THE VORTEX MODE OF INSTABILITY IN NATURAL CONVECTION FLOW ALONG INCLINED PLATES

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Abstract — The stability of natural convection flow along inclined plates has been investigated using the methods of linear perturbation theory. Attention has been focused on three dimensional spatially growing disturbances, which take the form of parallel rows of streamwise oriented vortices. Results are presented of the critical Rayleigh numbers versus plate inclination angle. The theory is in qualitative agreement with experiment. The angle at which the three dimensional instability crosses over into the two dimensional wave instability is fairly well predicted by the theory.

NOMENCLATURE

A. dimensionless lateral wave number:
B. dimensionless amplification parameter:
C. dimensionless pressure perturbation:
\( \delta \). specific heat at constant pressure:
\( g \). acceleration due to gravity:
G. dimensionless number \( G = \left( \frac{Gr_{\alpha}}{4} \right)^{1.4} \):
\( Gr_{\alpha} \), Grashof number \( Gr_{\alpha} = \frac{\alpha \beta \Delta T \lambda^3 \cos \varphi}{k^2} \):
\( k \). thermal conductivity:
Pr. Prandtl number \( Pr = \frac{\nu}{k} \):
Ra. Rayleigh number \( Ra = Pr Gr_{\alpha} \cos \varphi \):
\( \tau \). time:
\( \theta \). temperature:
\( \theta_s \). temperature at plate surface:
\( \theta_f \). temperature in free stream:
\( \alpha \). dimensionless perturbation velocity component in \( \alpha \) direction:
\( \alpha \). dimensionless perturbation velocity component in \( \alpha \) direction:
\( \alpha \). dimensionless perturbation velocity component in \( \alpha \) direction:
\( x \). streamwise coordinate parallel to plate surface:
\( \eta \). coordinate perpendicular to plate surface:
\( \zeta \). lateral coordinate.

Greek letters
\( \delta \). thermal diffusivity \( \delta = \frac{k}{\rho \beta} \):
\( \varphi \). lateral wavenumber:
\( \beta \). amplification parameter:
\( \beta \). temperature coefficient of volume expansion:
\( \beta = \frac{1}{\rho \sqrt{\rho \theta \lambda_{\alpha}}} \):
\( \varphi \). inclination angle of plate from vertical:
\( \theta \). temperature component \( \theta = \theta_s - \theta_f \):
\( \Delta \). temperature difference between plate surface and free stream \( \Delta T = \theta_s - \theta_f \):
\( \rho \). density:
\( \eta \). dimensionless \( \eta \) coordinate:
\( \delta \). characteristic thickness \( \delta = \frac{1}{2 \pi} \sqrt{Gr_{\alpha}} \):

Subscript
\( p \). refers (except in the case of \( C_{p} \)) to dimensional fluctuating components.

1. INTRODUCTION

Recent observations [1, 2] of natural convection flow along inclined plates have indicated that the flow exhibits an instability in the form of streamwise-oriented vortices whenever the inclination angle of the flow with the vertical exceeds a certain critical value. For smaller angles of inclination, it appears that the two-dimensional Tollmien Schlichting wave-type disturbances are the dominant mode of instability.
A proportion is a statement of equality between two ratios. It is written as \( \frac{a}{b} = \frac{c}{d} \) where \( a, b, c, d \) are real numbers and \( b \neq 0, d \neq 0 \).

The proportion can be used to solve for unknown values in the ratios.

\[ \frac{a}{b} = \frac{c}{d} \]

Given the values of three of the four unknowns, the fourth can be calculated.

1. \( \frac{a}{b} = \frac{c}{d} \)
2. \( \frac{a}{b} = \frac{c}{d} \)
3. \( \frac{a}{b} = \frac{c}{d} \)
4. \( \frac{a}{b} = \frac{c}{d} \)

Example:

Find the value of \( x \) in the proportion \( \frac{3}{4} = \frac{x}{8} \).

Solving for \( x \):

\[ \frac{3}{4} = \frac{x}{8} \]

\[ 3 \times 8 = 4 \times x \]

\[ 24 = 4x \]

\[ x = \frac{24}{4} \]

\[ x = 6 \]

The proportion can be used to solve for unknown values in the ratios.

Example:

Find the value of \( y \) in the proportion \( \frac{5}{7} = \frac{y}{35} \).

