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Multipulse Nd:YAG Kerr lens mode-locked laser

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Abstract

Mode locked lasers often oscillate with more than one pulse per round trip. In particular, we observe that double and triple pulses spontaneously appear in a diode-pumped Kerr lens mode-locked (KLM) Nd:YAG laser as the pumping power is increased. We find that this is caused by the cooperative induction, due to counter-propagating pulses in the Kerr medium, of an intensity-dependent lens which is stronger than the lens induced by a single pulse. We present a simple model to explain the measured duration and energy of the pulses and to take account of the observed stability of the pulse time separation. We also find that the system selects the mode of operation (i.e., single, double or triple pulse) in order to maximize the intracavity power. Finally, robust KLM is predicted to occur when the cavity length is a multiple integer of the distance of the Kerr medium to its nearest end mirror. To check this prediction, we demonstrate four pulses per round trip KLM cavity.

Keywords: Short pulse lasers; Diode-pumped Nd:YAG; Kerr lens mode-locking

1. Introduction

Short pulse lasers are tools of increasing importance, not only in the activities of scientific research but also in many other practical applications. There are several ways to produce short and ultrashort laser pulses. Most of them require some kind of nonlinear interaction in order to lock the phases of the oscillating cavity modes. One of the possible nonlinear interactions is the Kerr effect (or intensity-dependent index of refraction). It has been successfully exploited to generate short pulses in many solid state lasers. However, this nonlinear interaction also makes the pulses to be affected by many different dynamical instabilities. In fs pulse lasers, typical nonlinear systems' phenomena have been observed, as period doubling, multistability, quasiperiodic oscillations and even deterministic chaos; and they have been studied from different points of view [1–6]. Another nonlinear effect is the appearance of more than one pulse per round trip, which is often associated to a higher order soliton [7]. Lai et al. [8] have observed and explained two

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multipulse regimes in Ti:sapphire lasers. In one of them the pulse separation is in the range 0.1–1 ps, but it cannot be associated to a higher order soliton. In the other regime, the pulse separation is fixed, corresponding to the distance between the Ti:sapphire rod and the output coupler. In both cases, the pulse duration is reported to be the same than in the normal, single pulse regime.

A multipulse behavior is also observed in the diode-pumped Kerr lens mode-locked (KLM) Nd:YAG laser [9]. It shows both similarities and differences with the second regime reported in [8]. In the first place, the pulse duration in Nd:YAG (ps) indicates that the soliton dynamics should be absent. Besides, the satellite pulses appear at fixed time intervals, which are related to the position of the Kerr medium inside the cavity. However, the pulse duration in the single and the multipulse modes is different, and it increases with the number of pulses. Also in contrast with [8], we have not observed pulse walking on the onset of the multiple pulse mode-locking. We also found that the two pulses mode of operation is more robust against cavity misalignment than the perfect mode-locking (ML) regime.

In this paper, we present new experimental results on the diode-pumped KLM Nd:YAG laser dynamics, and submit a theoretical explanation to the observed different modes of operation (one, two, or three pulses). We elucidate the cause of the stability of the time distance between pulses, and we also find that the system selects the mode of operation in order to maximize the intracavity power. Finally, we modify the cavity in order to demonstrate a robust, high repetition rate KLM Nd:YAG laser with four pulses per round trip.

2. Multipulse mode-locking

As it has been previously reported [10], the diode-pumped, self starting KLM Nd:YAG laser (Fig. 1) provides transform limited pulses of 4.5 ps of duration, at a repetition rate near 100 MHz and an average output power of 800 mW (at 1064 nm) when pumped with 3 W (at 808 nm) and having a plane mirror with transmission of 6% as the output coupler. The pulse formation mechanism is dominated by the instantaneous variation of the losses due to the nonlinear focusing on the soft aperture caused by the active volume. A simple description in terms of a two-variables Poincaré map was developed [9]:

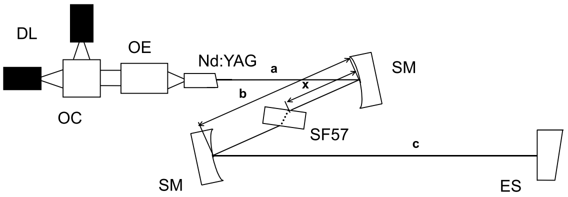

Fig. 1. Scheme of the laser cavity. DL, 2 W laser diodes at 808 nm; OC, combining optics; OE, focusing optics; SM, spherical mirrors (100 mm ROC); SF57, piece of SF57 glass (size: 8 mm); ES, wedged, 6% output coupler. Distances (approximate): $a = 343$ mm, $b = 104$ mm, $x = 56$ mm and $c = 1006$ mm.

$$\alpha_{n+1} = \frac{\alpha_n[1 + \mu r_n \exp(-r_n)]}{1 + 16g\alpha_n[1 + \mu r_n \exp(-r_n)]}, \tag{1a}$$

$$r_{n+1} = r_n \exp[2g - 2\mu \exp(r_n) + 2\ln(k)] \tag{1b}$$

whose fixed points provide a satisfactory prediction of the observed pulse duration and the energy values. In Eqs. (1a) and (1b), the variables are $\alpha = (\tau \cdot \Delta\omega)^{-2}$, where τ is the pulse duration (FWHM in intensity), $\Delta\omega$ is the bandwidth of the laser amplifier, and $r = 2\delta P_p / P_{crit}$, where P_p is the pulse peak power, P_{crit} is the critical power for self-focusing (in the SF57 glass) and δ is the (dimensionless) small signal relative spot size variation parameter [11], which measures the KLM pulse shaping strength (note that we define δ as a positive parameter: $\delta \equiv -1/w\,dw/dP$). The parameter μ is the ratio between the mode area (at the CW limit) and the (gain) aperture area, and k is the (field) round trip feedback factor, which accounts for the linear losses. The factor g is the saturated single-pass gain, and it includes the variable values at the n-round trip. In the early version of this map [9], and for simplicity reasons, g was assumed inversely proportional to the average intracavity intensity. This approximation worked well for the prediction of the values of the pulse variables, but it was unsatisfactory from other points of view. For example, it lead to the prediction of solutions different from zero for any value of pumping (i.e., it predicted no laser threshold). The map (1) showed only one type of instability, namely, a period doubling of an eigenvector practically collinear to the variable r, but this was not experimentally observed. A closer look to the equations showed that the instability was an artifact caused by the above mentioned approximation. In order to solve these inconveniences, a more accurate expression for g (at the cost of a more cumbersome expression for the map) was introduced later [12]:

$$g = \frac{g_{ss}}{1 + \frac{g_{ss} r \exp(r)}{\Gamma\sqrt{\alpha}}}, \tag{2}$$

where g_{ss} is the small signal gain and Γ, which joins several other parameters, holds to $\Gamma/g_{ss} \approx 0.89$. The improved map shows the correct behavior at the laser threshold.

But, both maps are unable to explain the observed multipulse mode-locking operation. As the pumping power is increased, it is observed that the number of mode-locking pulses per round trip increases to two, and then to three, before destabilizing to mode-locking with Q-switching (see Fig. 2). In the range where two different modes of operation coexist, the laser jumps spontaneously from one mode to the other, with no appreciable change in the average power, spectral distribution or mode shape. The pulse duration, however, increases from 4.5 (normal ML) to 6.3 ps (double pulse ML) to more than 15 ps (three pulses ML). The reason of this pulse lengthening, not observed in the pulse splitting of broadband lasers [8], is explained at the end of Section 4.

The cause of the multipulse operation becomes evident by noting that the pulse separation is locked to 2.73 ns, which is the time of flight from the Kerr element to its nearest cavity end. Therefore, in this mode of operation the KLM pulse shaping effect is not produced by a single pulse, as it happens in the case of normal ML, but by the cooperative effect of counter-propagating pulses (see Fig. 3). Now, two questions arise: one is related to the stability of the pulse separation, and is discussed in Section 3. The other one is what determines the region of stability of each of the possible modes of operation. This is considered in Section 4.

