



Saturation of Kerr-lens mode locking and the self-amplitude modulation coefficient

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Abstract

This work deals with the saturation of the Kerr-lens mode locking mechanism. A quantitative description of the self-amplitude modulation effect in a laser cavity is derived, considering both the nonlinear and geometrical (cavity) contributions to this nonlinear modulation. The loss of the pulse conformation strength in a cavity due to “bleaching” of the fast saturable absorber behavior of the Kerr lens together with an intracavity aperture produces the appearance of instabilities. The goal is then to elaborate a formalism that allows the prediction of these instabilities, and develops as useful for short pulse laser cavity design in general. The obtained self-amplitude coefficient is included within the *KLJL* ray-pulse matrix formalism, and a simple model for the temporal pulse parameters regarding only the amplifier bandwidth and the self amplitude modulation is studied. This model sets a limit for the maximum pulse energy of a stable solution for a given nonlinear modulation and bandwidth.

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1. Introduction

Short pulse lasers have developed as tools of major importance for countless applications in activities not only of scientific but also of technolog-

ical interests. One of the possible ways of obtaining short pulse operation of a laser is the Kerr-lens mode locking (KLM). This is a well-established technique for short and ultrashort pulse generation, based on the self-focusing effect of a high intensity beam that takes place when the light travels through an intracavity Kerr medium. This dynamic self-focusing has been successfully used for mode lock of several bulk-element solid-state lasers [1–4]. It is still one of the more practical ways to generate short and ultrashort pulses in the medium ($\sim W$) power range, since the advent of semiconductor

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saturable absorber mirrors [5] to produce pulsed lasers is still limited by the degradation of the saturable absorber with power. The complicated design of these latter devices restricts the full exploit of their advantages to lasers on the multiwatt scale. KLM lasers have been extensively studied in experiments (see [6] and references therein) and from different theoretical frameworks, including master equation models [7–10] and discrete map formalisms [11–13]. In these lasers however the nonlinear interaction can also produce pulse train instabilities. Complex behaviors such as period doubling, multistability and quasiperiodic oscillations have been observed and investigated from a variety of points of view [14–18]. The main purpose of this paper is to elaborate a formalism that allows taking into account the saturation effect of the KLM, in order to predict pulse instabilities in short pulse laser design and scale these systems with power.

The combination of the self-focusing Kerr effect with an intracavity aperture is equivalent to a fast response saturable absorber [8]. In this way, the system is expected to produce an amplitude modulation that follows the pulse with an instantaneous ($\sim 10^{-15}$ s) response. As the pulse power increases, the saturable absorber eventually bleaches, and this sets the limit of the pulse conformation (the reduction of the pulse duration in a single passage, or pulse shortening rate).

However, this analogy of the KLM with a fast saturable absorber is only partial. At this time it might be appropriated to recall that the aperture position within the cavity (and the choice of the geometric parameters of the latter) defines in a subtle manner the ability of the system to (a) favor the pulsed regime over the CW one and (b) produce a dynamic self-amplitude modulation (SAM) on the pulse in such a way that more losses are suffered on the tails of the pulse than on the central part of it. This last feature is what determines the pulse-shortening rate, since the curvature or parabolic term of the temporal modulation of the pulse is just the driving force for pulse conformation.

A very simplified scheme of the process can be sketched as follows, when a pulse train is focused through an intensity-dependent focal length lens (the Kerr lens): in the desired situation, the central part of each pulse is focused near the aperture

plane, whereas the (lower intensity) sides produce a weaker lens and consequently their focus plane are more distant from – after – the aperture plane, giving a larger spot on the aperture. The losses suffered in the aperture are greater in the sides than in the center of the pulse, giving the correct amplitude modulation shape (i.e. parabolic). If the pulse intensity is increased, the situation changes dramatically: the focal plane of the central part of the pulse can shift, focusing now before the aperture plane due to the stronger Kerr effect; this will produce a saturation of the Kerr effect and eventually may lead to the inversion of the SAM curvature. This is due to the fact that the minimum losses of the aperture are obtained for a region of the pulse between the center and the tails. In other words, for a given geometrical arrangement, a situation of optimum pulse conformation through SAM can turn into a detrimental condition just by increasing the pulse intensity.

