

25 Years of Progress and Future Challenges in Higher Order Computational Electromagnetics

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Abstract: A review of the past 25 years of progress and future challenges in the higher order computational electromagnetics (CEM) is presented. Higher order CEM techniques use current/field basis functions of higher orders defined on large (e.g., on the order of a wavelength in each dimension) curvilinear geometrical elements, which greatly reduces the number of unknowns for a given problem. The paper reviews and discusses generalized curved parametric quadrilateral, triangular, hexahedral, and tetrahedral elements and various types of higher order hierarchical and interpolatory vector basis functions, in both divergence- and curl-conforming arrangements, within the available and emerging higher order CEM methods and codes.

Key words: electromagnetic analysis, numerical techniques, higher order modeling, curved parametric elements, integral-equation techniques, differential-equation techniques, hybrid methods.

1. Introduction

Higher order (also known as large-domain or entire-domain) approach in computational electromagnetics (CEM) for antenna, microwave, and wireless technologies utilizes higher order basis functions for the approximation of currents and/or fields defined on large surface and/or volume geometrical elements (e.g., on the order of a wavelength in each dimension). This enables considerable reductions in the number of unknowns for a given problem, enhances the accuracy and efficiency of the CEM analysis, and results in faster (higher order) convergence of the solution, when compared to traditional low-order (also referred to small-domain or subdomain) CEM tools.

Over the past 25 years, the CEM community has developed an impressive body of knowledge in investigations and applications of higher order surface and volume elements and higher order basis functions. A large number of higher order techniques have been proposed and described in the frame of the method of moments (MoM), including surface integral equation (SIE) formulations [1]-[14], volume integral equation (VIE) formulations [15], [16], and volume-surface integral equation (VSIE) formulations [5], [17], [18], and finite element method (FEM) [3], [4], [17], [19]-[25], in addition to other approaches, such as the higher order finite-difference time-domain (FDTD) method [26]. They have already proved to be an efficient and reliable resource for solving large and complex electromagnetic problems in a variety of emerging areas of science and engineering. Higher order modeling is definitely becoming the mainstream of activity in CEM. However, many important research challenges are yet to be addressed and solved, with a common goal to create an ultimate, “ideal” analysis and design tool for each given class of real-world applications, and each given engineering problem.

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This paper reviews the past 25 years of progress and future challenges in the higher order CEM for antenna and microwave engineering applications. It discusses the state of the art in this important area, and aims at providing a representation of fundamental aspects across a spectrum of higher order CEM techniques, with a focus on geometrical modeling and basis functions within the available and emerging higher order CEM methods and codes. The core of the paper is an abridged version of the review in [27]. Section 2 presents most frequently used surface and volume curved geometrical elements for higher order CEM. Section 3 discusses higher order current and field modeling using different types of hierarchical and interpolatory vector basis functions, in both divergence- and curl-conforming arrangements. In Section 4, a characteristic numerical example is provided to illustrate higher order CEM modeling.

2. Higher Order Surface and Volume Geometrical Modeling of Arbitrary EM Structures

An arbitrary surface can be represented by a mesh with a basic building element in the form of a generalized curved parametric quadrilateral, shown in Fig.1(a). Its surface is defined by a parametric transformation (mapping) via Lagrange interpolation polynomials of arbitrary orders from a square parent domain in a local parametric coordinate system to a curved quadrilateral in the global 3-D coordinate system. Generalized quadrilaterals of higher geometrical orders are used in SIE-MoM techniques [2], [7], [9], VSIE techniques [17], [18], and MoM-PO modeling [8]. Their planar curvilinear version is used in 2-D FEM modeling [25]. Quadrilaterals of the first order (bilinear quadrilaterals) are used with higher order current approximations in SIE technique [1] and VSIE technique [5].

