Efficient Higher Order Finite Element–Moment Method Modeling of 3-D Radiation and Scattering Problems

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Abstract: A higher order hybrid finite element – method of moments (FEM-MoM) technique for three-dimensional analysis and design of radiating and scattering structures is presented. The technique uses generalized parametric hexahedral and quadrilateral elements of arbitrary geometrical orders in conjunction with curl- and divergence-conforming hierarchical polynomial vector basis functions of arbitrary approximation orders, and the Galerkin testing procedure. It enables very considerable reduction of the number of unknowns when compared to low-order hybrid solutions.

Key words: electromagnetic analysis, numerical techniques, hybrid methods, finite element method (FEM), method of moments (MoM), higher order modeling, curved parametric elements, scattering.

1. Introduction

Hybrid finite element-boundary integral (FE-BI) techniques have been extensively utilized for an exact truncation of the unbounded spatial domain in the finite element method (FEM) analysis for quite some time[1]-[5]. They basically divide the problem into an interior and exterior region. The field in the interior, usually inhomogeneous, region is expressed using FEM and the field in the exterior, homogeneous, region is represented by some sort of a boundary-integral equation (BIE). The fields in the interior and exterior regions are then coupled by the field continuity conditions at the FE-BI boundary surface. Excellent properties of both the FEM in dealing with inhomogeneities in the bounded domain and BIE in treating homogeneous open regions are thus seamlessly united into a powerful hybrid technique. Since the BIE computational methodologies correspond to solutions of surface integral equations (SIEs) based on the method of moments (MoM), the hybrid methods are also referred to as FEM-MoM techniques.

An impressive body of knowledge has been developed in the area of FE-BI modeling and applications in the past two decades [1]-[12]. However, in terms of the particulars of the numerical discretizations, most hybrid tools are low-order (or small-domain) techniques, which implies that the structure under consideration is modeled by volume and surface geometrical elements that are electrically very small, on the order of $\lambda/10$ in each dimension, λ being the wavelength in the medium, and the fields and currents within the elements are approximated by low-order basis functions. This results in a very large number of unknowns (unknown field/current distribution coefficients) needed to obtain results of satisfactory accuracy, with all the associated

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problems and enormous requirements in computational resources. An alternative is the higher order (or large-domain) computational approach [13], which utilizes higher order field/current basis functions defined on large (e.g., on the order of λ in each dimension) curvilinear geometrical elements. Only recently higher order FEM-MoM techniques have been proposed, developed, and employed in analysis of high-frequency unbounded electromagnetic structures [8]-[11]. However, none of the proposed techniques exploits the full potential of the higher order FE and BI modeling and all of the actually reported results are limited to the utilization of basis functions of the second or third order. In addition, these techniques still implement small finite and boundary elements for field/current modeling, and the higher order meshes reported actually represent small-domain solutions to the electromagnetic problems considered.

Our goal in this paper is to demonstrate the accuracy and efficiency of our novel 3-D fully higher order FEM-MoM method developed by hybridizing the higher order FEM [14]-[15] and MoM [16] techniques. In our analysis method we emphasize large-domain geometrical modeling by using large curvilinear Lagrange-type volume and surface elements in coarse meshes and hierarchical polynomial vector basis functions of high (arbitrary) orders. In this way, elements of different shapes and sizes and different orders of polynomial field- and current-approximations can be used in the same FEM-MoM mesh. Thus, we fully exploit the potential of the higher-order modeling, which can lead to the reduction in computation costs by one to two orders of magnitude when compared to low-order (small-domain) techniques, for the same or better accuracy [14]-[16]. Additionally, the new hybrid method can incorporate multiple MoM objects and FEM regions in a global unbounded MoM domain. Consequently, our method is not strictly dependent on the standard FE-BI scheme; its MoM part provides much greater modeling versatility and potential for applications than just as a BI closure to the FEM part.

In Section 2, we present the theoretical background of the new higher order FEM-MoM technique. In Section 3, we give a simple example to illustrate the large-domain hybrid modeling of open-region structures.