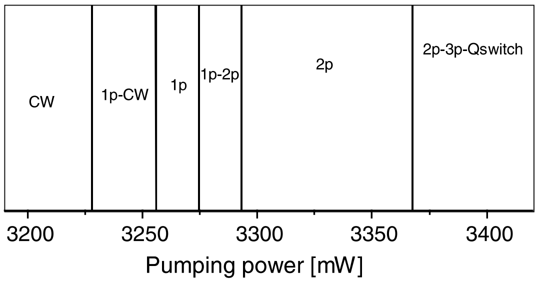

Fig. 2. Regions of the observed modes of operation as pumping power is increased.

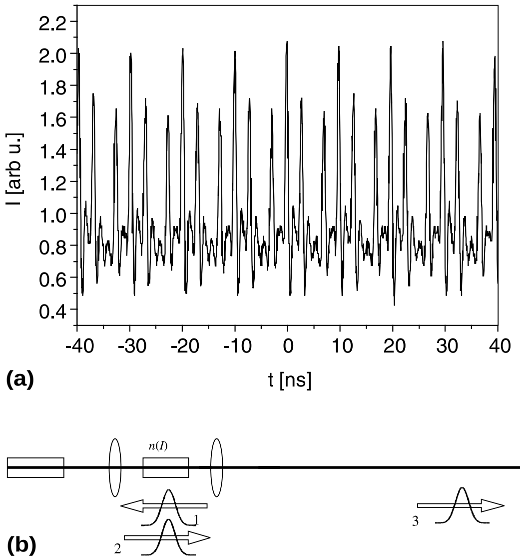

Fig. 3. (a) Fast photodiode oscilloscope trace of the three-pulses per round trip mode of operation, (b) scheme of the cavity showing how the pulses collide in the SF57 slab to produce the KLM pulse shaping effect.

3. Stability of pulse separation

At a first sight, the stable separation between pulses observed in our laser appears similar to the behavior reported in an Er/Yb fiber laser [13], where the stabilization of a multiple pulse mode-locked regime is associated to the dynamics of gain depletion and recovery. The pulses repeal each other due to the group-velocity drift induced by the gain dynamics, and the stable solution is reached when the pulses are equally spaced. However, we believe that the mechanism that stabilizes the pulse duration in our KLM Nd:YAG is different. Here, the key point to understand the stability of the pulse separation lies in the variation of the small signal relative spot size parameter (δ) with the position of the pulse collision. In order to evaluate this variation, we perform a calculation following Lai et al. [8]. The Kerr medium (the piece of SF57 glass) is divided into 10–20 slices, so that their length is roughly equal to the pulse length. The effect of each slice is taken into account by a simple ABCD matrix. The beam is numerically propagated back and forth in the cavity using the ABCD matrices formalism (starting with the CW values), until a stationary value is obtained. Then the spot size at the aperture (the active region) is calculated, and then its relative small signal variation δ. The results are plotted in Fig. 4.

For the single pulse case, the positive value of δ indicates that normal ML is favored against CW for the chosen values of the laser parameters.

For the two pulses case, the calculation is carried out by halving the pulse energy (for, the average power is the same for the single and the double pulse modes), but now the pulse collision is taken into account. The position where the pulses collide is identified by doubling the pulse power for that slice. The numerical iteration described for the single pulse case is repeated. In this way, the spot size on the aperture (hence, δ) is obtained as a function of the position of the slice where the pulses collide. We see (Fig. 4) that δ is strongly dependent of the position where the pulses collide. In the maximum, it is more than four times larger than the single pulse value, in spite of the fact that the pulse energy has been halved. This means that, for the double pulse mode, the KLM is strongly favored if the pulses collide in the Kerr medium in a certain position (which practically coincides with the waist of the CW solution), and with a tolerance below 0.5 mm (what amounts to about 1.5 ps, shorter than the pulse duration itself). A similar result had been obtained for the Ti:sapphire laser and fs pulses [8]. It is noteworthy that the cooperative nonlinear interaction is strong enough to “lock” the pulse separation also for the much longer pulses (two orders of magnitude, at least) generated by the Nd:YAG laser.