An increase of the pulse peak power can be produced either by a higher intracavity average power, $\langle P \rangle$ (by means of increasing the pump power, for example) or by lengthening the laser cavity L . This extended cavity arrangement is a well-known method to obtain high-power pulses [19]; however, these extended-cavity systems are likely to operate in undesired double pulse regimes. A quantitative description of the Kerr-lens saturation phenomenon is then a desirable tool for laser design.

The next section is devoted to the development of an explicit expression for the SAM in a general laser cavity, taking into account spatial, temporal and nonlinear effects on the pulse. In Section 3 the geometrical factor is studied in a specific laser cavity. Finally, the inclusion of this effect in the *KILL* temporal matrix formulation and the study of a simple “pure SAM” pulse-conformation laser model are discussed in Section 4.

2. Explicit calculation of the nonlinear amplitude modulation in a laser cavity

In what follows, this idea of saturation of the SAM is stated in a more formal way, in order to obtain a quantitative description of the effect and a

parameter that characterizes it. The goal is the development of new tools for aid in KLM cavity design.

The pulse amplitude modulation caused by the combination of the intracavity aperture and the self-focusing of the beam is not merely an intensity-dependent lens. In fact, it is this combination of two *spatial* effects (self-focusing and spatial losses) that establish the analogy of KLM with a fast saturable absorber, and hence it affects the *temporal* shape of the pulse.

Nonlinear focusing is stronger where the field intensity is higher (i.e. in the center of a gaussian beam). This produces a variation of the mode size along the length of the pulse, which in turn generates the temporal amplitude modulation when crossing the aperture. The transmission through such self-modulation element, T_{SAM} , can be estimated studying the transformation of a gaussian pulse of duration τ , energy U , and spatial parameters w_1, R_1 that propagates in the nonlinear medium of length z assuming a temporal envelope $P(t) = U/\tau \exp[-2(t/\tau)^2]$. The spatial parameters at the output of the nonlinear medium $w_2(t), R_2(t)$ can be obtained using the expressions [20]:

$$w_2^2(t) = w_1^2 \left[\left(1 + \frac{z}{R_1} \right)^2 + \left(\frac{\lambda z}{\pi n_0 w_1^2} \right)^2 \left(1 - \frac{P(t)}{P_c} \right) \right], \tag{1a}$$

$$\frac{1}{R_2(t)} = \left(\frac{w_1}{w_2(t)} \right)^2 \left[\frac{1}{R_1} + \frac{z}{R_1^2} + z \left(\frac{\lambda}{\pi n_0 w_1^2} \right)^2 \left(1 - \frac{P(t)}{P_c} \right) \right]. \tag{1b}$$

These expressions describe the nonlinear focusing effect on the beam, taking into account the instantaneous power of the beam normalized to the critical power of self focusing in the nonlinear material, P_c . Far from the peak ($P(t) \ll P_c$), Eq. (1) gives a linear spot-size propagation; however, the spot size at $t = 0$ is smaller by a factor

$$\left(\frac{\lambda z}{\pi n_0 w_1 w_2^0} \right)^2 \frac{U}{2\tau P_c}, \tag{2}$$

where w_2^0 is the small-signal ($P = 0$) spot size at the exit of the nonlinear medium. Expressions (1) for

the beam parameters after the nonlinear medium are exact within the parabolic approximation for the temporal pulse envelope, and numerical values can be calculated performing a self-consistent calculation, that can be resolved in an iterative way. The next step is to propagate the beam through the cavity up to the position of the aperture. This is a linear propagation, which can be calculated in a straightforward way using the *ABCD* matrix formalism. The new beam parameters are $w_3(t), R_3(t)$ at the aperture plane. An explicit expression for the nonlinear modulation can then be obtained, calculating the losses suffered by the beam at the aperture: we can assume that the aperture size, w_{ab} , is very close to the small signal (CW) spot size on that plane (this is a reasonable assumption since the cavity is designed for KLM operation). The losses are then proportional to the ratio $\mu(t) = (w_3(t)/w_{ab})^2$ [13]. In this situation, the self-modulation is described by the curvature of the nonlinear loss modulation, and it is proportional to the second time derivative of the beam area, normalized to the aperture area on the aperture plane, μ .