Solving for \( y \):

\[ \frac{5}{7} = \frac{y}{35} \]

\[ 5 \times 35 = 7 \times y \]

\[ 175 = 7y \]

\[ y = \frac{175}{7} \]

\[ y = 25 \]

The proportion can be used to solve for unknown values in the ratios.

Example:

Find the value of \( z \) in the proportion \( \frac{2}{3} = \frac{z}{12} \).

Solving for \( z \):

\[ \frac{2}{3} = \frac{z}{12} \]

\[ 2 \times 12 = 3 \times z \]

\[ 24 = 3z \]

\[ z = \frac{24}{3} \]

\[ z = 8 \]
has obtained solutions to the mean velocity and temperature profiles for natural convection along inclined plates. However, his work apparently contains a basic error which restricts the range of validity of his results to inclination angles of less than about 30°. However, his solutions even then indicate that only minor changes in velocity profiles occur with increasing angle of inclination, while the mean temperature profile which after all is the basic driving mechanism of the instability, is almost independent of the inclination angle. Herein, we have assumed that the similarity solutions for vertical heated plates would be applicable when the definition of Grashof number is modified to incorporate the effect of inclination angle by the factor cos φ. This assumption is not without justification [15]. Furthermore, it is known a priori that the results of linear stability analyses, provide main qualitative information. It appears hardly justifiable therefore, to resort to time consuming numerical calculations of the developing profiles on inclined heated plates, especially since in the absence of similarity solutions, it would be necessary to integrate the full boundary layer equations in finite difference form.

3. METHOD OF SOLUTION

At this stage, several choices for solving the system of equations (3) are available. Approximate analytic techniques such as the Galerkin method described in Kato et al. [26] can be employed. However, due to the complexity and number of dependent variables in the equations, Galerkin’s procedure would involve an excessive amount of algebra resulting from the several analytical integrations required. A further disadvantage to using methods such as Galerkin’s is the difficulty in obtaining accurate eigenfunctions. This has been discussed by Smith [31]. It was therefore decided to solve the system of differential equations by numerical integration. In the present problem, all mean flow quantities except the velocity component perpendicular to the wall are zero in the region outside the boundary layer. The differential equations in the free stream therefore simplify to the constant coefficient type, which are readily solved in terms of elementary functions. The method of solution employed was to integrate the equations numerically from the edge of the boundary layer to the wall, using the exponentially decaying far field solutions to provide initial conditions at the boundary layer edge (taken as \( r = 6 \) for \( Pr = 0.72 \) and \( r = 8 \) for \( Pr = 1.06 \)). Since the system of differential equations contain exponentially growing “parasitic” solutions, it was necessary to integrate them as a system of four initial value problems, using the Gram-Schmidt orthomization process to maintain linear independence between the four homogeneous solution vectors. The general method is well described in [37], while the details as applied to the specific problem here is available in [28].

Results were obtained for inclination angles from the vertical of (5°, 30°, 45° and 60°) at Prandtl numbers of 0.72 and 100. In addition, amplification curves were computed for a fixed angle of inclination of 30° for different values of the amplification parameter BG.

4. RESULTS AND DISCUSSION

The results obtained are presented in Figs. 1-7. Figures 2 and 3 are the neutral amplification curves for the Prandtl numbers 0.72 and 100, respectively. Both figures have been plotted, using for the abscissa the quantity \( G \), which Nachstein (1963) interprets as a characteristic Reynolds number of the flow. As expected, the flow is increasingly susceptible to the vortex instability at the higher inclination angles. It is interesting that the critical dimensionless wavenumbers appear unaffected by inclination angle, in agreement with the experiments of Sparrow and Huser [1]. At the higher Prandtl number, the critical wavenumber is increased to about 1.17 which is more than a factor of two over its value at the lower Prandtl number of 0.72. The value of \( G \) is consistently lower at the higher Prandtl number as would be expected, since increasing Prandtl numbers result in steeper mean temperature gradients. Lee and Lock (10) obtained critical wavenumber of zero indicating vortices of infinite extent. As mentioned in the introduction, we believe that the discrepancy is due to their use of the parallel flow assumption. At small wavenumbers, the coefficient \( G \) of the first derivatives in equation (13) becomes increasingly important with respect to the wavenumber. For this reason our curves in Figs. 2 and 3 display definite minima, indicating that the vortices remain of controlled size. Furthermore, our numerical scheme did not exhibit any problems of convergence to the eigenvalue as theirs apparently did.

