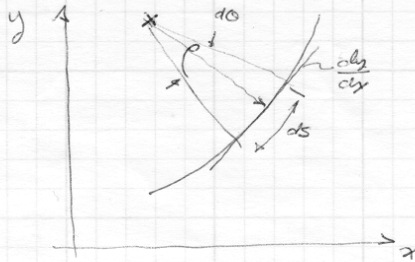


RADIUS OF CURVATURE

$$y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

$$\frac{d^2y}{dx^2} = f''(x)$$



Cartesian

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Polar or Cylindrical

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\theta}\right)^2\right]^{3/2}}{r^2 + 2\left(\frac{dr}{d\theta}\right)^2 - r\frac{d^2r}{d\theta^2}}$$

note in $\begin{cases} \text{Pure} \\ \text{cylindrical} \\ \text{motion} \end{cases} \frac{dr}{d\theta} = 0 \quad \frac{d^2r}{d\theta^2} = 0 \quad \rho =$

$$\rho = \frac{[r^2 + 0]^{3/2}}{r^2 + 2(0)^2 - r(0)} = \frac{r^2 \cdot 3/2}{r^2} = \frac{r^3}{r^2} = r$$

Derivation (contension)

$$\rho d\theta = ds$$

$$\rho = \frac{ds}{d\theta}$$

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (A)$$

$$d\theta = \theta_2 - \theta_1 = \tan^{-1}\left(\frac{dy}{dx}\right)_2 - \tan^{-1}\left(\frac{dy}{dx}\right)_1$$

Taylor series expansion

$$\tan^{-1}\left(\frac{dy}{dx}\right)_2 = \tan^{-1}\left(\frac{dy}{dx}\right)_1 + \frac{d\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right)}{ds} ds + \dots$$

$$d\theta = \cancel{\tan^{-1}\left(\frac{dy}{dx}\right)_1} + \frac{d\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right)}{ds} ds + \dots - \cancel{\tan^{-1}\left(\frac{dy}{dx}\right)_1}$$

$$\rho = \frac{ds}{\frac{d\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right)}{ds} ds} = \frac{1}{\frac{d\left(\tan^{-1}\left(\frac{dy}{dx}\right)\right)}{dx} \left(\frac{dx}{ds}\right)}$$

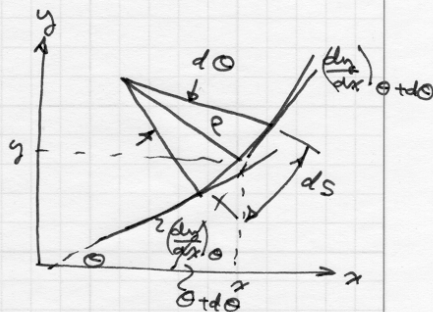
but $\frac{dx}{ds} = \frac{1}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \quad (\text{see A above})$

$$\frac{d \tan^{-1} u}{dx} = \frac{1}{1+u^2} \frac{du}{dx} \quad \text{or}$$

$$\frac{d \tan^{-1}\left(\frac{dy}{dx}\right)}{dx} = \frac{1}{1 + \left(\frac{dy}{dx}\right)^2} \frac{d^2y}{dx^2}$$

so

$$\rho = \frac{[1 + \left(\frac{dy}{dx}\right)^2]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$



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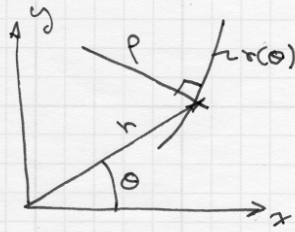
$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\frac{dx}{d\theta} = -r \sin \theta + \cos \theta \frac{dr}{d\theta}$$

$$\frac{dy}{d\theta} = +\cos \theta r + \sin \theta \frac{dr}{d\theta}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta} \frac{d\theta}{dx}}{\frac{dx}{d\theta}} = \frac{+r \cos \theta + \frac{dr}{d\theta} \sin \theta}{-r \sin \theta + \frac{dr}{d\theta} \cos \theta}$$



