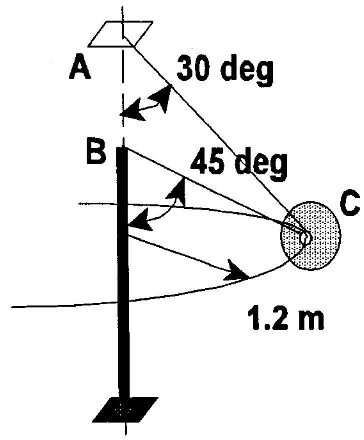
Name ~~MASTER~~

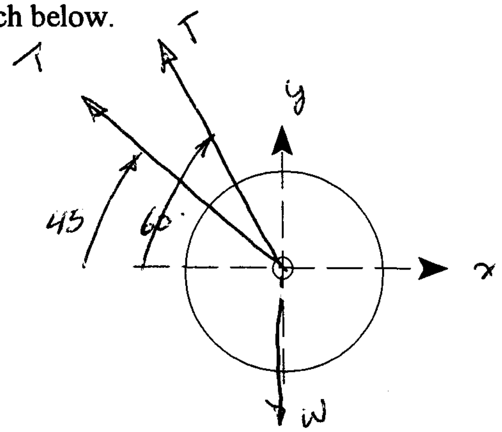
CE 261 DYNAMICS
MIDTERM 2: CHAPTERS 3/5-3/13
Thursday, March 27, 2003. Time: 5:00-6:30 pm

~~SHOW ALL WORK OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS~~
ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY

1. A single wire ACB passes through a ring at C attached to a sphere which revolves at the constant speed V in the horizontal circle shown. The mass of the sphere is 4 kg and the radius of the circle is 1.2 m.



a. (10 points) Draw a free body diagram of the sphere on the sketch below.



b. (15 points) Calculate the speed, V, of the sphere and the tension, T, in the cord.

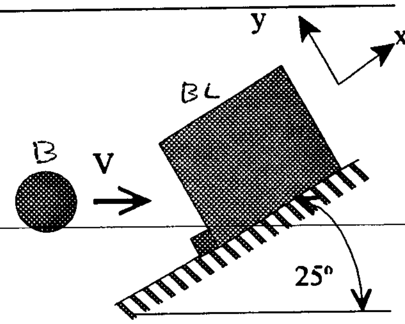
$$\Sigma F_y = 0 = T\sin 45 + T\sin 60 - W = 0$$

$$T = \frac{W}{\sin 45 + \sin 60} = \frac{4(9.816)}{\sin 45 + \sin 60} = \boxed{24.96 \text{ N}}$$

$$\Sigma F_x = 0 = -T\cos 45 - T\cos 60^\circ = -m\frac{V_\theta^2}{R}$$

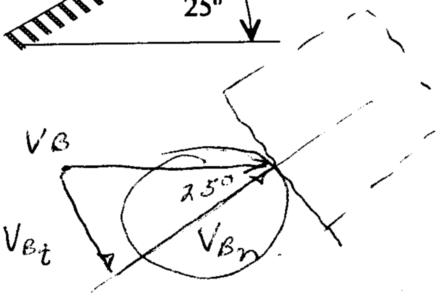
$$V_\theta = \sqrt{\frac{R}{m}T(\cos 45 + \cos 60)}$$

$$= \sqrt{\frac{1.2}{4}\left(\frac{4g)(\cos 45 + \cos 60)}{(\sin 45 + \sin 60)}\right)} = \boxed{3.006 \text{ m/s}}$$



2. A 1-kg ball moving horizontally at 12 m/s strikes a 10-kg block. The coefficient of restitution of the impact is e = 0.6, and the coefficient of kinetic friction between the block and the inclined surface is $\mu_k = 0.4$.

a. (15 points) Calculate the velocity of ball and the block just after impact. Give both magnitudes and directions.



$V_{B_t} = V_{B_y} = -V_B \sin 25^\circ = -12 \sin 25 = -5.07\ m/s$

$V_{Bx} = V_B \cos 25 = 12 \cos 25 = 10.88\ m/s$

$e = \dfrac{V_{Bx}' - V_{BLx}'}{V_{BLx}^{\,0} - V_{Bx}} \Rightarrow +10.88(.6) = -V_{Bx}' + V_{BLx}'$

$m_B V_{Bx} + m_{BL} V_{BL}^{\,0} = m_B V_{Bx}' + m_{BL} V_{BLx}'$

$10.88 = V_{Bx}' + 10 V_{BLx}'$

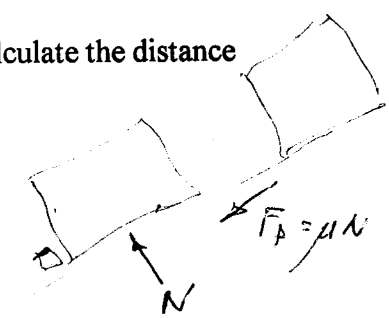
add $1.6(10.88) = 11 V_{BLx}'$

$V_{BLx}' = 1.6 \dfrac{(10.88)}{11} =$ **1.58 m/s up slope**

$V_{Bx}' = 1.58 - 0.6(10.88) = -4.95\ m/s$

$V_{Bx}' = -4.95\,\vec{i} - 5.07\,\vec{j}$
$= 7.08\ m/s\ @\ \theta = 45.64^\circ$

b. (10 points) **Assuming the velocity of block is 2 m/s** just after impact, calculate the distance the block slides before stopping.