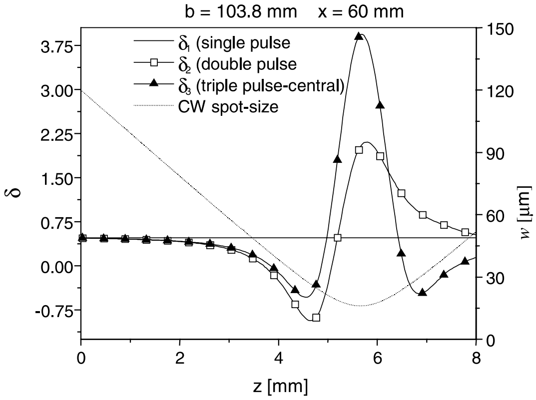

Fig. 4. Variation of the parameter δ (which measures the KLM pulse shaping strength) as a function of the position of the pulse collision inside the Kerr medium, for the different modes of operation. The beam waist for the CW mode is also shown. Note the strong localization of the favorable region for the two and three pulses ML modes near the waist of the CW mode.

For the three pulses mode the calculation is performed in an analogous way, by dividing the single pulse peak power by three, and by taking into account a “favored” slice twice per round trip (because in this case the central pulse of the train collides when propagating in both directions, see Fig. 3). The localization and magnitude of the favorable values of δ is even stronger. The peak value is more than eight times the single pulse value. Note also that, for the multipulse modes, δ even changes sign for some collision positions, what means a very strong barrier against pulse drift.

4. Selection of the mode of operation

Now, we face the problem of explaining why the system prefers one mode of operation to another, depending on the values of the parameters. As a starting guess, one is tempted to use the customary approach (which has worked so well for Ti:sapphire [5]) of writing down an iterative map and calculating the regions of linear stability of the fixed points. However, in this case, the map is unable to describe such transitions, because it predicts that the single pulse mode-locking solution has no instabilities (other than the trivial one, i.e., the laser threshold). A hint for the solution to this problem is obtained from the lab by noting that, in the conditions of self starting ML, the ML operation extracts more power from the medium than the coexistent CW mode. This is illustrated (Fig. 5) with oscilloscope traces of the laser output showing the switching from the ML to the CW mode. The gray, wide trace corresponds to the signal obtained from a fast photodiode. Superimposed with it, the signal obtained with a power meter (a thermopile) is plotted (its scale is on the rightward side). A mechanical perturbation interrupts the ML operation (the large amplitude gray regions correspond to ML), and the laser goes to the CW operation. After a while, the ML self starts. After taking into account the slow reaction time of the thermopile, we see that the self starting ML operation corresponds to an output power about 1% higher than the CW operation. In consequence, we hypothesize that the system self-stabilizes in the mode of operation that maximizes the intracavity power.

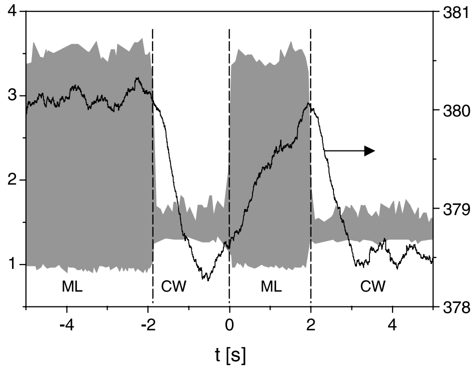

Fig. 5. Oscilloscope trace of the laser output as obtained with a fast photodiode (gray line), and average output power, as obtained with a slow thermopile (black line), in the condition of self starting ML. The ML mode extracts more power from the laser than the CW one.