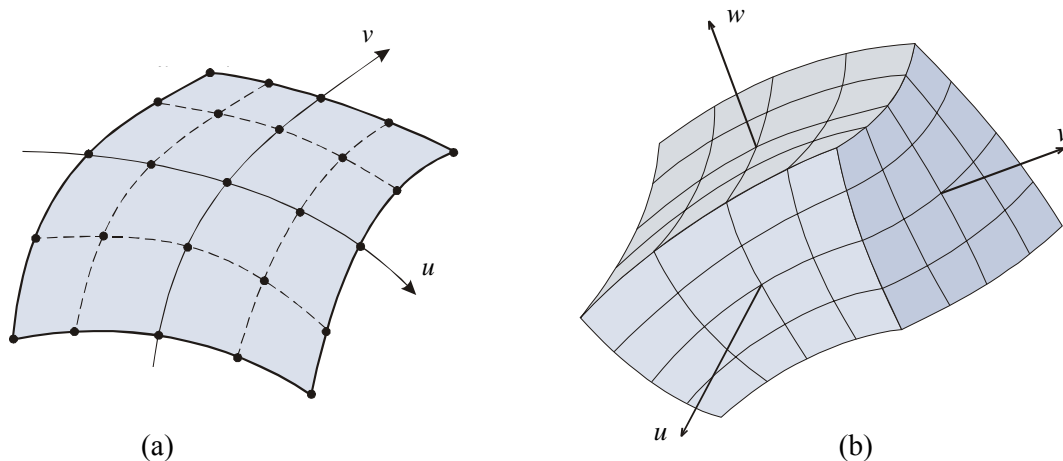


Fig.1. Lagrange-type curved parametric elements for surface and volume geometrical modeling of arbitrary 3-D electromagnetic structures: (a) generalized quadrilateral and (b) generalized hexahedron.

Another attractive and effective geometrical modeling technique for arbitrary surfaces is based on using generalized curved parametric triangles as basic building blocks for mesh generation. A generalized triangle is defined by a mapping from the planar equilateral unit-height triangle in local simplex coordinates (normalized area coordinates) to a curved triangle in the global 3-D coordinate system via Silvester-Lagrange interpolation polynomials of an arbitrary order. Generalized triangles of higher geometrical orders are used in SIE techniques [2]-[4], [10], [12], [14], SIE-Green's function technique for PEC structures in planar multilayer dielectric media [11], and finite element – boundary integral (FE-BI) techniques [19], [20]. Their planar curvilinear version is used in 2-D FEM technique [21].

Volumetric modeling of arbitrary electromagnetic structures can be carried out using generalized curved Lagrange-type interpolation hexahedra of arbitrary geometrical orders, shown in Fig.1(b), representing a volume (3-D) generalization of quadrilateral patches in Fig.1(a). These elements are used

in VIE techniques [16], VSIE techniques [17], [18], FEM techniques [23]-[24], and FE-BI modeling [17]. Trilinear hexahedra (of the first order) are used with higher order current approximations in VIE [15] and VSIE [5] modeling.

Extending the triangular curved surface to a volumetric element, a generalized curved parametric tetrahedron is obtained. Generalized tetrahedra of higher geometrical orders are used for FEM modeling in [4], [19], [20].

3. Higher Order Basis Functions for the Approximation of Currents and Fields

The first class of basis functions on a generalized quadrilateral in Fig.1(a) is a set of divergence-conforming hierarchical-type vector basis functions representing a higher order generalization of rooftop functions, constructed from simple power functions in parametric coordinates u and v . With them, the electric surface current density vector over generalized quadrilaterals (in the MoM-SIE model) is approximated as [27]

$$\mathbf{J}_s = \frac{1}{\mathfrak{S}} \left[\sum_{i=0}^{N_u} \sum_{j=0}^{N_v-1} \alpha_{ij}^{(u)} f_{ij}^{(u)}(u,v) \mathbf{a}_u + \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} \alpha_{ij}^{(v)} f_{ij}^{(v)}(u,v) \mathbf{a}_v \right], \quad f_{ij}^{(u)} = \hat{P}_i(u) P_j(v), \quad f_{ij}^{(v)} = P_i(u) \hat{P}_j(v),$$

$$\hat{P}_i(u) = \begin{cases} 1-u, & i=0 \\ u+1, & i=1 \\ u^i-1, & i \geq 2, \text{ even} \\ u^i-u, & i \geq 3, \text{ odd} \end{cases}, \quad P_j(v) = v^j, \quad \mathfrak{S} = |\mathbf{a}_u \times \mathbf{a}_v|, \quad \mathbf{a}_u = \frac{\partial \mathbf{r}}{\partial u}, \quad \mathbf{a}_v = \frac{\partial \mathbf{r}}{\partial v}, \quad -1 \leq u, v \leq 1, \quad (1)$$

where N_u and N_v are the adopted degrees of the polynomial current approximation, and $\mathbf{r} = \mathbf{r}(u,v)$ is the position vector of the point determined by parameters u and v on the quadrilateral surface. These basis functions are used for SIE modeling of both electric and magnetic surface currents on generalized quadrilaterals of higher geometrical orders in [7], and in conjunction with bilinear quadrilaterals in [1], as well as in VSIE modeling [5]. Orthogonality and conditioning properties of hierarchical higher order basis functions in (1) can be significantly improved if basis functions are constructed from standard orthogonal polynomials instead of power expansions. Notable examples are classes of Legendre basis functions [9] and functions constructed from Chebyshev polynomials [6].

