CE 261 DYNAMICS
MIDTERM 3: CHAPTERS 5 & 6
Thursday, April 24, 5:00-6:30pm

SHOW ALL WORK OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS
ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY

1. The crank AB has a constant clockwise angular velocity of 100 rpm.

\[ \omega = \omega_{AB} \times \vec{r}_{AB} = -10 \text{ rad/sec} \times (2 \text{ in.}) = -21 \text{ rad/sec} \]

\[ \vec{V}_B = \vec{V}_A + \vec{L}_A \times \vec{r}_{AB} = \vec{V}_A + \vec{L}_A \times (-21 \hat{\theta}) \]

\[ V_p \hat{\theta} = -21 \hat{\theta} + 21 \hat{\theta} + \omega_p \hat{\theta} \times (6 \hat{\theta} - 2 \hat{\theta}) \]

\[ = -21 \hat{\theta} + 21 \hat{\theta} + \omega_p \hat{\theta} \times 4 \hat{\theta} \]

\[ V_p = 21 + 2 \omega_p \hat{\theta} \]

\[ \omega_p = 6 \text{ rad/sec} \]

\[ V_p = 21 + 2 \times 6 = 33 \text{ ft/sec} \]

a. (5 points) Draw a free body diagram of the forces on the piston.

b. (15 points) What is the velocity of the Piston P?

\[ \vec{V}_B = \vec{V}_A + \vec{L}_A \times \vec{r}_{AB} = -10 \hat{\theta} \times (2 \hat{\theta}) = -21 \hat{\theta} + 21 \hat{\theta} \times (2 \hat{\theta}) \]

\[ \vec{V}_P = \vec{V}_B + \vec{L}_B \times \vec{r}_{PB} \]

\[ V_p \hat{\theta} = -21 \hat{\theta} + 21 \hat{\theta} + \omega_p \hat{\theta} \times (6 \hat{\theta} - 2 \hat{\theta}) \]

\[ = -21 \hat{\theta} + 21 \hat{\theta} + \omega_p \hat{\theta} \times 4 \hat{\theta} \]

\[ V_p = 21 + 2 \omega_p \hat{\theta} \]

\[ \omega_p = 6 \text{ rad/sec} \]

\[ V_p = 21 + 2 \times 6 = 33 \text{ ft/sec} \]

(c. (15 points) What is the acceleration of the Piston P? (Assume: \( V_p = 100 \text{ ft/sec} \) and \( \omega_{BP} = 100 \text{ rad/sec} \))

\[ \vec{a}_B = \vec{a}_{AB} \times \vec{r}_{AB} = \vec{a}_{AB} \times (-10 \hat{\theta} \times (2 \hat{\theta}) = (-10 \hat{\theta} \times (-10 \hat{\theta}) \times (2 \hat{\theta}) = -220 \hat{\theta} \times 2 \hat{\theta} \]

\[ = 220 \times 2 \hat{\theta} = 440 \hat{\theta} \]

\[ \vec{a}_P = \vec{a}_B + \vec{a}_{PB} = \vec{a}_B + \vec{a}_{PB} + \omega_p \hat{\theta} \times \vec{a}_{PB} \]

\[ = -220 \hat{\theta} \times 2 \hat{\theta} + 100 \hat{\theta} \times 100 \hat{\theta} \times (6 \hat{\theta} - 2 \hat{\theta}) + \omega_p \hat{\theta} \times (6 \hat{\theta} - 2 \hat{\theta}) \]

\[ = -220 \hat{\theta} \times 2 \hat{\theta} + 100 \hat{\theta} \times 100 \hat{\theta} \hat{\theta} + 6 \hat{\theta} \times 2 \hat{\theta} + \omega_p \hat{\theta} \times 2 \hat{\theta} \]

\[ = -220 \hat{\theta} \times 2 \hat{\theta} + 6 \times 10 \hat{\theta} + 2 \hat{\theta} \hat{\theta} + 2 \hat{\theta} \hat{\theta} \]

\[ \omega_p = 19500 \text{ ft/sec} \]

\[ \omega_p = 6 \times 10^5 \text{ ft/sec} \]

\[ \omega_p = -60, 220 + 2 \omega_p \]

\[ = 19, 800 + 6 \hat{\theta} \hat{\theta} \]

\[ \omega_p = 53, 620 \text{ ft/sec} \]

\[ \omega_p = 4468 \text{ ft/sec} \]
2. The 0.1 kg slender bar and the 0.2 kg cylindrical disk are released from rest with the bar horizontal. The disk rolls on the curved surface.

(a. (5 points) Draw free body diagrams of all forces on the slender bar and rolling disk. Indicate locations of center of gravities, and directions of rotation for angular velocity and acceleration.

\[ \mathbf{R}_{AB} = 0.12 \hat{i} + 0 \hat{j}. \quad \mathbf{R}_{PB} = 0.04 \hat{i} + 0 \hat{j}. \]

\[ \omega_{AB} = 0 \hat{k}. \quad \omega_{PB} = 0 \hat{k}. \]

(b. (5 points) Fill in the following table:

\[ \mathbf{C}_{PB} = \mathbf{C}_{AC} \times \mathbf{r}_{PB} = \mathbf{C}_{AO} \times \mathbf{r}_{BO} \]

\[ + \mathbf{C}_{AC} \mathbf{r}_{PB} \]

\[ + \mathbf{C}_{AC} \mathbf{r}_{PB} \]

(c. (10 points) How do the angular acceleration of the bar, \( \alpha_{AB} \), and the angular acceleration of the disk, \( \alpha_{D} \), relate to one-another? (Hint: acceleration of point B must be same for bar and disk).