The general expression for the modulation curvature is then:

$$\begin{aligned} \rho_{SAM} &\equiv \frac{1}{2} \frac{\partial^2 \mu}{\partial t^2} \Big|_{t=0} \\ &= \frac{U}{\tau^3 w_{ab}^2 P_c} \left(\frac{\lambda z}{\pi w_1} \right)^2 \left\{ \left(A + \frac{B}{z} \right)^2 - B^2 \left[\left(\frac{1}{z} - \frac{1}{R_2} \right)^2 + \left(\frac{\lambda}{\pi w_2^2} \right)^2 \right] \right\}. \end{aligned} \tag{3}$$

At this point, the assumption that the only time-dependent variable is P is made. ρ_{SAM} is then a first-order approximation on the temporal variable. The temporal shape of the modulation can be obtained in a straightforward manner, by evaluating the second derivative in (3) on the center of the pulse; w_1 and w_2 (R_1 and R_2) correspond to the spot-sizes (curvature radii) at the entrance and the exit of the nonlinear medium, respectively. The coefficients A and B are the matrix elements of the linear propagation from the nonlinear medium plane to the aperture plane.

3. Geometric (cavity configuration) effects

The transmission through a self-amplitude modulator is

$$T_{\text{SAM}} = 1 - \rho_{\text{SAM}} t^2. \tag{4}$$

This expression can be compared with the transmission for a fast saturable absorber, introduced by Ippen [6]:

$$T_{\text{fs}} = 1 - \left(\frac{\gamma U}{2\tau}\right) \left(\frac{t^2}{\tau^2}\right). \tag{5}$$

Comparing these two expressions, a first approximation for an explicit SAM factor, γ , can be obtained. This coefficient takes into account both the geometrical and nonlinear factors, for a general cavity with an aperture and a nonlinear Kerr medium:

$$\gamma = \frac{2}{w_{ab}^2 P_c} \left(\frac{\lambda z}{\pi w_1}\right)^2 \left\{ \left(A + \frac{B}{z}\right)^2 - B^2 \left[\left(\frac{1}{z} - \frac{1}{R_2}\right)^2 + \left(\frac{\lambda}{\pi w_2^2}\right)^2 \right] \right\}. \tag{6}$$

Here, all the spatial parameters (w_i , R_i , A and B) are to be obtained through a self-consistent calculation, taking into account the Kerr medium and peak intracavity power. The SAM coefficient is inversely proportional to the critical power and the aperture size. It is worth to notice that γ can be negative. This is due to the fact that the effect of the nonlinear medium on the aperture plane is influenced by the unavoidable beam propagation through the optics between them.

In a specific cavity, a change of sign or a large fluctuation of the SAM parameter will influence the performance of the pulsed laser: the γ factor takes into account the cavity effects. In other words, γ represents the curvature of the loss modulation in the center of the pulse. If this factor is positive, the SAM favors the pulse formation. If, on the contrary, it is negative, the nonlinear modulation prevents the correct pulse formation, since the aperture introduces more losses on the center of the pulse than on the tails.