$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(\frac{+r \cos \theta + \frac{dr}{d\theta} \sin \theta}{-r \sin \theta + \frac{dr}{d\theta} \cos \theta}\right)^2$$

$$= \left[r^2 \sin^2 \theta + 2r \frac{dr}{d\theta} \sin \theta \cos \theta + \left(\frac{dr}{d\theta}\right)^2 \cos^2 \theta + r^2 \cos^2 \theta + 2r \frac{dr}{d\theta} \sin \theta \cos \theta + \left(\frac{dr}{d\theta}\right)^2 \sin^2 \theta \right] /$$

$$\left[r^2 \sin^2 \theta + 2r \frac{dr}{d\theta} \sin \theta \cos \theta + \left(\frac{dr}{d\theta}\right)^2 \cos^2 \theta \right]$$

$$= \frac{\left[r^2 (\sin^2 \theta + \cos^2 \theta) + \left(\frac{dr}{d\theta}\right)^2 (\sin^2 \theta + \cos^2 \theta) \right]}{\left(-r \sin \theta + \frac{dr}{d\theta} \cos \theta\right)^2}$$

$$= \frac{\left[r^2 + \left(\frac{dr}{d\theta}\right)^2 \right]}{\left(-r \sin \theta + \frac{dr}{d\theta} \cos \theta\right)^2}$$

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d}{d\theta} \left(\frac{dy}{dx}\right) \frac{d\theta}{dx}$$

$$\frac{d\left(\frac{dy}{dx}\right)}{d\theta} = \frac{d}{d\theta} \left[\frac{+r \cos \theta + \frac{dr}{d\theta} \sin \theta}{-r \sin \theta + \frac{dr}{d\theta} \cos \theta} \right]$$

$$= \frac{\left(+\frac{dr}{d\theta} \cos \theta - r \sin \theta + \frac{d^2r}{d\theta^2} \sin \theta + \frac{dr}{d\theta} \cos \theta \right) \left(-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right)}{\left(-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right)^2}$$

$$= \frac{\left[+r \cos \theta + \frac{dr}{d\theta} \sin \theta \right]}{\left(-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right)^2} \left(-\frac{dr}{d\theta} \sin \theta + r \cos \theta + \frac{d^2r}{d\theta^2} \cos \theta - \frac{dr}{d\theta} \sin \theta \right)$$

$$= \left\{ \begin{aligned} & \cancel{-r \frac{dr}{d\theta} \sin^2 \theta} + r^2 \sin^2 \theta - \cancel{r \frac{dr^2}{d\theta^2} \sin^2 \theta} - \cancel{r \frac{dr}{d\theta} \cos^2 \theta} \sin \theta \\ & + \left(\frac{dr}{d\theta} \right)^2 \cos^2 \theta - \cancel{r \frac{dr}{d\theta} \sin \theta \cos \theta} + \frac{dr}{d\theta} \frac{dr^2}{d\theta^2} \sin \theta \cos \theta + \\ & \left(\frac{dr^2}{d\theta} \right) \cos^2 \theta \end{aligned} \right\} =$$

$$\begin{aligned} & \left[\cancel{-r \frac{dr}{d\theta} \sin^2 \theta} - r^2 \cos^2 \theta + r \frac{dr^2}{d\theta^2} \cos^2 \theta - \cancel{r \frac{dr}{d\theta} \cos^2 \theta} \sin \theta \right. \\ & \left. - \left(\frac{dr}{d\theta} \right)^2 \sin^2 \theta + r \frac{dr}{d\theta} \sin \theta \cos \theta + \frac{dr}{d\theta} \frac{dr^2}{d\theta^2} \sin \theta \cos \theta \right. \\ & \left. - \left(\frac{dr^2}{d\theta} \right) \sin^2 \theta \right] \left[-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right]^2 \end{aligned}$$

$$= \frac{r^2 (\sin^2 \theta + \cos^2 \theta) - r \frac{dr}{d\theta} (\sin^2 \theta + \cos^2 \theta) + 2 \left(\frac{dr}{d\theta} \right)^2 (\cos^2 \theta + \sin^2 \theta)}{\left[-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right]^2}$$

∴

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{3/2}}{\left[-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right]^3}$$

$$= \frac{\left[r^2 + 2 \left(\frac{dr}{d\theta} \right)^2 - r \frac{dr^2}{d\theta^2} \right]}{\left[-r \sin \theta + \frac{dr}{d\theta} \cos \theta \right]^2}$$

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{3/2}}{\left[r^2 + 2 \left(\frac{dr}{d\theta} \right)^2 - r \frac{dr^2}{d\theta^2} \right]}$$