In order to check this hypothesis, we enlarge the iterative map (1) to take into account multipulse operation. The generalization is straightforward, by adding terms which represent the nonlinear contributions of the adjacent pulses:

$$\alpha_{n+1}^{(i)} = \frac{\alpha_n^{(i)} + (\mu/2)\mathrm{e}^{-r_n^{(i)}}\left[\alpha_n^{(i)} r_n^{(i)} \mathrm{e}^{-\left(r_n^{(i+1)}+r_n^{(i-1)}\right)} + \alpha_n^{(i+1)} r_n^{(i+1)} \mathrm{e}^{-r_n^{(i+1)}} + \alpha_n^{(i-1)} r_n^{(i-1)} \mathrm{e}^{-r_n^{(i-1)}}\right]}{1 + 16g^{(i)}\left\{\alpha_n^{(i)} + (\mu/2)\mathrm{e}^{-r_n^{(i)}}\left[\alpha_n^{(i)} r_n^{(i)} \mathrm{e}^{-\left(r_n^{(i+1)}+r_n^{(i-1)}\right)} + \alpha_n^{(i+1)} r_n^{(i+1)} \mathrm{e}^{-r_n^{(i+1)}} + \alpha_n^{(i-1)} r_n^{(i-1)} \mathrm{e}^{-r_n^{(i-1)}}\right]\right\}}, \tag{3a}$$

$$r_{n+1}^{(i)} = r_n^{(i)} \exp\left[2g^{(i)} - \mu \mathrm{e}^{-\left(r_n^{(i)}+r_n^{(i+1)}\right)} - \mu \mathrm{e}^{-\left(r_n^{(i)}+r_n^{(i-1)}\right)} + 2\ln(k)\right], \tag{3b}$$

where the i (upper index) corresponds to each pulse in the same n (lower index) transit time. Each i-pulse contributes to the nonlinear effect with its own power and the power of the previous $(i-1)$ and next $(i+1)$ pulses. If the two-pulses mode is considered, there is no $i-1$ $(i+1)$ pulse for the first (second) pulse in the train. If the three pulses mode is considered, the central pulse has contributions from the other two, but the first and the third pulses have additional contributions from the central pulse only. The gain factor is assumed to be shared among the pulses, so that each pulse sees a gain

$$g^{(i)} = \frac{1}{N} \frac{g_{\mathrm{ss}}}{1 + \frac{g_{\mathrm{ss}} r^{(i)} \exp(r^{(i)})}{\Gamma\sqrt{\alpha^{(i)}}}}, \tag{3c}$$

where N is the total number of pulses per round trip.

The fixed points of (3) for the different modes of operation, and for typical parameters' values [9] $k = 0.96$, $g_{\mathrm{ss}} = 5.6$ and $\mu = 0.9$, coincide quite well with the observed values. For single pulse ML, the predicted pulse duration and energy are 6.8 ps and 154 nJ, against observed values of 4.5 ps and 125 nJ. For the two pulses mode, the predictions are 8.5 ps and 81 nJ against observed values of 6.3 ps and 63 nJ. For the three pulses mode, the prediction is of 13.6 ps and 38 nJ for the central pulse, and 14.6 ps and 35 nJ for the other two; while the observed values are longer than 15 ps (for the three pulses), 51 nJ for the central pulse and 37 nJ for the other two. The numerical agreement can be improved by fine tuning of the parameters, but we prefer using typical tabulated values, because the approach is meant to obtain a qualitative rather than a quantitative description.

In Fig. 6 the intracavity power predicted for each mode of operation is plotted as a function of the small signal gain (i.e, pumping power). The mode of operation that provides the maximal power extraction is not always the same. It starts with the one pulse ML for small values of the gain, then follows the two pulses regime and finally the three pulses ML. This is, the same sequence of Fig. 2. As a further evidence, the curve for the two pulse ML operation with noninteracting pulses (i.e., when they collide far from the favorable region in Fig. 4) is always below the curve of some other mode of operation. In agreement with this result, this mode of operation (two pulses at a distance different from 2.73 ns) is not obtained experimentally.