As stated above, in order to obtain a numerical value of γ per round trip, the spatial cavity pa-

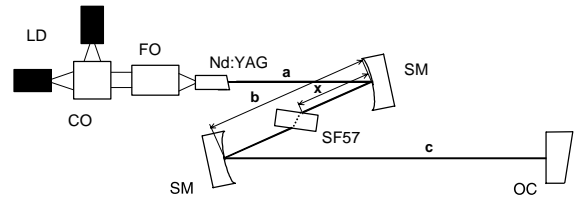


Fig. 1. Basic layout of the KLM Nd:YAG laser cavity. LD: laser diodes; CO: combining optics; FO: focusing optics; SM: spherical mirrors; OC: output coupler. Cavity dimensions are $a = 340$ mm, $b \cong 100$ mm, $c = 1000$ mm, $x \cong 60$ mm. The active medium is a 3 mm diameter, 5 mm length, 1.1% atm Nd-doped YAG rod, while the nonlinear medium is a 8 mm thick SF57 glass slab.

rameters are to be numerically calculated before. As an example, this was performed for a KLM Nd:YAG laser cavity (Fig. 1) which is fully described in a previous work [21], using an iterative method. In this laser, an 8 mm-thick SF57 acts as the nonlinear medium, while the gain-guiding together with the thermal lens aberrations produced by the pump beam define a soft aperture on the Nd:YAG rod. The control parameter for this cavity is the position of the nonlinear medium relative to the two 100 mm ROC spherical mirrors. Fig. 2 shows the variation of the SAM coefficient with the position of the nonlinear medium along the intracavity beam waist. For this

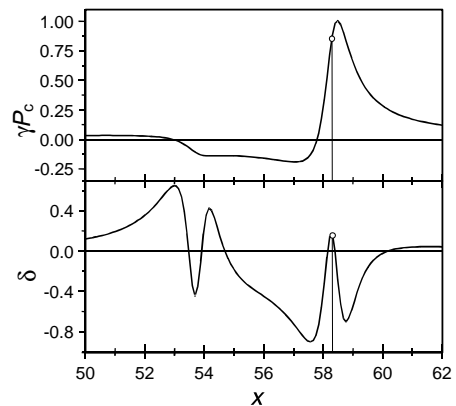


Fig. 2. SAM coefficient (scaled with the critical power of self-focusing, P_c) for different cavity configurations, obtained varying the position of the nonlinear medium between the two spherical mirrors (top). Behavior of the small-signal spot size relative variation, δ , for the same situation (bottom).

calculation, the distance between mirrors was fixed in 104 mm, and the other geometrical parameters are as described in [21]. The shown values are obtained for $P = 0$, thus indicating that a strong dependence of the cavity geometry is present in the system, and variation of the self amplitude modulation coefficient is present even at small-signal powers. In the same figure, the small signal relative spot size variation $\delta \equiv -1/w(\partial w/\partial P)_{P=0}$ is also shown.

With the explicit formulation for γ , Eq. (6), and close inspections of graphics like the one in Fig. 2, another tool for KLM laser design is available. If the δ parameter, reveals favorable regions to obtain good discrimination between low and high power solutions [22], the γ parameter calculated in this way can prove useful to discover which are the regions where the pulse conformation is good, and the other regimes where the system could eventually go into pulse splitting. It is worth to note in Fig. 2 that several regions where δ is positive – hence favorable to KLM – exist. However, the SAM coefficient is large and positive only in a small interval around $x = 58\text{--}60$ mm (x is the distance between the nonlinear medium and one of the spherical mirrors). This position constitutes a good region to experimentally explore for KLM, since both δ and γ are positive. On the other hand, the region around $x = 52\text{--}54$ mm, where δ is large (and at first sight favorable), has a γ value small or negative, being thus detrimental for stable ML operation.

The single-passage SAM coefficient can also be calculated for a higher intracavity power. Fig. 3 shows the behavior of γ at a fixed geometrical condition where $\delta > 0$ and $\gamma(P=0) > 0$. The SAM coefficient begins to grow with the increasing power; for $P/P_c \sim 0.5$ the slope changes sign and γ decreases, until it reaches a negative value – saturation of the nonlinear modulation – for $P/P_c \sim 0.8$.