2. Higher Order FEM-MoM Analysis

In our hybrid higher order FEM-MoM technique, the solution in the exterior region is obtained utilizing higher order MoM for discretizing the set of coupled electric/magnetic field integral equations (EFIE/MFIE) with electric and magnetic surface currents as unknowns. The solution in the interior region of the problem is obtained utilizing higher order FEM for discretizing the curl-curl electric-field vector wave equation. The two regions are denoted as region *a* (exterior or MoM region) and region *b* (interior or FEM region), as shown in Fig. 1. The two methods are coupled at the boundary of the interior (FEM) region via MoM electric surface currents, which are directly related to the boundary conditions required for closing of the FEM region. The

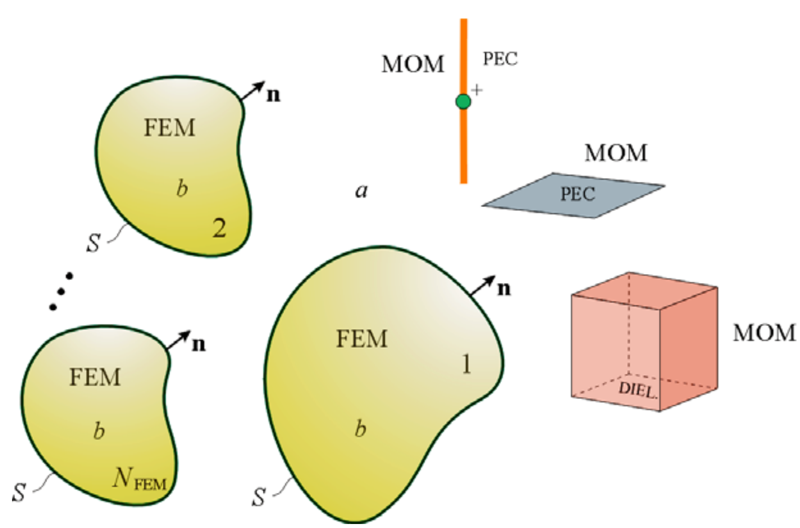

Fig.1. Exterior region (region a) modeled by MoM and interior region (region b) modeled by FEM, representing a general 3-D electromagnetic structure.

FEM-MoM boundary can be moved some distance away from the actual objects within the FEM domain (e.g., when the objects are metallic), or it can coincide with the object boundary surface (e.g., for dielectric objects). In Fig. 1, multiple MoM objects and multiple FEM regions can be present in an overall MoM environment. In this arrangement, for example, piecewise homogeneous dielectric domains can be modeled as MoM objects (via surface equivalence theorem) or as FEM regions. Metallic objects, on the other hand, can be modeled as MoM objects (via surface electric currents) or they can be enclosed in the virtual dielectric domain and treated as FEM regions.

The versatility of this concept, combined with the higher order large-domain modeling framework, allows for efficient modeling of complex EM structures provided that basic geometrical modeling building blocks, surface and volume elements, can be large and flexible. Hence, generalized curvilinear quadrilateral surface elements and hexahedral volume elements of arbitrary geometrical orders are employed for the tessellation of MoM and FEM regions, respectively. Generalized curvilinear quadrilateral, like the one shown in Fig. 2(a), has been chosen for surface geometrical modeling in the exterior (MoM) region. Similarly, the element type used for volume geometrical modeling in the interior (FEM) region is the generalized curvilinear hexahedron, shown in Fig. 2(b). Lagrange interpolating polynomials [14]-[16] are used to define mappings from the square/cubical parent domains to the Lagrange-type curved quadrilateral/hexahedron. The geometrical flexibility of these elements is illustrated in Fig. 3, in a model of a dielectric sphere enclosing a PEC plate. Finally, polynomials of arbitrary orders are used in a hierarchical fashion in the construction of vector basis functions to approximate unknown currents and fields. With such exact compatibility of volume and surface geometrical elements, and field and current approximations, the hybridization of the two methods is performed in a true higher order fashion, with respect to both geometrical modeling and field/current modeling, in both regions. In addition, the modeling flexibility and computational efficiency of the hybrid method is further enhanced by the hierarchical nature of both techniques. Namely, hierarchical higher order basis functions enable utilization of different orders of field/current approximation in different elements in the model for efficient selective discretization of the solution domain, since each lower-order set of functions is a subset of higher-order sets. Higher order large-domain hexahedral mesh generation is carried out by a semi-automatic algorithm combining the domain decomposition and mapped meshing techniques.