$N = 10 g \cos 25 = 88.96\ N$

$F_f = \mu N = 0.4(88.9) = 35.59\ N$

$T_1 + U_{g_1} = T_2 + U_{g_2} + W_2$

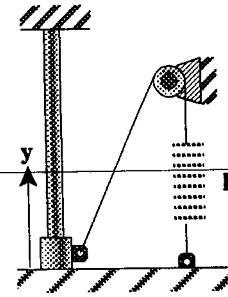
$\frac{1}{2} m_{BL} V_{BLx}'^{2} + 0 = 0 + W S \sin 25^\circ + F_f \cdot S$

$S = \dfrac{\frac{1}{2} m_{BL} V_{BLx}'^{2}}{(F_f + W \sin 25^\circ)} = \dfrac{\frac{1}{2} m_{BL} V_{BLx}'^{2}}{(\mu g m_{BL} \cos 25 + g m_{BL} \sin 25)}$

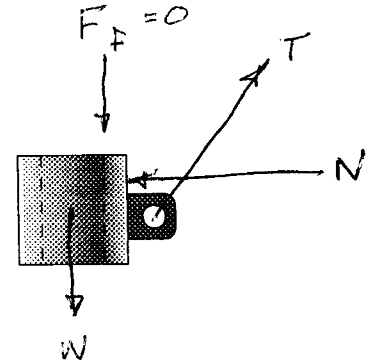
$= \dfrac{\frac{1}{2}(2)^2}{(0.4(9.816)\cos 25 + 9.816 \sin 25^\circ)} =$ **0.260 m**

$9.816(0.4 \cos 25 + \sin 25)$

3. The 1-kg collar A is attached to the linear spring (k = 300 N/m) by a 2m long string. The collar starts from rest in the position show, and the initial tension in the string is 100 N.



~~a. (10 points) Draw a free body diagram of the forces on the~~ collar.

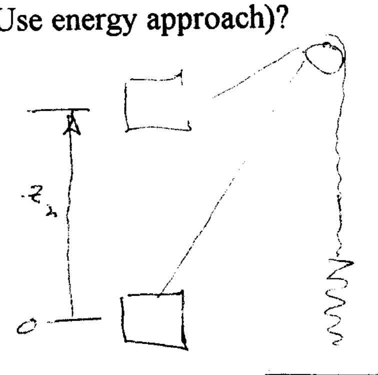


$F = k x_s$

$x_s = \frac{100}{300} = 0.33\,m$

b. (10 points) What distance does the collar slide up the smooth bar (Use energy approach)? State your assumptions.

Assume no friction, $V_1 = V_2 = 0$



$$\tfrac{1}{2} m_1 v_1^2 + g m_1 z_1 + \tfrac{1}{2} k (x_{s_1})^2 = \tfrac{1}{2} m_1 v_2^2 + g m_1 z_2 + \tfrac{1}{2} k (x_{s_2})^2$$

$$g z_2 m_1 = \tfrac{1}{2} k (x_{s_1})^2$$

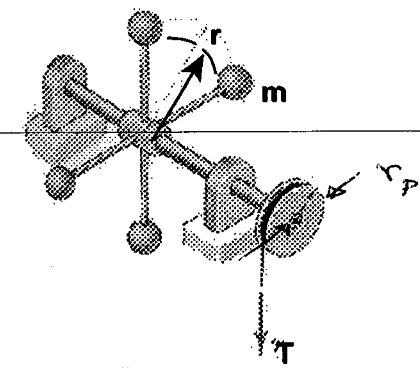
$$z_2 = \frac{\tfrac{1}{2} k (x_{s_1})^2}{g m_1} = \frac{\tfrac{1}{2}(300)(0.33)^2}{9.816(1)} = \boxed{1.66}\ m$$

c. (5 points) If there was friction between the collar and the rod, say μ_k, what additional term would there be in the energy equation? What additional information would you need to solve problem?

add Friction Work Term $\int F_f dz_2 = \int_{z_1}^{z_2} \mu N(z)\, dz$

Need μ need geometry to calculate T to get N.

4. The assembly to the right starts from rest and reaches an angular speed of 20 rad/sec under the action of a 10 Newton force T applied to a 0.4 m radius pulley applied to the string for t seconds. Neglect friction and all masses except those of the four spheres, m = 1 kg/each, located at r = 0.5 m which may be treated as particles.



A. (9 points) Calculate the time t the tension T is applied.

$$\int_0^t T r_p\,dt = H_2 - H_1^{\,\nearrow 0}$$

$$(T r_p)t = 4 r m (r\omega_2)$$

$$t = \frac{4 r^2 m \omega_2}{r_p T} = \frac{4(0.5)^2(1)(20)}{(0.4)(10)}$$

$$= \boxed{5 \text{ sec}}$$

B. (8 points) What is the rate of angular acceleration, α, at time t when the angular speed is 20 rad/sec?

$$\Sigma T \cdot r_p = I_{oo}\alpha \qquad \text{but } I_{oo} = 4mr^2$$

$$\alpha = \frac{T \cdot r_p}{4 \cdot m r^2} = \frac{10(0.4)}{4(1)(0.5)^2} = \boxed{4 \text{ rad}/s^2}$$

C. (8 points) What distance did the force T move to produce an angular rotation rate of 20 rad/sec?

$$W_{ab} = T_2 - T_1^{\,\nearrow 0} + U_{g2} - U_{g1}^{\,\nearrow 0}$$

$$T \cdot \Delta s = \frac{1}{2}(m)V^2 = \frac{1}{2}(4m_{sp})(r\omega)^2$$

$$\Delta s = \frac{1}{2}\frac{4m_{sph}(r\omega)^2}{T}$$

$$= \frac{1}{2}\frac{4(1)(0.5 \cdot 20)^2}{10}$$

$$= \boxed{20 \text{ m}}$$