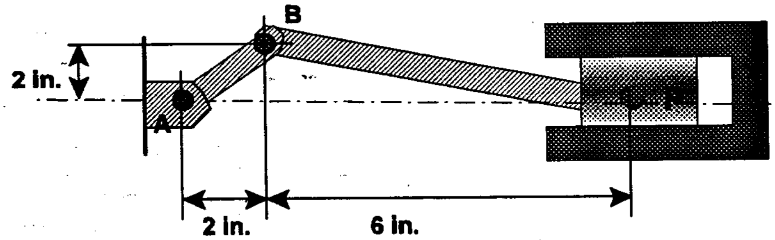
Name $MASTER$.

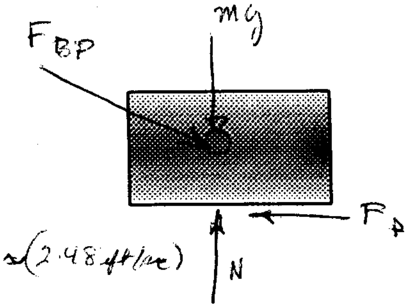
CE 261 DYNAMICS
MIDTERM 3: CHAPTERS 5 & 6
Thursday, April 24, 5:00-6:30pm

SHOW ALL WORK OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS
ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY

1. The crank AB has a constant clockwise angular velocity of 100 rpm.



a.. (5 points) Draw a free body diagram of the forces on the piston.



b. (15 points) What is the velocity of the Piston P?

$\vec V_B = \vec\omega_{AB}\times\vec r_{AB} = -\frac{100(2\pi)}{60}\hat k\times(2\hat i+2\hat j) = -21\hat j + 21\hat i \sim (2.48\ ft/sec)$

$\vec V_P = \vec V_B + \vec V_{P/B} = \vec V_B + \vec\omega_D\times\vec r_{PB}$

$V_P\hat i = -21\hat j + 21\hat i + \omega_D\hat k\times(6\hat i - 2\hat j)$

$= -21\hat j + 21\hat i + \omega_D 6\hat j + 2\omega_D\hat i$

$V_P = 21 + 2\omega_D$

$0 = -21 + 6\omega_D \Rightarrow \omega_D = \frac{21}{6} = 3.5\ rad/sec$

$V_P = 21 + 2(3.5) = \boxed{28\ in/sec = 2.33\ ft/sec}$

c. (15 points) What is the acceleration of the Piston P? (Assume: $V_P = 100$ ft/sec and $\omega_{BP} = 100$ rad/sec

$\vec a_B = \vec\omega_{AB}\times\vec\omega_{AB}\times\vec r_{AB} + \cancel{\vec\alpha_{AB}}^{0}\times\vec r_{AB} = (-10.5\hat k\times(-10.5\hat k)\times(2\hat i+2\hat j)$

$\simeq -220.5\hat i - 220.5\hat j \sim (26\ ft/sec^2)$

$a_P\hat i = \vec a_B + \vec a_{P/B} = \vec a_B + \vec\omega_D\times\vec\omega_D\times\vec r_{PB} + \vec\alpha_D\times\vec r_{PB}$

$= -220.5\hat i - 220.5\hat j + 100\hat k\times 100\hat k\times(6\hat i - 2\hat j) + \alpha\hat k\times(6\hat i - 2\hat j)$

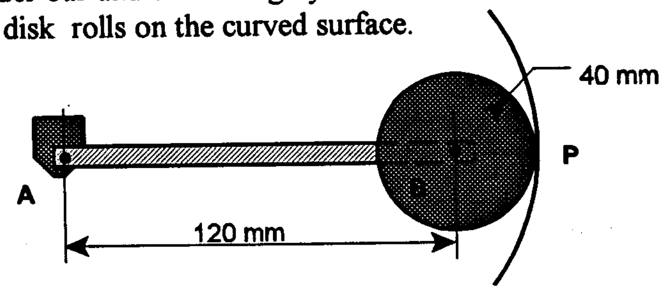
$= -220.5\hat i - 220.5\hat j - 6\times10^4\hat i + 2\times10^4\hat j + 6\alpha_D\hat j + 2\alpha_D\hat i$

$a_P = -220.5 - 6\times10^4 + 2\alpha_D = -60,220 + 2\alpha_D$

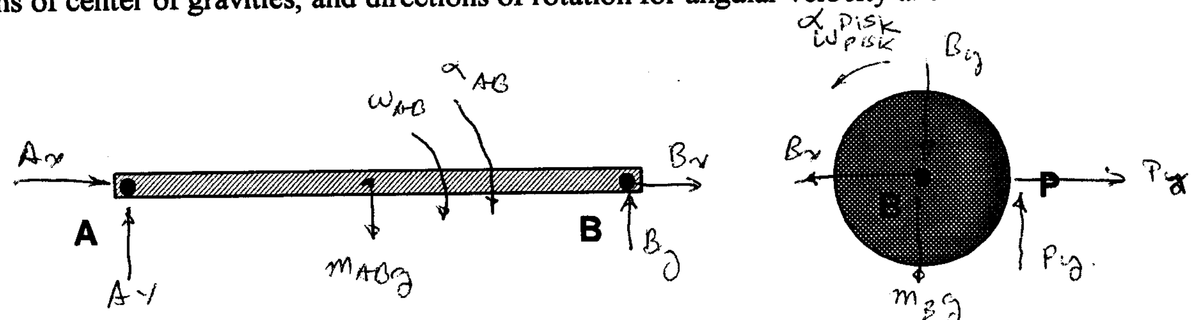
$0 = -220.5 + 2\times10^4 + 6\alpha_D = -19,800 + 6\alpha \Rightarrow \alpha = \frac{19800}{6} = 3300\ rad/sec^2$

