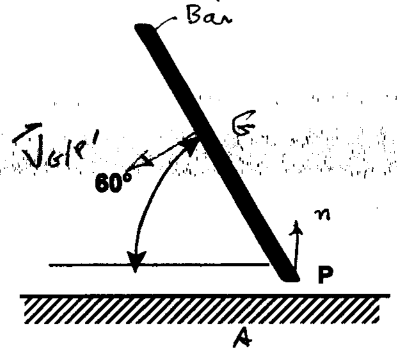
Name MASTER

CE 261 DYNAMICS
MIDTERM 3
Thursday, May 11, 1:30-3:30 pm

SHOW ALL WORK OPEN BOOK, CLOSED HOMEWORK OR SOLUTION MANUALS ONE PAGE NOTES PERMITTED, NO CREDIT FOR ANSWER ONLY



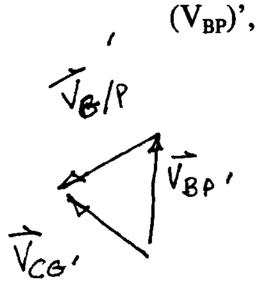
1. The slender bar is shown just before it hits the smooth floor. The length of the bar is 1 m and its mass is 2 kg. The bar's initial angular velocity is zero, and it is moving downward at 4 m/sec.

a. If the coefficient of restitution of the impact is e = 0.2, what is the velocity of the bar impact point, $(V_{BP})'$, immediately after impact?

$$e = \frac{(V_{Bar}')_n - (\cancel{V_{wall}})_n}{(\cancel{V_{wall}})_n - (V_{Bar})_n} = -\frac{V_{Bar\,P\,n}'}{V_{Bar\,P\,n}}$$

$$0.2 = \frac{-V_{Bar\,P\,n}'}{-4} \quad \therefore \quad V_{Bar\,P\,n}' = 0.2(4) = \boxed{0.8 \text{ m/sec}}$$

b. Assuming the answer to part a. is 1 m/sec, write an expression for the vector velocity of the center of gravity of the bar after impact, $(V_{CG})'$ in terms of the velocity of the bar at point P after impact, $(V_{BP})'$, and the angular velocity of the bar after impact, ω'. (Hint this is straight kinematics)



$$\vec{V}_{CG}' = \vec{V}_{BP}' + \vec{V}_{CG/P}' = \vec{V}_{BP}' + \vec{\omega}_B' \times \vec{r}_{PG}$$

c. Using conservation of angular momentum of the bar about the impact point, P, write an expression that relates the velocity of gravity of the bar after impact, $(V_{CG})'$, and the angular velocity of the bar after impact, ω'.

$$\Sigma M_P dt = 0 = H_P' - H_P = \tilde{I}\omega_B' + m\vec{r}_{PG} \times \vec{V}_{CG}' - m\vec{r}_{PG} \times \vec{V}_{CG}$$

$$0 = \frac{1}{12} m_B l^2 \omega_B' + m_B\left(-V_{BP}' \frac{l}{2}\cos 60° + \left(\frac{l}{2}\right)^2 \omega_B'\right) - m_B V_{CG} r_{PG} \cos 60$$

d. Combine equations from part b and c to determine the angular velocity of the bar after impact, ω'.

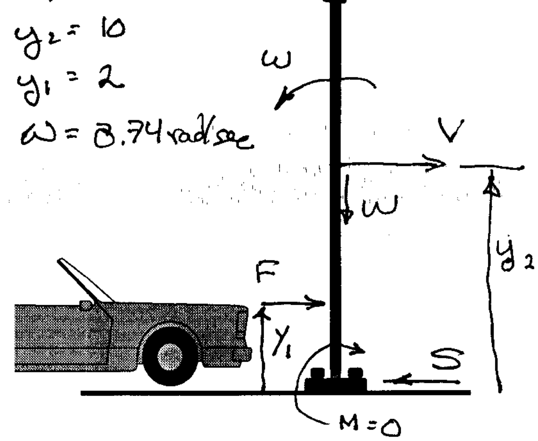
$$\omega_B' = \frac{(V_{CG} + V_{BP}') r_{PG} \cos 60}{\frac{1}{12} l^2 + (l/2)^2} = \frac{(4+1)(1)\cos 60}{\frac{1}{12}(1)^2 + (0.5)^2}$$

$$= \boxed{7.5 \text{ rad/sec}}$$

Name________________________

2. Engineers desire tp design a streetlight pole to shear off at ground level when struck by a vehicle. They desire to estimate the force S required to shear off the ground support bolts. From videotapes of a test impact they know the angular velocity of the pole is $\omega = 0.74$ rad/sec and the horizontal velocity of its centerpoint is V = 22 ft/sec after impact, whose duration is $\Delta t = 0.01$ sec. The pole can be modeled as a 20-ft, 140 lb pole. The car impacts the pole 2 ft above the ground, the couple exerted on the pole by its support can be neglected.