The next class of higher order basis functions are divergence-conforming interpolatory vector basis functions on generalized triangles, developed starting with Rao-Wilton-Glisson (RWG) basis functions, which are a simplex counterpart of rooftop functions on quadrilateral patches. They have excellent orthogonality properties, producing well-conditioned MoM/FEM matrices, and are used in SIE techniques for modeling of PEC structures in free space [2]-[4], [10], and in layered media [11], as well as for FE-BI modeling [19]. In higher order TDIE techniques, a higher order spatial discretization of currents is combined with a temporal discretization based on the marching-on-in-time scheme [12]. In point-based higher order SIE techniques based on the Nyström discretization, the unknown surface currents are represented by their samples at a set of discrete points on the surface geometrical elements (quadrilateral or triangular patches) in the model [13], [14].

The curl-conforming version of hierarchical higher order basis functions on generalized quadrilaterals in (1) is used, for instance, in 2-D FEM computation [25]. Based on the curl-conforming version of the RWG bases, which is known as Whitney forms, the corresponding functions on generalized triangles are those used in 2-D FEM modeling [21]. Higher order sets of divergence- and curl-conforming interpolatory basis functions defined on generalized curved quadrilaterals, analogous to those on triangles, are also available [4], [2], as well as hierarchical higher order basis functions for triangular elements [2].

For VIE and VSIE hierarchical modeling using generalized hexahedral elements in Fig.1(b), a 3-D generalization of divergence-conforming higher order 2-D vector bases in (1) is used, with power expansions in (1) [5], [15] or Legendre expansions [16]. In the curl-conforming form, we have the following approximation for the electric field vector in the elements for FEM computation:

$$\mathbf{E}_u = \sum_{i=0}^{N_u-1} \sum_{j=0}^{N_v} \sum_{k=0}^{N_w} \alpha_{ijk}^{(u)} P_i(u) \hat{P}_j(v) \hat{P}_k(w) \mathbf{a}'_u, \quad \mathbf{a}'_u = \frac{\mathbf{a}_v \times \mathbf{a}_w}{\mathfrak{I}}, \quad -1 \leq u, v, w \leq 1, \quad (2)$$

with versions combining power functions [23] and Legendre polynomials [24]. FEM models with 3-D curl-conforming bases on higher order interpolatory tetrahedral elements are given in [4], [19]. Such functions for higher order time-domain FEM formulations are proposed in [20].

Finally, the accuracy and efficiency of higher order basis functions near sharp edges can be improved by explicitly incorporating the terms that exactly model the singular edge currents/field behavior [1], [22].

4. Numerical Results

As the numerical example of higher order CEM modeling, we consider a dielectric ($\epsilon_r = 4$ and $\mu_r = 1$) coated PEC spherical scatterer, of radii $a = 0.3423\lambda_0$ (PEC) and $b = 0.444\lambda_0$ (the entire scatterer), λ_0 being the free-space wavelength, as shown in Fig.2(a). This corresponds to the frequency of the internal resonance of the sphere, and it is always a good check to investigate the method performance in such cases. This example is also illustrative because it is a structure with pronounced curvature. We employ the higher order hybrid FEM-MoM technique [28], with as simple as possible geometrical mesh consisting of only 6 curvilinear hexahedral FEM elements in Fig.1(b) and 6 attached curvilinear quadrilateral MoM patches in Fig.1(a) on the outer boundary, all of the second geometrical orders, which is sketched in Fig.2(a). Shown in Fig.2(b) is the normalized $\theta\theta$ bistatic radar cross section of the coated sphere for the plane wave incidence indicated in the figure inset. The FEM field approximation orders in (2) are adopted to be 2 in the radial direction and 4 in the other two directions in hexahedra, with the corresponding orders for MoM current approximations in (1), so that the numbers of unknowns in the hybrid solution are 580 and 216 in the FEM and MoM regions, respectively, and the total computational time is 9 seconds. The results are compared with the exact Mie's solution and with numerical results obtained by a low-order symmetric FEM-IE [29], and a good agreement of the three sets of results is observed.