Up to this point, it becomes clear that the appropriate conditions for obtaining KLM are fulfilled via a fine design and adjust of geometrical variables of the cavity; however, this condition is by no means independent of the laser power. As the pulse power varies, the nonlinear effect changes and the cavity should be readjusted to adapt itself

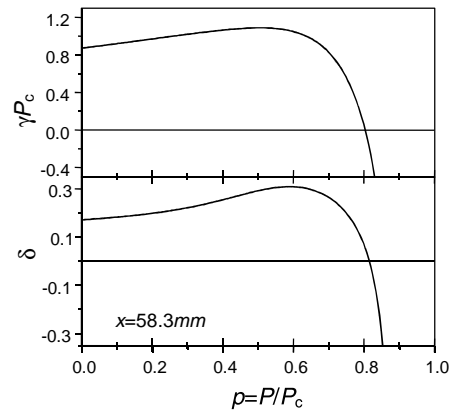


Fig. 3. Evolution of the γ and δ factors for increasing intracavity power, for the condition marked in Fig. 2.

to the new condition. An excessive self-focusing that saturates or inverts the sign of the Kerr-lens effect can produce a pulse train instability. If enough nonlinearity, gain and bandwidth exist in the system, the laser can sustain pulses with a complex temporal structure, or even a double pulse (pulses with separation of the order of the picosecond). Such is the case of Ti:Sapphire lasers [17]. Instead, in narrowband lasers like Nd:YAG, the pulse train can lose stability against the CW solution, or degenerate into a multiple pulse operation, as it was described and discussed in [23].

In the following section, the preceding results will be stated within the *KILL* temporal ray-pulse matrices and a simple iterative map will be studied, in order to establish a limit for the amount of SAM that a KLM laser can sustain.

4. Temporal matrix for the self-amplitude modulation. Application to a simple model

The transmission of a pulse through the “non-linear amplitude modulator”, that is the Kerr medium followed by an aperture, is described up to the quadratic term by (4). In terms of the temporal gaussian parameter, p [24], the expression reads:

$$\frac{1}{p'} \approx \frac{1}{p} - i \frac{\gamma U}{\tau^3 k}, \quad (7)$$

where k is the absolute value of the wave vector. The transformation rule for the temporal parameter p) in a generic through a generic $KIJL$ matrix is

$$\frac{1}{p'} \equiv \frac{\lambda J + L \frac{1}{p}}{K + I \frac{1}{\lambda} \frac{1}{p}}. \quad (8)$$

It is then straightforward to obtain the temporal matrix that accounts for the SAM:

$$M_{\text{SAM}} = \begin{pmatrix} 1 & 0 \\ -i \frac{\gamma U}{2\pi\tau^3} & 1 \end{pmatrix}. \quad (9)$$

Expression (9) is nonlinear in τ and includes spatial effects through γ . In this matrix the KLM effect is summarized: this SAM matrix is the temporal counterpart of the spatial $ABCD$ matrix that accounts for a gaussian aperture. In other words, the effect of this element is to reshape the pulse, cutting its tails without altering its chirp. A pulse of duration τ_1 and chirp S_1 propagating in a “fast saturable absorber” like this, suffers a transformation given by

$$S_2 = S_1, \quad (10a)$$

$$\tau_2 = \tau_1 \frac{1}{\sqrt{1 + \frac{\gamma U}{2\tau_1}}}. \quad (10b)$$

The pulse dispersion remains unchanged, while its duration is reduced by a factor $\approx \gamma U / 4\tau$. This factor increases with the pulse energy and is inversely proportional to the pulse duration, meaning that the shorter the pulse, the higher the single passage pulse shortening rate, as it is expected for a fast absorber.

Next, a minimum hypothesis model of a KLM laser is studied, using the ray-pulse matrix formulation. The model takes into account only the SAM effect and the spectral narrowing in the amplifier.

