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THE VORTEX MODE OF INSTABILITY IN NATURAL CONVECTION FLOW ALONG INCLINED PLATES

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Abstract—The stability of natural convection flow along inclined plates has been investigated using the methods of linear perturbation theory. Attention has been focussed on three dimensional spatially growing disturbances, which take the form of parallel rows of streamwise oriented vortices. Results are presented of the critical Rayleigh numbers versus plate inclination angle. The theory is in qualitative agreement with experiment. The angle at which the three dimensional instability crosses over into the two dimensional wave instability is fairly well predicted by the theory.

NOMENCLATURE

A, dimensionless lateral wave number;
B, dimensionless amplification parameter;
C_p, dimensionless pressure perturbation;
$\hat{c}$, specific heat at constant pressure;
g, acceleration due to gravity;
G, dimensionless number $G = 4\left(\frac{Gr_x}{4}\right)^{1/4}$;
Gr_x, Grashof number $Gr_x = \frac{g\beta\Delta T x^3 \cos\varphi}{\nu^2}$;
k, thermal conductivity;
Pr, Prandtl number $Pr = \frac{\mu\hat{c}}{k}$;
Ra, Rayleigh number, $Ra = Pr \, . \, Gr_x/\cos\varphi$;
t, time;
T, temperature;
T_w, temperature at plate surface;
T_∞, temperature in free stream;
u, dimensionless perturbation velocity component in x direction;
v, dimensionless perturbation velocity component in y direction;
w, dimensionless perturbation velocity component in z direction;
x, streamwise coordinate parallel to plate surface;
y, coordinate perpendicular to plate surface;
z, lateral coordinate.

Greek letters

$\hat{\alpha}$, thermal diffusivity $\hat{\alpha} = \frac{k}{\rho\hat{c}}$;
α, lateral wavenumber;
β, amplification parameter;
$\hat{\beta}$, temperature coefficient of volume expansion $\hat{\beta} = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\theta}\right)_p$;
φ, inclination angle of plate from vertical;
θ, temperature component $\theta = T - T_\infty$;
ΔT, temperature difference between plate surface and free stream $\Delta T = T_w - T_\infty$;
ρ, density;
η, dimensionless y coordinate;
δ, characteristic thickness $\delta = \sqrt{2}x/(Gr_x)^{1/4}$.

Subscript

p, refers (except in the case of C_p) to dimensional fluctuating components.

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1. INTRODUCTION

RECENT observations [1, 2] of natural convection flow along inclined plates have indicated that the flow exhibits an instability in the form of streamwise oriented vortices whenever the inclination angle of the flow with the vertical exceeds a certain critical value. For smaller angles of inclination, it appears that the two dimensional Tollmein Schlichting wave-type disturbances are the dominant mode of instability.

The occurrence of streamwise oriented vortices in other stratified flows is by no means uncommon. Gage and Reid [3]. Kuo [4] and more recently Meroney *et al.* [5] have indicated that unstable stratification in various shear flows can cause the appearance of three dimensional disturbances. Furthermore. Gage and Reid [3] have shown that for thermally stratified plane Poiseuille flow. Squire's Theorem is inapplicable when the Richardson number (defined by them as $Ra/64Rd$, where Ra, Rd are the Rayleigh and Reynolds numbers, respectively) goes below a certain small negative value. This implies that when the Rayleigh number of the flow is greater than some critical value, three dimensional rolls will form regardless of the magnitude of the shear.

For the case of inclined plate natural convection, it appears reasonable to suggest therefore, that whenever the component of the body force at right angles to the plate exceeds a certain critical value, streamwise oriented vortices are generated. For smaller inclination angles, the body force component along the plate serves to accelerate the flow streamwise, until the critical Reynold's number governing the stability of two dimensional waves is exceeded. The dominant instability mechanism for the transition process is therefore dependent on two competing factors, the first being the critical Reynold's number for wave disturbances, while the second is the critical Rayleigh number for longitudinal rolls.

