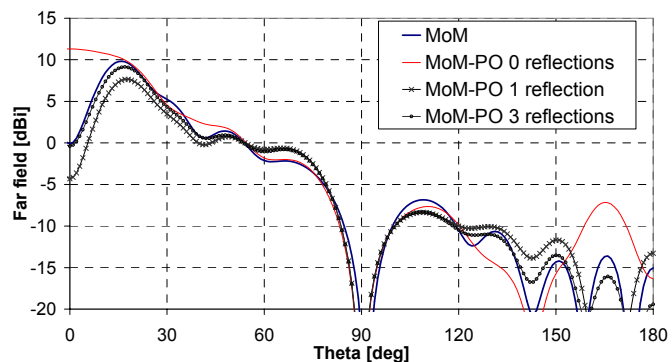
**Fig.1.** MoM-PO model of an automobile with an oblique monopole rooftop antenna.



**Fig.2.** Radiation pattern and impedance of the antenna in Fig.1: pure MoM and hybrid MoM-PO solutions.



**Fig.3.** MoM-PO model of a tri-plate reflector excited by a short dipole antenna.



**Fig.4.** Far field of the antenna in Fig.3: pure MoM and MoM-PO solutions with 0, 1, and 3 PO-PO reflections.

#### 4. Conclusions

This paper has presented higher order hybrid numerical-asymptotic electromagnetic simulations of electrically large structures based on an iterative MoM-PO technique employing multiple reflections between currents in the PO region nested within an overall iterative MoM-PO multiple-reflection process. The results demonstrate the advantage of using hybrid techniques over pure numerical ones at higher frequencies and the effectiveness of the new higher order technique with multiple PO-PO reflections.

#### References

- [1] M. Djordjevic and B. M. Notaros, "Higher Order Hybrid Method of Moments-Physical Optics Modeling Technique for Radiation and Scattering from Large Perfectly Conducting Surfaces," IEEE Transactions on Antennas and Propagation, Vol. 53, No. 2, February 2005, pp.800-813.
- [2] J. M. Taboada and F. Obelleiro, "Including Multibounce Effects in the Moment-Method Physical-Optics (MMPO) Method," Microwave and Optical Technology Letters, Vol. 32, No. 6, March 20 2002, pp.435-439.