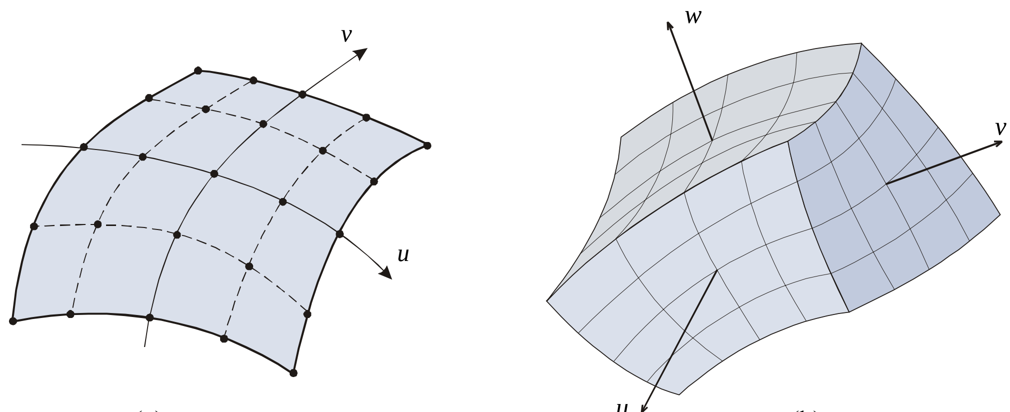

Fig.2. Lagrange-type curved parametric elements for surface and volume geometrical modeling of the structure in Fig. 1: (a) generalized quadrilateral and (b) generalized hexahedron.

Basic field equations in the domains *a* and *b* in Fig. 1 can be written as follows:

$$\mathbf{E}^a = \mathbf{E}_\mathrm{J}(\mathbf{J}_\mathrm{S}) + \mathbf{E}_\mathrm{M}(\mathbf{M}_\mathrm{S}) + \mathbf{E}^\mathrm{inc}, \quad \mathbf{E}^b = \mathbf{E}^b(\mathbf{J}_\mathrm{S}), \quad \mathbf{H}^a = \mathbf{H}_\mathrm{J}(\mathbf{J}_\mathrm{S}) + \mathbf{H}_\mathrm{M}(\mathbf{M}_\mathrm{S}) + \mathbf{H}^\mathrm{inc}, \quad \mathbf{H}^b = \mathbf{H}^b(\mathbf{E}^b), \quad (1)$$

where $\mathbf{J}_\mathrm{S}$ and $\mathbf{M}_\mathrm{S}$ represent electric and magnetic surface current vectors, respectively, and $\mathbf{E}^\mathrm{inc}$ and $\mathbf{H}^\mathrm{inc}$ are incident electric and magnetic field intensities. The above equations are coupled through boundary conditions at the region *b* boundary,

$$\mathbf{E}^a{}_\mathrm{tan} = \mathbf{E}^b{}_\mathrm{tan} = \mathbf{n} \times \mathbf{M}_\mathrm{S}, \qquad \mathbf{H}^a{}_\mathrm{tan} = \mathbf{H}^b{}_\mathrm{tan} = \mathbf{J}_\mathrm{S} \times \mathbf{n}, \quad (2)$$

with **n** standing for the outward unit normal vector, which results in a hybrid system of equations:

$$-\mathbf{E}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})_{\tan} - \mathbf{E}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})_{\tan} + \mathbf{E}^{b}{}_{\tan} = \mathbf{E}^{\mathrm{inc}}{}_{\tan}, \qquad -\mathbf{H}_{\mathrm{J}}(\mathbf{J}_{\mathrm{S}})_{\tan} - \mathbf{H}_{\mathrm{M}}(\mathbf{M}_{\mathrm{S}})_{\tan} + \mathbf{J}_{\mathrm{S}} \times \mathbf{n} = \mathbf{H}^{\mathrm{inc}}{}_{\tan}. \tag{3}$$