The stability of natural convection flow to wave-type disturbances has been analytically investigated by Plapp [6]. Nachsteim [7] and Gebhart [8] among others. The three dimensional instability mode has only recently received attention (Haaland [9], Lee and Lock [10]). Haaland [9] used a form of base flow profile and disturbance different from that of the present paper. Lee and Lock [10] who consider both two as well as three dimensional perturbations assumed that the base flow was parallel. This type of assumption has been questioned by Haaland [9]. The present paper is a linear analysis of the three dimensional instability in a growing natural convection boundary layer.

2. FORMULATION

Consider an isothermal plate inclined at some angle φ to the vertical as depicted in Fig. 1. The coordinate axes and velocities are assumed conventionally as x, y, z and u, v, w respectively, with x being measured in streamwise direction from the leading edge of the plate. The standard methods of linear perturbation theory in which the instantaneous values of the velocity, temperature and pressure components are perturbed by disturbances of small amplitude and the mean flow components subtracted, with terms higher than first

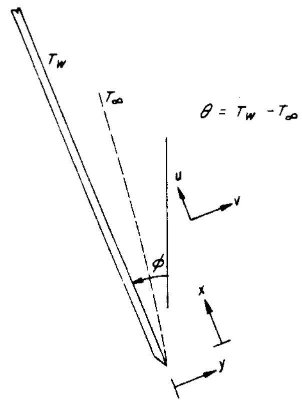

FIG. 1. Natural convection from an inclined plate.

order in perturbation quantities being neglected results in the following system of differential equations:

$$\frac{\partial\tilde u}{\partial t}+\bar u\frac{\partial\tilde v}{\partial y}+\bar u\frac{\partial\tilde u}{\partial x}+\tilde v\frac{\partial\bar u}{\partial y}+\bar v\frac{\partial\tilde u}{\partial y}=g\beta\tilde\theta\cos\varphi+\nu\nabla^2\tilde u-\frac{1}{\rho}\frac{\partial\tilde p}{\partial x}$$

$$\frac{\partial\tilde v}{\partial t}+\bar u\frac{\partial\tilde v}{\partial x}+\tilde v\frac{\partial\bar v}{\partial y}+\bar v\frac{\partial\tilde v}{\partial y}=g\beta\tilde\theta\sin\varphi+\nu\nabla^2\tilde v-\frac{1}{\rho}\frac{\partial\tilde p}{\partial y}$$

$$\frac{\partial\tilde w}{\partial t}+\bar u\frac{\partial\tilde w}{\partial x}+\bar v\frac{\partial\tilde w}{\partial y}=\nu\nabla^2\tilde w-\frac{1}{\rho}\frac{\partial\tilde p}{\partial z} \qquad (1)$$

$$\frac{\partial\tilde u}{\partial x}+\frac{\partial\tilde v}{\partial y}+\frac{\partial\tilde w}{\partial z}=0.$$

$$\frac{\partial\tilde\theta}{\partial t}+\bar u\frac{\partial\tilde\theta}{\partial x}+\tilde v\frac{\partial\bar\theta}{\partial y}+\bar v\frac{\partial\tilde\theta}{\partial y}=\hat\alpha\nabla^2\tilde\theta.$$

where ν = kinematic viscosity

$$\hat\alpha=\frac{k}{\rho\bar c},\quad \hat\beta=-\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\theta}\right)_p.$$

$\sim$ signify fluctuating components.

$$t = \text{time} \quad\text{and}\quad \theta = T-T_\infty.$$

In writing the above equations, the assumption of an incompressible, Boussinesq fluid has been made. Since the scope of the present investigation is confined to three dimensional disturbances, and in particular longitudinal vortices, the following representation of the perturbations are postulated.

$$\tilde u = u_p(y)e^{\beta x}\cos\alpha z$$
$$\tilde v = v_p(y)e^{\beta x}\cos\alpha z$$
$$\tilde w = w_p(y)e^{\beta x}\sin\alpha z \qquad (2)$$
$$\tilde p = p_p(y)e^{\beta x}\cos\alpha z$$
$$\tilde\theta = T_p(y)e^{\beta x}\cos\alpha z.$$

A spacewise growth of the disturbance has been assumed on the basis of the experimental observations [1, 2] reported in the literature. In these experiments, the vortices were seen to appear at some distance from the leading edge of the plate and amplify with increasing distance downstream. At any given x, the vortices appeared stationary with a fixed lateral wavenumber α. The mathematical assumption made here, is that a spacewise growth is more unstable than a timewise growth and is identical with the assumption made by Smith [11] for Taylor–Goertler vortices along concave curved walls.