A comparison of the neutral amplification curve from the present theory for \( \alpha = 15° \), and the corresponding curve for the two dimensional wave instability on a vertical heated plate calculated by Nachstein [7] have been drawn in Fig. 4. The two curves from Nachstein’s work are the result of two different stability calculations, with and without velocity-temperature coupling effects. When temperature fluctuations are taken into consideration, the critical values of \( G \) and the wavenumber is sharply reduced. However, both neutral amplification curves of the wave instability have critical \( G \) values above that of the vortex instability. Using Nachstein’s [7] results for the wave disturbances, a curve has been constructed of critical
The water marker is...

[Diagram]

The water marker is used to indicate the level of water in a container. It is a simple and effective way to ensure that the water level is maintained at a desired level. The water marker is typically made of plastic or metal and is marked with graduation lines to indicate the level of water.

The water marker can be used in various applications, such as in coffee makers, tea pots, and other kitchen appliances. It is also used in industrial applications, such as in storage tanks and cooling towers.

The water marker is easy to use and can save time and effort in maintaining the correct water level. It is also useful in situations where manual monitoring of the water level is not feasible, such as in large storage tanks or in remote locations.

The water marker is an important tool in maintaining the quality and safety of water in various applications. It is a simple and effective solution to a common problem and is widely used in different industries.
laminar boundary layer stability in free convection,
7. P. Nachtsheim, Stability of free-convection boundary-
8. B. Gettis, Natural convection flow, instability and transition,
9. S. Hadland, Contributions to linear stability theory on
nearly parallel flows, Ph.D. Thesis, University of Min-
nesota (1972).
10. J. B. Lee and G. S. H. Lock, Instability in boundary
layer free convection along an inclined plate, presented
at Fourth Western Canadian Heat Transfer Conference,
Winnipeg, Manitoba (1972).
11. A. M. O. Smith, On the growth of Taylor-Görtler
vortices along highly concave walls, Q. Appl. Math.
33(2), 235-2055.
12. T. S. Chang and W. K. Sartory, Hydromagnetic
Görtler instability in a boundary layer on a concave
wall. Developments in Theoretical and Applied Mechanics,

L'INSTABILITÉ DE TYPE TONBILLOUERNE DANS LA
CONVECTION NATURELLE LE LONG DE PLAQUES INCLINÉES

Résumé—On a étudié la stabilité d'un écoulement de convection naturelle le long de plaques inclinées à
l'aide des méthodes de perturbation linéaire. L'attention a été portée sur des perturbations indi-
centrales croissantes dans l'espace et qui prennent la forme de rangées parallèles de tourbillons orientés
dans le sens de l'écoulement. On présente les résultats concernant les nombre critiques de Rayleigh
en fonction de l'angle d'inclinaison de la plaque. La théorie est en accord avec l'expérience.
L'angle pour lequel l'instabilité tridimensionnelle devient une instabilité bidimensionnelle ondulatoire
est assez bien estimé par la théorie.

INSTABILE WIRBEL BEI FREIER KONVEKTION LÄNGS GENEIGTER PLATTEN

Zusammenfassung—Die Stabilität der Strömung bei freier Konvektion längs geneigter Platten wurde
mit Hilfe der linearen Strömungstheorie untersucht. Besondere Aufmerksamkeit wurde auf dreidi-
censionale, räumlich wachsende Störungen gerichtet, die die Form von parallelen Reihen in Strömungsrichtung
orientierter Wirbel annehmen. Als Ergebnisse werden die kritischen Rayleigh-Zahlen über den Plattene-
neigungswinkel angegeben. Die Theorie stimmt qualitativ mit dem Experiment überein. Der Winkel, bei
der die dreidimensionale Instabilität übergeht in eine zweidimensionale wellenartige Instabilität, wird
von der Theorie recht gut vorausgesagt.

ВИХРЕВАЯ НЕУСТОЙЧИВОСТЬ ТЕЧЕНИЯ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ
ВДОЛЬ НАКЛОННОЙ ПЛАСТИНЫ

Аннотация — С помощью методов теории линейных возмущений исследовалась устойчивость
tечения при естественной конвекции вдоль наклонной пластинки. Рассматривались трехмерные
вращающиеся возмущения, которые занимают вид параллельных рядов вихрей, ориентиро-
vанных по течению. Результаты представлены в виде зависимости критического числа Рейли
от угла наклона пластинки. Теория качественно согласуется с экспериментом. Теоретически
хорошо предсказывается угол, при котором трехмерная неустойчивость переходит в двумер-
ную волнообразную неустойчивость.