$V = 22$ ft/sec
$\Delta t = 0.1$
$m_{pole} = 140$ lb
$y_2 = 10$
$y_1 = 2$
$\omega = 0.74$ rad/sec



a. Draw a free body diagram on the sketch to right of the pole including impact force, shear force, weight, and indicate directions of motion, V and ω.

a. From the principle of linear impulse and momentum write an equation including the shear force S.

$$\int \Sigma F dt = m_g V_2 - m_g \cancel{V_1}^{0}$$

$$(F - S)\Delta t = mV$$

$$S = F - \frac{mV}{\Delta t} = F - \frac{(140)(22)}{(32.2)(0.1)} = F - 9656$$

b. From the principle of angular impulse and angular momentum write an equation for angular impulse about the pole base. (Be careful with your negative and positive sense of rotation of the angular rotations associated with the center velocity and pole rotation about its center.)

$$\int M_o dt = H_2 - H_1 = [\bar{I}\omega + (\bar{r} \times m\bar{V})\cdot k]_1^2$$

$$-y_1 F \Delta t = \bar{I}\omega - y_2 mV$$

$$F = \frac{-\bar{I}\omega + y_2 mV}{y_1 \Delta t} = \frac{-\frac{1}{12}\left(\frac{140}{32.2}\right)(20)^2(0.74) + 10\left(\frac{140}{32.2}\right)(22)}{2(0.01)}$$

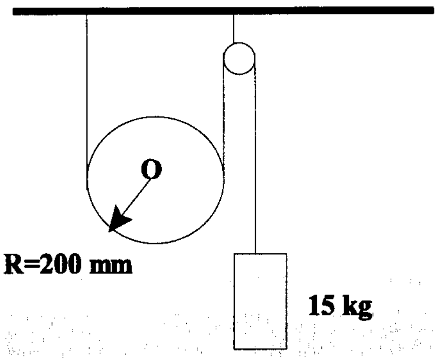
$$= 42{,}466 \text{ lbf}$$

c. Solve these equations for the average shear force S required.

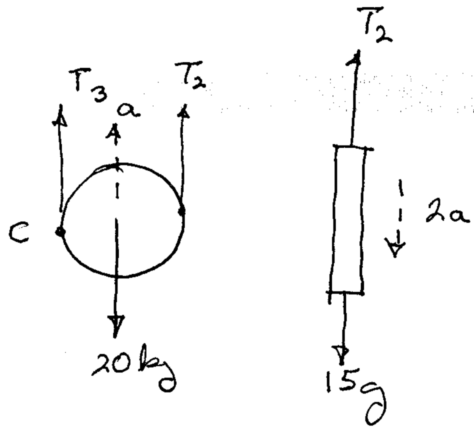
$$S = \frac{y_2 mV - \bar{I}\omega}{y_1 \Delta t} - \frac{mV y_1}{\Delta t\, y_1} = \frac{(y_2 - y_1)mV - \bar{I}\omega}{y_1 \Delta t}$$

or $$S = F - 9565 = 42{,}466 - 9565 = \boxed{32{,}901 \text{ lbf}}$$

3. Given the 20 kg solid disk of radius 200 mm suspended by cables and the 15 kg counterweight at the end of the cable.



a. Draw the free body diagram for disk and counterweight.



b. Calculate the upward acceleration, a_0 of the disk center 0 and the cable tension, T, between disk and counterweight.

Counterweight $\quad \Sigma F = ma$

$$T_2 - 15(9.816) = -15(2a)$$

Cylinder

$$\Sigma M_c = I_c \alpha \qquad I_c = \frac{1}{2}mr^2 + mr^2$$

$$\text{but } \alpha = \frac{a}{r} \qquad = \frac{1}{2}(20)(0.2)^2 + (20)(0.2)^2 = 1.2\ kg\,m^2$$

$$0.4T_2 - 20(9.816)(0.2) = 1.2\left(\frac{a}{0.2}\right)$$

$$T_2 - 147.2 = -30a$$

$$0.4T_2 - 39.2 = 6a$$

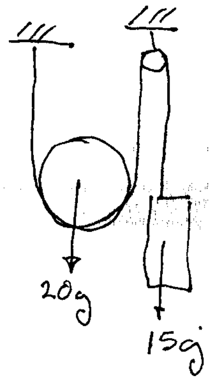
$$0.4(147.2 - 30a) - 39.2 = 6a$$

$$a(6 + 12) = 0.4(147.2) - 39.2 = 19.68$$

$$a = \frac{19.68}{18} = \boxed{1.09\ m/sec^2}$$

4. In the previous problem assume that the 15 kg counterweight has fallen 1 meter.

a. Draw an "active force" diagram for the disk-counterweight system.