Substitution of equations (2) into the set (1) results in:

$$-u_p\frac{d\bar v}{dy}+\bar u u_p\beta+\bar v u_p'+v_p\frac{d\bar u}{dy}=-\frac{1}{\rho}\beta p_p+g\beta T_p\cos\varphi+\nu[u_p''-(\alpha^2-\beta^2)u_p]$$

$$\bar u v_p\beta+\bar v v_p'+v_p\frac{d\bar v}{dy}=g\beta T_p\sin\varphi-\frac{1}{\rho}p_p'+\nu[v_p''-(\alpha^2-\beta^2)v_p]$$

$$\bar u\beta w_p+\bar v w_p'=\alpha\frac{p_p}{\rho}+\nu[w_p''-(\alpha^2-\beta^2)w_p]$$

$$\beta u_p+v_p'+\alpha w_p=0$$

$$\bar u\beta T_p+v_p\frac{\partial\bar\theta}{\partial y}+\bar v T_p'=\alpha[T_p''-(\alpha^2-\beta^2)T_p].$$

The temperatures and velocities are now generalized and made dimensionless in the manner usually employed in boundary layer and stability theory. In accordance with Gebhart [8], the following scaling lengths and velocities are defined:

$$\text{Grashof number } Gr_x=\frac{g\hat\beta x^3\Delta T\cos\varphi}{\nu^2}$$

$$\Delta T=T_w-T_\infty$$
$$G=4\left(\frac{Gr_x}{4}\right)^{1/4}$$
$$\eta=\frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$
$$\delta=\sqrt{2}x/(Gr_x)^{1/4}$$
$$U^*=\frac{2\nu}{x}(Gr_x)^{1/2}$$
$$B=\beta\delta$$
$$A=\alpha\delta$$
$$C_p=\frac{p_p}{\rho U^{*2}}.$$

Temperatures are scaled with ΔT.

The following system of differential equations results

$$\frac{d^2u}{d\eta^2}-GV\frac{du}{d\eta}+\left[B^2-A^2-G\left(BU-\frac{dV}{d\eta}\right)\right]u-G\frac{dU}{d\eta}v=GBC_p-T$$

$$\frac{d^2v}{d\eta^2}-GV\frac{dv}{d\eta}+\left[B^2-A^2-G\left(BU+\frac{dV}{d\eta}\right)\right]v=G\frac{dC_p}{d\eta}-T\tan\varphi$$

$$\frac{d^2w}{d\eta^2}-GV\frac{dw}{d\eta}+[B^2-A^2-BGV]w=-GAC_p$$

$$Bu+Aw+v'=0 \qquad (3)$$

and finally

$$\frac{d^2T}{d\eta^2}-PrGV\frac{dT}{d\eta}+[B^2-A^2-PrBGU]T=PrGv\frac{dT_0}{d\eta}.$$

Here $Pr=\nu/\alpha$ is the Prandtl number. U, V and T_0 refer to the dimensionless base flow quantities.

It should be noted that the parallel flow assumption, viz. $V=dV/d\eta=0$ has not been made, since there is increasing evidence [9, 12] that such an assumption can cause significant variation in the results obtained. The boundary conditions for the perturbations are:

$$\eta=0,\ u=v=w=T=\frac{dv}{d\eta}=0$$

$$\eta\to\infty,\ u,v,w,T\to 0,$$

where the auxiliary boundary condition, $\eta=0$, $dv/d\eta=0$ is obtained from the equation of continuity.

The equations are homogeneous and linear with homogeneous boundary conditions and therefore constitute an eigenvalue problem. The information sought in solving the system (3) is the variation of G with the remaining three parameters A, B and φ.

The linearized theory presented here, requires an analytical representation of the mean flow (also referred to as the "base flow") components. In this analysis, the similarity solutions of Ostrach [13] for the natural convection flow along a vertical heated plate were approximated by high order polynominals. Seventh degree polynominals were used to represent the base flow profiles for $Pr=0{\cdot}72$ while thirteenth degree polynominals were employed for $Pr=10{\cdot}0$. In order to obtain improved accuracy of representation, derivatives of the profiles were approximated by separate polynominals. Calculations were made for Prandtl numbers of 0·72 and 10·0 at four different plate inclination angles.