Finally, some comments on the causes of the pulse lengthening and the mechanism of multipulse generation are in order. In a relatively narrowband laser as Nd:YAG, soliton shaping effects are absent. The main factor that determines the pulse duration is the self amplitude modulation (SAM) [12]. This temporal effect is generated by the combination of two spatial processes, namely, the self-focusing on the nonlinear medium and the losses due to the gain aperture [9]. Contrasting with self phase modulation (which is essential to soliton dynamics), the SAM strength depends not only on the nonlinear medium and the pulse peak power, but also on geometrical characteristics. A detailed analysis [12] shows that the single pulse mode-locking regime can lose stability because an excess of SAM. As the pump power increases, the pulse energy and in consequence the SAM becomes stronger. A stronger SAM produces a sharper pulse modulation and a narrower pulse. This is the reason why, in our laser, a more energetic pulse is also shorter. Eventually, the pulse duration reaches the limit imposed by the amplifier's bandwidth. The reaction of the system is to split the energy between two (or more) pulses. These lower energy pulses induce a lower SAM and in consequence have a longer duration, so that they can now be accommodated into the amplifier's bandwidth. The mode-locking operation is still possible because the weaker SAM is balanced by an enhancement of the nonlinearity (δ) thanks to the colliding pulses effect. As we have shown, the resulting increase in the factor δ is enough to support a regime of two (or more) colliding pulses, in spite of their lower peak power.

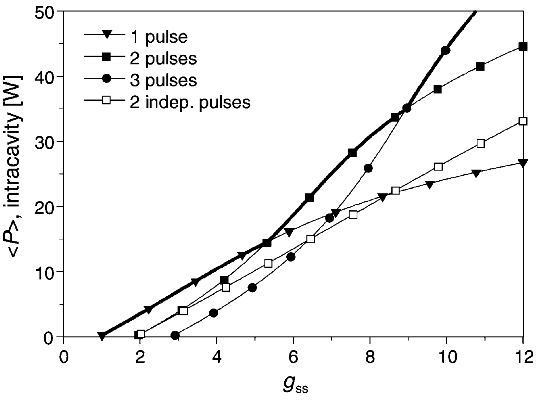

Fig. 6. Intracavity power, according to the fixed points of the map (3), for the different possible modes of operation. Note that the observed sequence of modes of operation (see Fig. 2) can be explained by the criterion of the maximization of the intracavity power. The curve for the mode of operation with two noninteracting pulses is always below some other curve. In agreement with this result, this mode of operation is never obtained experimentally.

In Ti:sapphire multipulse ML, pulse splitting is often followed by an effect of pulse "walking", in which the two pulses separate until reaching their final positions, in a process which is reported to last a fraction of a second [8]. We have not observed pulse walking in our laser, so that we conclude that it is much faster or, conversely, that the new pulse grows from noise already at the correct separation. It is conceivable that, when the ML state becomes unstable by an excess of SAM, the pulse is severely distorted and its energy spread. Then new pulses would rapidly grow from the resulting background, but only at the privileged positions where δ is large. Which one of the two mentioned processes is the actual one in our laser is a subject that remains to be studied in more detail.

5. High repetition rate KLM

If the cavity length is adjusted to a multiple integer of the distance between the Kerr medium