Formal current- and field-expansions that are used for the approximation of unknowns in the domains *a* and *b* can be represented as:

$$\mathbf{J}_{\mathrm{S}} = \sum_{j=1}^{N_{\mathrm{MOM}}} \alpha_j \, \mathbf{j}_{\mathrm{S}_j}, \qquad \mathbf{M}_{\mathrm{S}} = \sum_{j=1}^{N_{\mathrm{MOM}}} \beta_j \, \mathbf{j}_{\mathrm{S}_j}, \qquad \mathbf{E}^{b} = \sum_{l=1}^{N_{\mathrm{FEM}}} \gamma_l \, \mathbf{e}_l. \tag{4}$$

where $\mathbf{j}_{\mathrm{S}j}$ are the divergence-conforming hierarchical polynomial basis functions [16], $\mathbf{e}_l$ are the curl-conforming hierarchical polynomial basis functions [14], α_j and β_j are unknown current-distribution coefficients, and γ_l are unknown field-distribution coefficients. Scattered electric and magnetic field **E** and **H** in region *a* are expressed using electromagnetic potentials with the Green's function evaluated separately for each of the homogeneous domains of this region, and the resulting equations are discretized by the substitution of the formal current-expansions (4) into the expressions for potentials. For region *b*, the field expansion (4) is substituted in the curl-curl electric-field vector wave equation

$$\nabla \times \mu_{\mathrm{r}}^{-1} \nabla \times \mathbf{E} - k_0^2 \varepsilon_{\mathrm{r}} \mathbf{E} = 0, \tag{5}$$

where ε_{r} and μ_{r} are complex relative permittivity and permeability of the inhomogeneous (possibly lossy) internal medium, $k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$ is the free-space wave number, and ω the angular frequency of the implied time-harmonic variation.

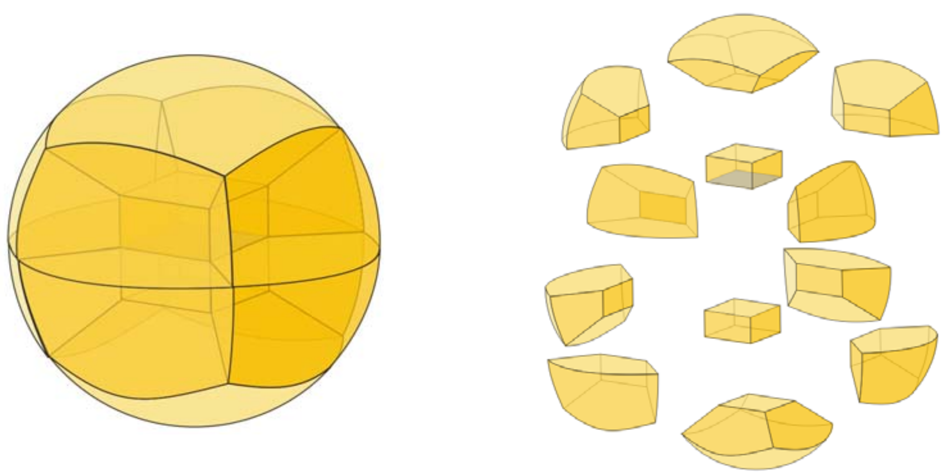

Fig.3. Higher order model of a dielectric sphere enclosing a rectangular thin PEC plate, illustrating versatility of generalized parametric elements in Fig. 2.

Galerkin-type solutions for unknown current distribution coefficients, α_j and β_j, in region *a* and field distribution coefficients, γ_l, in region *b* are explained in [16] and [14], respectively, whereas the details of the hybridization process are given in [17].

3. Numerical Results

Consider a square metallic (PEC) plate scatterer in free space shown in Fig. 4. The plate edges are λ_0 long, λ_0 being the free-space wavelength. For the purpose of the FEM-MoM analysis, the plate is encased in an air box of side lengths λ_0, λ_0, and $0.2\lambda_0$, respectively, as depicted in Fig. 5. In Fig. 6, the FEM-MoM results for the $\phi\phi$ bistatic radar cross section of the plate are compared with the results obtained by the pure MoM [16] and low-order symmetric FEM-IE [12]. The higher order FEM-MoM model uses elements of the first geometrical order, with the air layer around the plate represented by two rectangular FEM elements, and 5 MoM patches are attached onto the outer boundary of each of the FEM hexahedra. The field approximation orders in the FEM elements are adopted to be 6 in x and y directions, and 3 in the z direction, whereas the corresponding current approximation orders on MoM patches are set to be 5 and 2, so that the numbers of unknowns in the hybrid solution are 798 and 520 in the FEM and MoM regions, respectively. The results are obtained using an AMD Athlon™ 64 3500+ processor running at 2.21 GHz and with 2 GB of RAM under Microsoft Windows XP operating system. The FEM and MoM systems are solved using sparse and regular Gaussian elimination solvers, respectively. The total storage required for the filling and solving of the systems is 24 MB. The total computational time is 50 seconds, with the total matrix filling time being 43 s and solving time 7 s. The pure MoM solution uses 5 PEC patches to model the plate, and a total of 624 unknowns. Note that this simple purely metallic target is a typical example where pure MoM techniques will outperform FEM-based methods in terms of the efficiency. The FEM-IE solution is obtained using 4,876 or 10,132 unknowns [12] (computational time is not provided in [12]). We observe a very good agreement of the three sets of results, and a very considerable reduction in the number of unknowns with the higher order hybrid solution.