A refinement in the analysis would be to use base flow profiles obtained for inclined plates. Kierkus [14] has obtained solutions to the mean velocity and temperature profiles for natural convection along inclined plates. However, his work apparently contains a basic error which restricts the range of validity of his results to inclination angles of less than about 30°. However, his solutions even then, indicate that only minor changes in velocity profiles occur with increasing angle of inclination, while the mean temperature profile which after all is the basic driving mechanism of the instability, is almost independent of the inclination angle. Herein, we have assumed that the similarity solutions for vertical heated plates would be applicable when the definition of Grashof number is modified to incorporate the effect of inclination angle by the factor $\cos\varphi$. This assumption is not without justification [15]. Furthermore, it is known *a priori*, that the results of linear stability analyses, provide mainly qualitative information. It appears hardly justifiable therefore, to resort to time consuming numerical calculations of the developing profiles on inclined heated plates, especially since in the absence of similarity solutions, it would be necessary to integrate the full boundary layer equations in finite difference form.

3. METHOD OF SOLUTION

At this stage, several choices for solving the system of equations (3) are available. Approximate analytic techniques such as the Galerkin method (described in Kantorovich and Krylov [16]) could be employed. However, due to the complexity and number of dependent variables in the equations, Galerkin's procedure would involve an excessive amount of algebra resulting from the several analytical integrations required. A further disadvantage to using methods such as Galerkin's is the difficulty in obtaining accurate eigenfunctions. This has been discussed by Smith [11]. It was therefore decided to solve the system of differential equations by numerical integration. In the present problem, all mean flow quantities except the velocity component perpendicular to the wall are zero in the region outside the boundary layer. The differential equations in the free stream therefore simplify to the constant coefficient type, which are readily solved in terms of elementary functions. The method of solution employed, was to integrate the equations numerically from the edge of the boundary layer to the wall, using the exponentially decaying far field solutions to provide initial conditions at the boundary layer edge (taken as $\eta = 6$ for $Pr = 0{\cdot}72$ and $\eta = 8$ for $Pr = 10{\cdot}0$). Since the system of differential equations contain exponentially growing "parasitic" solutions, it was necessary to integrate them as a system of four initial value problems, using the Gram–Schmidt orthonormalization process to maintain linear independence between the four homogeneous solution vectors. The general method is well described in [17], while the details as applied to the specific problem here is available in [18].

Results were obtained for inclination angles from the vertical of 15, 30, 45 and 60° at Prandtl numbers of 0·72 and 10·0. In addition, amplification curves were computed for a fixed angle of inclination of 30° for different values of the amplification parameter BG.

4. RESULTS AND DISCUSSION

The results obtained are presented in Figs. 2–7. Figures 2 and 3 are the neutral amplification curves for the Prandtl numbers 0·72 and 10 respectively. Both figures have been plotted, using for the abscissca the quantity G, which Nachsteim (1963) interprets as a characteristic Reynolds number of the flow. As expected, the flow is increasingly susceptible to the vortex instability at the higher inclination angles. It is interesting that the critical dimensionless wavenumbers appear unaffected by inclination angle, in agreement with the experiments of Sparrow and Husar [1]. At the higher Prandtl number, the critical wavenumber is increased to about 1·17 which is more than a factor of two over its value at the lower Prandtl number of 0·72. The value of G is consistently lower at the higher Prandtl number as would be expected, since increasing Prandtl numbers result in steeper mean temperature gradients. Lee and Lock [10] obtained critical wavenumbers of zero indicating vortices of infinite extent. As mentioned in the introduction, we believe that the discrepancy is due to their use of the parallel flow assumption. At small wavenumbers, the coefficient GV of the first derivatives in equation (3) become increasingly important with respect to the wavenumber. For this reason our curves in Figs. 2 and 3 display definite minima, indicating that the vortices remain of controlled size. Furthermore, our numerical scheme did not exhibit any problems of convergence to the eigenvalue as theirs apparently did.