$a_P = -60,220 + 2(3297) = \boxed{53,620\ in/sec^2 = 4468\ ft/sec^2}$

2. The 0.1 kg slender bar and the 0.2 kg cylindrical disk are released from rest with the bar horizontal. The disk rolls on the curved surface.



a. (5 points) Draw free body diagrams of all forces on the slender bar and rolling disk. Indicate locations of center of gravities, and directions of rotation for angular velocity and acceleration.



b. (5 points) Fill in the following table:

$\mathbf{R}_{AB} = \underline{0.12}\ \mathbf{i} + \underline{0}\ \mathbf{j}$. $\mathbf{R}_{PB} = \underline{-0.4}\ \mathbf{i} + \underline{0}\ \mathbf{j}$.

$\omega_{AB} = \underline{0}\ \mathbf{k}$. $\omega_{PB} = \underline{0}\ \mathbf{k}$.

c. (10 points) How do the angular acceration of the bar, α_{AB}, and the angular acceleration of the disk, α_D, relate to one-another? (Hint acceleration of point B must be same for bar and disk).

$$\vec{a}_{P_t} = \vec{\alpha}_{DISK} \times \vec{r}_{PB} = \vec{\alpha}_{AB} \times \vec{r}_{AB}$$

$$+\alpha_{DISK}\, r_{PB}\, \vec{j} = a_{P_t}\vec{j} = -\alpha_{AB}\, r_{AB}\, \vec{j}$$

$$\boxed{\alpha_{DISK} = -\alpha_{AD}\frac{r_{AB}}{r_{PB}}}$$

d. (20 points) What is the bar's angular acceleration, α_{AB}, at the instant of release? (Hint: Use moments about A and W)

$$\Sigma M_A = I_A \alpha_{AB} \Rightarrow -0.1g(0.06) + B_y(0.12) = \tfrac{1}{3}(0.1)(0.12)^2 \alpha_{AB}$$

$$\Sigma M_P = I_P \alpha_{DISK} \Rightarrow 0.04 B_y + 0.2g(0.04) = \left(\tfrac{1}{2}(0.2)(0.04)^2 + 0.2(0.04)^2\right)\alpha_D$$

Solve
$$\begin{cases} -0.06 + 0.12 B_y = 4.8\times10^{-4}\alpha_{AB} \\ 0.079 + 0.04 B_y = 4.8\times10^{-4}\alpha_{DISK} = 4.8\times10^{-4}(-3\alpha_{AB}) \end{cases}$$

$$\boxed{\alpha_{AB} = -62\ \text{rad/sec}^2}$$

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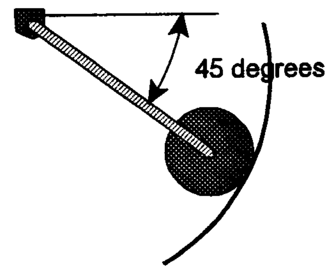

3. In the previous problem assume the horizontal bar falls to an angle of 45 degrees. The disk rolls along the wall, but assume no rolling friction along the wall and no air drag.

a. (5 points) What is the relationship between the velocity of the end of the bar (center of the disk) V_B and the angular velocity of the bar, ω_{AB}?

$$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{AB} = -\omega_{AD} r_{AD} \vec{t}$$

b. (5 points) What is the relationship between the angular velocity of the disk, ω_{disk}, and the angular velocity of the bar, ω_{AB}?

$$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{AB} = \vec{\omega}_{DISK} \times \vec{r}_{PB}$$

$$-\omega_{AB} r_{AB} \vec{t} = \omega_{Disk} r_{PB} \vec{t}$$

$$\boxed{\omega_{DISK} = -\omega_{AB}\left(\frac{r_{AB}}{r_{PB}}\right) = -3\omega_{AB}}$$

$$I_D = \frac{1}{2} m r^2$$

c. (20 points) What is the magnitude of the angular velocity of the bar, ω_{AB}?

$$\text{Work} = \Delta T + \Delta V_g.$$

$$0 = \frac{1}{2} I_A \omega_{AB}^2 + \frac{1}{2} I_D \omega_D^2 + \frac{1}{2} m_D V_B^2 - 0.1g(0.06)\sin 45 - 0.2g(0.12)\sin 45$$

$$0 = \frac{1}{2}\left(\frac{1}{3}(0.1)(0.12^2)\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{1}{2}(0.2)(0.04)^2\omega_D^2\right) + \frac{1}{2}(0.2)\left[(0.12)(0.12)\right]^2$$
$$- 0.1g(0.06)\sin 45 - 0.2g(0.12)\sin 45$$

$$0 = 2.4\times10^{-4}\omega_{AB}^2 + 8.0\times10^{-5}(\omega_{AB})^2(9) + 1.44\times10^{-3}\omega_{AB}^2 - 4.24\times10^{-3}g - 1.7\times10^{-2}g.$$

$$0 = 2.4\times10^{-3}\omega_{AB}^2 - 2.12\times10^{-2}g.$$

$$\omega_{AB}^2 = \frac{2.12\times10^{-2}g}{2.4\times10^{-3}}$$

$$\boxed{|\omega_{AB}| = 9.31 \text{ rad/sec}}$$