5. Conclusions

This paper has reviewed the higher order CEM for antenna and microwave engineering applications. These techniques use higher order current/field basis functions defined on large (e.g., on the order of a wavelength in each dimension) curvilinear geometrical elements, which greatly reduces the number of unknowns and enhances the accuracy and efficiency of the analysis. There is a great diversity of higher order formulations, elements, bases, and solution techniques. Although all these components, as well as their many working combinations resulting in higher order CEM codes, seem to be completely different, they all have a lot in common. On the other side, they all show some advantages and deficiencies. The choice of the "best" method depends on the particular problem that needs to be solved. Therefore, all presented and/or referenced higher order formulations, elements, bases, and solutions, as well as many others that could not be addressed and referenced here due to space limitations, are important and constitute a body of knowledge in the higher order CEM area. Moreover, it is likely that practically all future CEM techniques and codes will have some higher order properties, because such elements and bases exhibit excellent convergence, flexibility, and suitability for refinements and adaptive simulations. Finally, low-order modeling approach can be considered to be a special case of the higher order modeling.

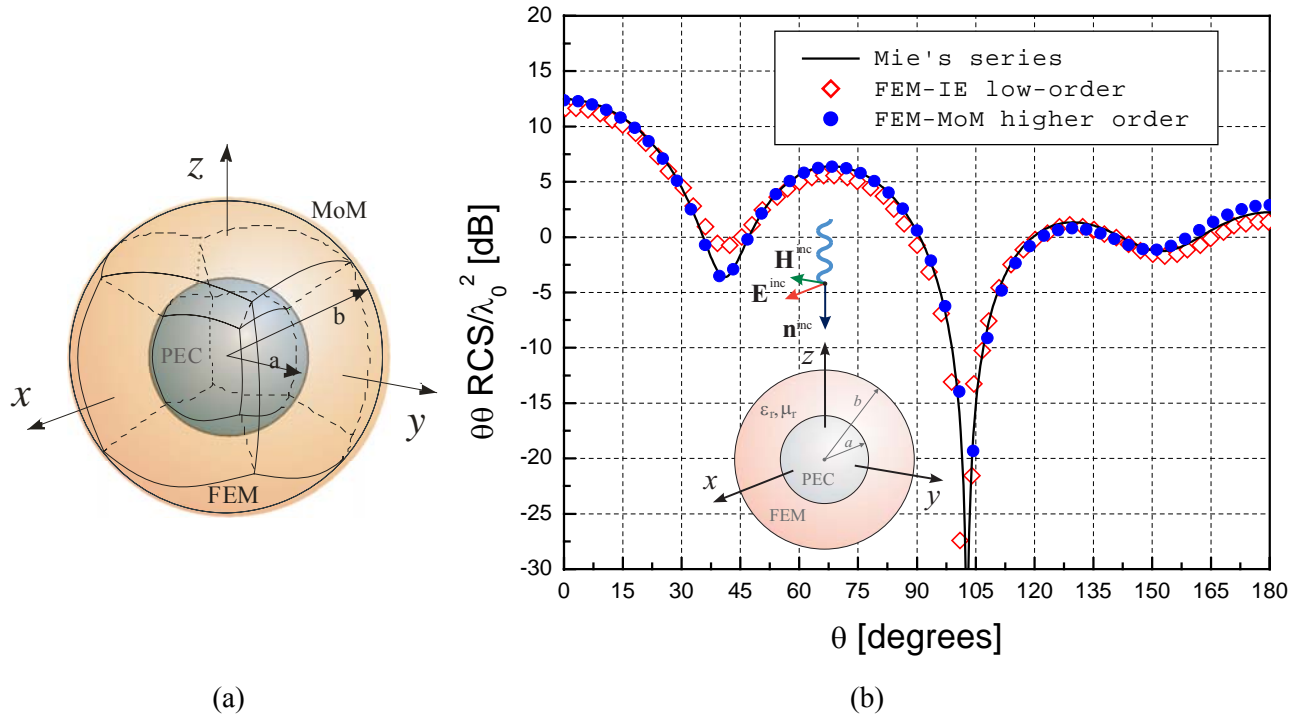


Fig. 2. FEM-MoM analysis [28] of a dielectric coated PEC spherical scatterer at its internal resonance (courtesy of Milan Ilić): (a) higher order model with 6 curved FEM hexahedral elements and 6 MoM quadrilateral patches and (b) normalized $\theta\theta$ bistatic radar cross section – comparison of the higher order FEM-MoM solution with numerical results using a low-order FEM-IE [29] and the exact Mie’s solution.

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