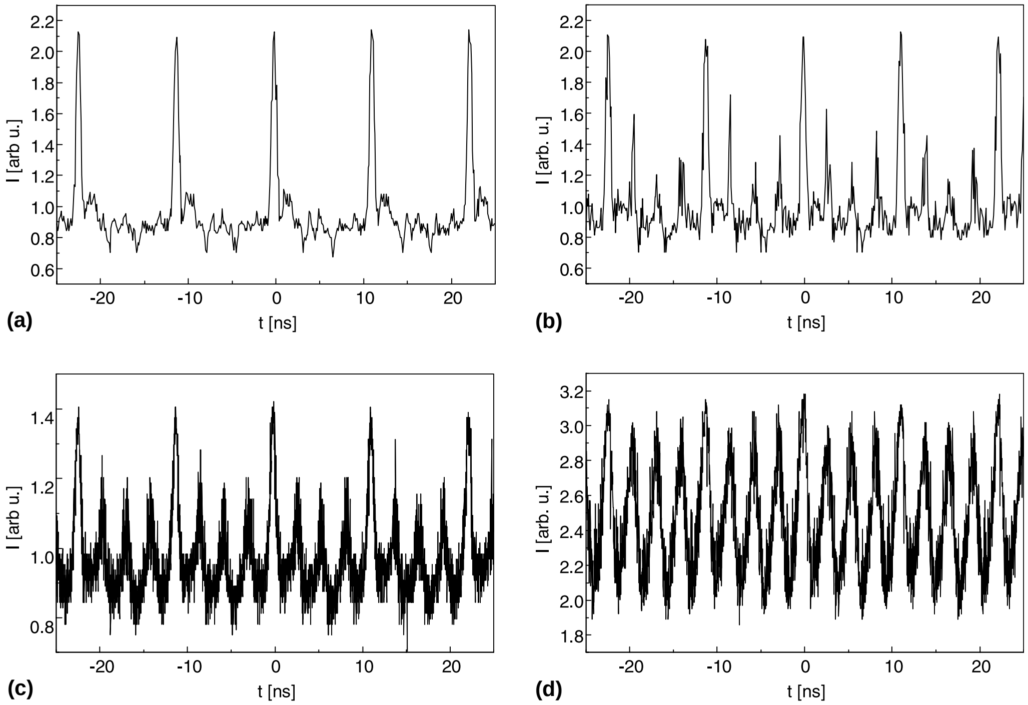

Fig. 7. Different ML regimes as the pumping power is increased, obtained in the cavity adjusted to sustain the 4-pulses per round trip mode: (a) normal ML, (b) almost 3-pulses per round trip, (c) nonequal 4-pulses per round trip, (d) equal 4-pulses per round trip.

and its nearest end mirror, then the pulse shaping effect will arise as the cooperation between colliding pulses for all the pulses in the train. This mode of operation (called "degenerate P2" in [9]) is presumably more robust than the usual single pulse ML regime. Besides, it is a simple way of increasing the repetition rate (what is of interest for some applications) keeping the cavity length in comfortable values.

In order to demonstrate this point, we modify the cavity in Fig. 1 by lengthening the distance c in such a way that $4(a+x) = a+b+c$. We choose to do it in this way, because the stability condition is less sensitive to changes in the distance c than to changes in the other cavity parameters. At low pumping regimes the single pulse ML is obtained (see Fig. 7). As the pumping power is increased, more pulses incorporate to the train until that, at the highest pumping level, the regime with four equal pulses is finally achieved (Fig. 7(d)). The repetition rate is about 360 MHz. This regime is more robust than the normal ML but, as a drawback, the pulses are longer (about 17 ps).

6. Summary

The key of the operation of KLM lasers is the insertion of some material with a large intensity-dependent index of refraction inside the cavity. This affects the optical field in two ways. In the spatial domain, the beam is focused. In the temporal domain, the optical carrier frequency is chirped. These effects change the beam size and pulse duration, hence the field intensity, which in turn affects the strength of the Kerr effect in a nonlinear way. In Nd:YAG the temporal effect is negligible, and the system can be described by the sole effect of the SAM caused by the combination of self-focusing, appropriate cavity design, and the soft aperture due to the finite transversal size of the active region. As a consequence, Nd:YAG KLM does not show the complex instabilities observed in

Ti:sapphire. Instead, a robust multipulse ML operation is conspicuous. The pulse splitting occurs when the SAM is so strong that the sharply modulated pulse cannot accommodate into the (relatively narrow) amplifier's bandwidth. The system's reaction is to divide the energy among two or more pulses of lower energy and longer duration.

The multipulse ML is then stabilized by the cooperative action of the pulses colliding in the Kerr medium. The value of the small signal relative spot size parameter δ is larger for the two and three pulses mode of operation than for the perfect ML regime, and besides, it is strongly dependent on the position where the pulses collide. This allows to explain the notable stability of the time distance between pulses. An interesting and nontrivial result is that the optimal collision point coincides with the waist of the CW mode.

A previously reported description of KLM in Nd:YAG with a two variables Poincaré map is enlarged to include the multipulse regime. The fixed points of the enlarged map predict the observed values of pulse energy and duration with acceptable accuracy. The model is also used to establish that the laser oscillates in one ML regime or another maximizing the intracavity power.

Finally, by adjusting the cavity length, a robust four equal pulses per round trip ML mode of operation is achieved, as a way to illustrate how to take advantage of what might be called "colliding-pulse KLM effect".

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