The effect of a finite bandwidth gain medium is described in [25]. The equivalent temporal matrix is

$$M_{\text{amplif}} = \begin{pmatrix} 1 & i \frac{16\pi g}{\Delta\omega_a^2} \\ 0 & 1 \end{pmatrix}, \quad (11)$$

with g the saturated gain per passage, and $\Delta\omega_a$ the amplifier bandwidth. The gain medium tends to broaden the pulse by tailoring its bandwidth. The shorter pulses that can emerge of such media are given by

$$\tau_{\min} = \frac{4\sqrt{g\pi}}{\Delta\omega}. \quad (12)$$

An iterative map is built by simply performing the product of the two matrices $M_{\text{SAM}} \times M_{\text{amplif}}$. The resulting matrix is

$$M_{\text{SAM}} \times M_{\text{amplif}} = \begin{pmatrix} 1 & i \frac{16\pi g}{\Delta\omega_a^2} \\ -i \frac{\gamma U}{2\pi\tau^3} & 1 + \frac{8\gamma U g}{\tau^3 \Delta\omega_a^2} \end{pmatrix}; \quad (13)$$

and, according to the $KIJL$ transformation (8), the following map is obtained:

$$S_{n+1} = \frac{S_n}{\left(1 + \frac{16g}{\tau_n^2 \Delta\omega_a^2}\right)^2 + \left(\frac{8gS_n}{\Delta\omega_a^2}\right)^2}, \quad (14a)$$

$$\tau_{n+1} = \tau_n \cdot \left[\frac{\gamma U}{2\tau_n} + \frac{1 + \frac{16g}{\tau_n^2 \Delta\omega_a^2} + \frac{4g}{\Delta\omega_a^2} S_n^2 \tau_n^2}{\left(1 + \frac{16g}{\tau_n^2 \Delta\omega_a^2}\right)^2 + \left(\frac{8gS_n}{\Delta\omega_a^2}\right)^2} \right]^{-1/2}. \quad (14b)$$

The steady-state solutions are found imposing the conditions $S_{n+1} = S_n$, $\tau_{n+1} = \tau_n$. In order to analyze the evolution of the solutions with the different parameters, the pulse energy is assumed independent of the pulse duration. This can be justified by noting that the pulse energy is essentially the mean laser power (which is constant in the steady state) times the round-trip time, T_{RT} . Also, in a first approximation, only the small-signal value of the γ factor (i.e. independent of the pulse duration) is considered. First, S is obtained from (14a), and it is then replaced in (14b) to find the fixed point of the variable τ :

$$S = 0, \quad (15a)$$

$$\tau = \frac{16g}{\gamma U \Delta\omega_a^2} \left(1 \pm \sqrt{1 - \frac{\gamma^2 U^2 \Delta\omega_a^2}{16g}} \right). \quad (15b)$$

The first inspection of these expressions shows that the solutions have no chirp. This is a reasonable result, since neither the gain medium nor the SAM introduces dispersion to the pulse.

The dependence of the pulse duration with the control parameters of the system – gain, bandwidth and SAM – are depicted in the graphics of

Fig. 4. There are two possible solutions for the pulse duration. The one that gives the shorter pulse is always unstable. The stable solution gives large pulse duration for small SAM values and approaches to the transform limited value as the nonlinear modulation increases. If the cavity lacks a mechanism to increase the pulse duration, like the dispersion in femtosecond lasers or fiber lasers, then the solution collapses due to an excess of nonlinear modulation. This is an interesting result, as it sets a limit for the SAM (and the pulse energy, since (15b) scales identically with γ and U). The following relation gives that limit:

$$(\gamma U)_{\max} \cong \frac{4\sqrt{g}}{\Delta\omega_a}. \quad (16)$$

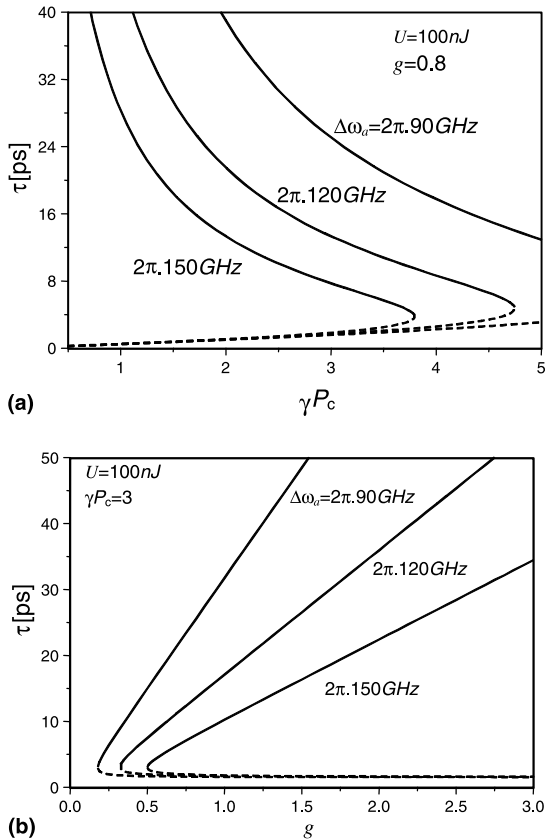


Fig. 4. Dependence of the steady-state solution of the pulse duration [Eq. (15b)] with the change of (a) the nonlinear SAM coefficient γ ; (b) the gain g , for different values of the amplifier bandwidth $\Delta\omega_a$. The dotted lines represent the unstable solutions.

In this extreme situation, the single-passage absolute pulse reduction is of the order of the pulse duration itself:

$$(\Delta\tau)_{\max} \cong \frac{\sqrt{g}}{\Delta\omega_a}. \quad (17)$$

As it is expected, no solution exists for negative SAM values.

The effect of the gain produces an almost linear increase of the pulse duration, while an increase of the bandwidth causes a decrease of τ , as observed in the practice.

This simple model condenses the general features of KLM pulses, when the SAM is the nonlinear dominant mechanism. The pulses are chirp-free and get shorter as the modulation depth increases. Furthermore, the limit given by the amplifier bandwidth restricts the amount of nonlinearity that the system can hold.

It is then clear how this self-modulation effect can be incorporated in KLM cavity design and study. This analysis can be made extensive to more complex systems such as ultrashort pulse lasers, but additional work is needed. In such systems, like the Ti:Sapphire laser, other effects should be taken into account: material dispersion and self-phase modulation (SPM) are relevant on femtosecond time scales. Together with SAM and gain broadening, they give place to a rich variety of solutions for the output pulse train. Hence, for these systems, a more detailed description that is beyond the scope of this work is needed. However, two different limit situations can be discussed: if the operation mode of the laser implies that the pulse is almost transform-limited on the active medium, the results of this work hold, provided there is no additional dispersion between the Kerr medium and the aperture. If, on the contrary, the pulse is highly chirped on the active medium, a relaxation of the maximum energy limit can be expected, since the peak power is lower for that state of the pulse than for the transform-limited state.

5. Summary

An explicit expression for the calculation of the amount of SAM present in a laser cavity was

derived. The SAM factor considers the geometrical effect that arises from the spatial separation between the nonlinear medium and the aperture. A specific laser cavity is studied, mapping the γ factor for different cavity configurations and intracavity powers. This calculation shows that the pulse conformation coefficient gives sensible information, which is complementary with the small-signal spot size variation, δ . While this figure gives the affinity of the cavity to operate in CW or pulsed regime, the SAM factor reveals the sign and strength of the pulse conformation, given by the curvature of the amplitude modulation.

Finally, the SAM given by the combination of a Kerr-lens and an aperture is stated within the ray-pulse matrix formalism, as a nonlinear 2×2 matrix of practical interest in laser cavity design. In this way, the nonlinear temporal effects can be now taken fully into account, at least up to the quadratic order, using the matrix introduced here and the self phase modulation matrix [26,27]. A simple nonlinear self-consistent Poincaré map for the temporal pulse parameters is calculated, considering only the amplifier gain and the SAM. The steady state solution sets a limit for either the pulse energy or the strength of the pulse conformation, essentially given by the bandwidth of the gain.

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