$\Delta z_W = -2\Delta z_D$

$V_D = -\frac{1}{2} V_W$

$V_W = -2 V_D$

b. Using the energy method calculate the resultant velocity, V_o of the disk center of gravity and its angular velocity, ω, after the counterweight falls 1 meter.

$$0 = \Delta T + \Delta V_g$$

$$0 = \frac{1}{2} I \omega_D^2 + \frac{1}{2} m_D V_D^2 + \frac{1}{2} m_W V_W^2 + m_D g \Delta h_D - m_W g \Delta h_W$$

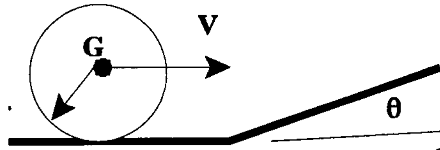
$$0 = \frac{1}{2}\left(\frac{1}{2} m_D r_D^2\right)\omega_D^2 + \frac{1}{2} m_D (\omega_D r_D)^2 + \frac{1}{2} m_W (2\omega_D r_D)^2 + m_D g\left(\frac{\Delta h_W}{2}\right) - m_W g \Delta h_W$$

$$0 = \frac{1}{2}\left(\frac{1}{2} 20 (0.2)^2\right)\omega_D^2 + \frac{1}{2}(20)(0.2)^2\omega_D^2 + \frac{1}{2}(15)(4)(0.2)^2\omega_D^2 + 20g\left(\frac{1}{2}\right) - 15g(1)$$

$$0 = 0.2\,\omega_D^2 + 0.4\,\omega_D^2 + 1.2\,\omega_D^2 + 98.1 - 147.2$$

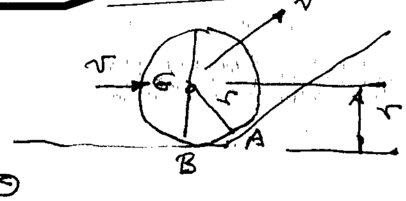
$$\omega_D = \left[\frac{147.2 - 98.1}{(1.8)}\right]^{1/2} = \boxed{5.22 \text{ rad/sec.}}$$

5. A uniform circular disk of mass 1 kg and radius 0.5m rolls without slipping with a velocity V_o = 1m/s. It encounters an abrupt change in slope of θ= 10°.



a. Using the methods of conservation of momentum find the new velocity of the cneter of the disk as it starts up the incline.

Consider angular momentum about point A for instant of contact.



$$\overset{\to 0}{\int M_A dt} = H_{A\,final} - H_{A\,initial}$$

$$H_{A\,INITIAL} = \tilde{I}\omega + m v r\cos\theta = \frac{1}{2}mr^2\left(\frac{V}{r}\right) + mVr\cos\theta$$

$$= \frac{1}{2}(1)(0.5)^2\left(\frac{1}{0.5}\right) + 1(1)(0.5\cos 10^\circ)$$

$$= 0.25 + 0.492 = 0.742 \text{ kg m}^2/\text{sec}.$$

$$H_{A\,final} = \tilde{I}\frac{v'}{r} + m v' r = \frac{1}{2}mr^2\frac{v'}{r} + mr^2\frac{v'}{r} = \frac{3}{2}mr^2\frac{v'}{r}$$

$$= \frac{3}{2}(1)(0.5)v' = 0.75$$

$$v' = \frac{0.742}{0.75} = \boxed{0.989 \text{ m/sec}}$$

b. What fraction of the initial energy is lost due to the contact with the incline? Assume that the vlelocity of the mass center after contact is 0.9 m/sec.

$$\eta = \frac{\frac{1}{2}I_B\omega^2 - \frac{1}{2}I_A\omega'^2}{\frac{1}{2}I_B\omega^2} = 1 - \frac{\omega'^2}{\omega^2} = 1 - \frac{v'^2}{v^2}$$

$$= 1 - \frac{(0.989)^2}{1^2} = 1 - 0.980 = \boxed{0.02}$$