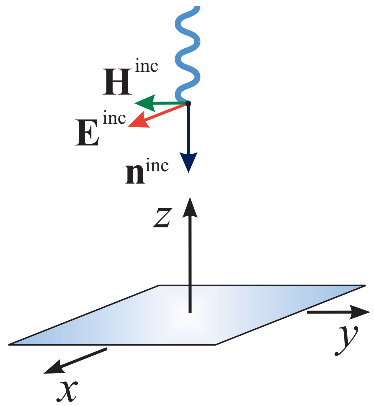

Fig.4. Scattering by a square metallic plate scatterer, λ_0 on a side, in free space.

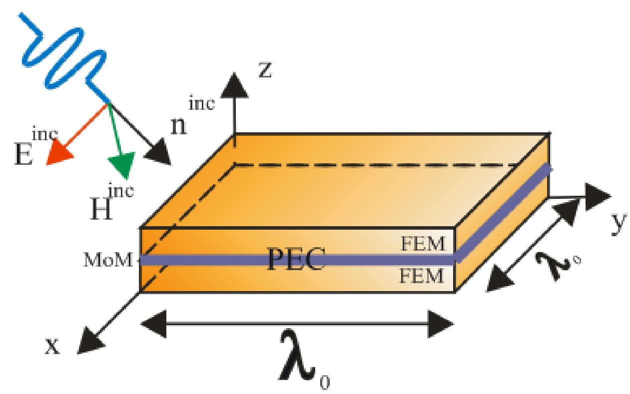

Fig.5. Higher order FEM-MoM model of the scatterer in Fig. 4 consisting of 2 FEM elements with 10 attached MoM patches on the outer boundary.

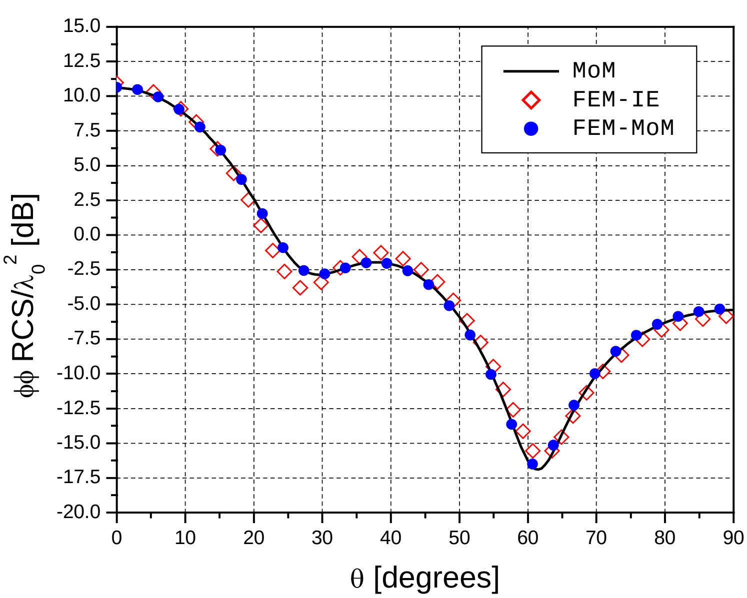

Fig. 6. Normalized ϕϕ bistatic radar cross section of the plate in Fig. 4: comparison of a higher order FEM-MoM solution (model in Fig. 5) with pure MoM [16] and low-order FEM-IE [12] solutions.

4. Conclusions

This paper has presented a higher order hybrid FEM-MoM technique for analysis and design of radiating and scattering structures. It combines the advantages and avoids the drawbacks of each of the two methods, FEM and MoM, alone, and adds to that the flexibility and efficiency of the higher order modeling methodology. Our future work will include the development of effective large-domain hexahedral meshing techniques to enhance modeling of complex real-world problems.

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