\[ \mathbf{C}_{PB} = \mathbf{C}_{AC} \mathbf{r}_{PB} \]

\[ + \mathbf{C}_{AC} \mathbf{r}_{PB} \]

(d. (20 points) What is the bar's angular acceleration, \( \alpha_{AB} \), at the instant of release? (Hint: Use moments about A and W)

\[ \sum \mathbf{M}_{A} = I_{AI} \alpha_{AB} \Rightarrow -0.10 (0.06) + 0.12 \mathbf{y} = \frac{1}{2} (0.1)(0.12) \alpha_{AB} \]

\[ \sum \mathbf{M}_{P} = I_{PAC} \alpha_{AC} \Rightarrow 0.04 \mathbf{y} + 0.20 \mathbf{y} = \frac{1}{2} (0.2)(0.093) + 0.20 (0.043)^{2} \alpha_{P} \]

Solve:

\[ \begin{cases} -0.06 + 0.12 \mathbf{y} = 4.8 \times 10^{-4} \alpha_{AB} \\ 0.079 + 0.04 \mathbf{y} = 4.8 \times 10^{-4} \alpha_{AC} \end{cases} \]

\[ \alpha_{AB} = -62 \text{ rad/s}^{2} \]
3. In the previous problem assume the horizontal bar falls to an angle of 45 degrees. The disk rolls along the wall, but assume no rolling friction along the wall and no air drag.

a. (5 points) What is the relationship between the velocity of the end of the bar (center of the disk) \( \vec{V}_B \) and the angular velocity of the bar, \( \omega_{AB} \)?

\[
\vec{V}_B = \omega_{AB} \times \vec{r}_{AB} = -\omega_{AO} \cdot \vec{r}_{AB} \frac{t}{t}
\]

\[
\vec{V}_B = \omega_{AB} \times \vec{r}_{AB} = \omega_{disk} \times \vec{r}_{PB}
\]

\[
-\omega_{AB} \cdot \vec{r}_{AB} \frac{t}{t} = \omega_{disk} \cdot \vec{r}_{PB} \frac{t}{t}
\]

\[
\omega_{disk} = -\omega_{AB} \left( \frac{\vec{r}_{AB}}{\vec{r}_{PB}} \right) = -3 \omega_{AB}
\]

\[
I_D = \frac{1}{2} m s^2
\]

b. (5 points) What is the relationship between the angular velocity of the disk, \( \omega_{disk} \), and the angular velocity of the bar, \( \omega_{AB} \)?

\[
\omega_{disk} = \omega_{AB} + \Delta \omega_B \cdot \vec{V}_B.
\]

\[
0 = \frac{1}{2} I_B \omega_{AB}^2 + \frac{1}{2} I_B \omega_B^2 + \frac{1}{2} m_B V_B^2 - 0.1 g (0.06) \sin 45 - 0.2 g (0.12) \sin 45
\]

\[
0 = \frac{1}{2} \left( \frac{1}{3} (0.1) (0.12)^2 \right) \omega_{AB}^2 + \frac{1}{2} \left( \frac{1}{2} (0.04) \omega_B^2 + \frac{1}{2} (0.12) \left[ (0.12) (0.12) \right] \right)
\]

\[
-0.1 g (0.06) \sin 45 - 0.2 g (0.12) \sin 45
\]

\[
0 = 2.4 \times 10^{-4} \omega_{AB}^2 + 8.0 \times 10^{-5} (\omega_{AB})^2 + 1.4 \times 10^{-3} \omega_{AB}^2 - 4.2 \times 10^{-3} \omega_{AB}^2 - 1.7 \times 10^{-2} \omega_{AB}^2
\]

\[
0 = 2.1 \times 10^{-3} \omega_{AB}^2 - 2.1 \times 10^{-2} \omega_{AB}^2
\]

\[
\omega_{AB}^2 = \frac{2.12 \times 10^{-2} \omega}{2.1 \times 10^{-3}}
\]

\[
|\omega_{AB}| = 9.31 \text{ rad/sec}
\]

c. (20 points) What is the magnitude of the angular velocity of the bar, \( \omega_{AB} \)?

\[
\omega_{disk} = \omega_{AB} + \Delta \omega_B \cdot \vec{V}_B.
\]

\[
0 = \frac{1}{2} I_B \omega_{AB}^2 + \frac{1}{2} I_B \omega_B^2 + \frac{1}{2} m_B V_B^2 - 0.1 g (0.06) \sin 45 - 0.2 g (0.12) \sin 45
\]

\[
0 = \frac{1}{2} \left( \frac{1}{3} (0.1) (0.12)^2 \right) \omega_{AB}^2 + \frac{1}{2} \left( \frac{1}{2} (0.04) \omega_B^2 + \frac{1}{2} (0.12) \left[ (0.12) (0.12) \right] \right)
\]

\[
-0.1 g (0.06) \sin 45 - 0.2 g (0.12) \sin 45
\]

\[
0 = 2.4 \times 10^{-4} \omega_{AB}^2 + 8.0 \times 10^{-5} (\omega_{AB})^2 + 1.4 \times 10^{-3} \omega_{AB}^2 - 4.2 \times 10^{-3} \omega_{AB}^2 - 1.7 \times 10^{-2} \omega_{AB}^2
\]

\[
0 = 2.1 \times 10^{-3} \omega_{AB}^2 - 2.1 \times 10^{-2} \omega_{AB}^2
\]

\[
|\omega_{AB}| = 9.31 \text{ rad/sec}
\]