A comparison of the neutral amplification curve from the present theory for $\varphi = 15°$, and the corresponding curves for the two dimensional wave instability on a vertical heated plate calculated by Nachsteim [7] have been drawn in Fig. 4. The two curves from Nachsteim's work are the result of two different stability calculations, with and without velocity–temperature coupling effects. When temperature fluctuations are taken into consideration, the critical values of G and the wavenumber is sharply reduced. However, both neutral amplification curves of the wave instability have critical G values above that of the vortex instability.

Using Nachsteim's [7] results for the wave disturbances, a curve has been constructed of critical

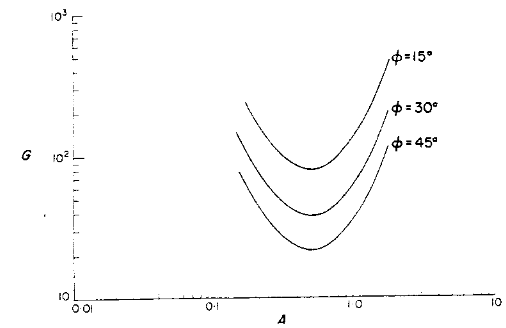

FIG. 2. Neutral stability curves for different inclination angles. Prandtl number = 0·72.

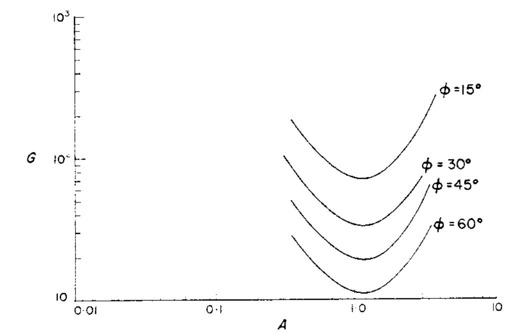

FIG. 3. Neutral stability curves for different inclination angles. Prandtl number = 10·0.

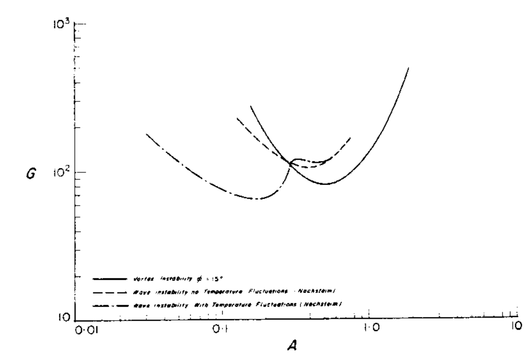

FIG. 4. Comparison of neutral amplification curves for the two and three dimensional instabilities. Prandtl number = 0·72.

Rayleigh number (calculated from $Ra = 4\,Pr([G/4]^4)$ vs inclination angle φ. His results were extrapolated to non-zero inclination angles by multiplying his calculated critical Rayleigh number by the cosine of the inclination angle. For angles up to about 20° this is probably a good approximation since intuitively, one would expect a lower value of the critical Rayleigh number for two dimensional instability. The experiments reported in [2], appear to confirm this.

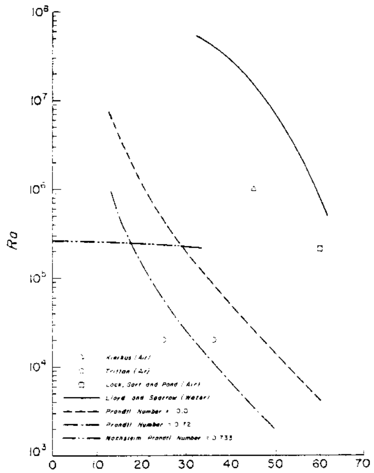

Fig. 5. Comparison between theory and experiment.

The curve obtained is shown in Fig. 5 together with similar curves of the critical Rayleigh number for the vortex mode of instability. Here, the Rayleigh number is based on g, not $g\cos\varphi$. Nachsteim's results were evaluated for a Prandtl number of 0·733. The intersection point of his curve with the $Pr = 0{\cdot}72$ curve of the present calculation should mark the inclination angle at which transition between the wave and vortex modes of instability occurs. The rationale for this conclusion is based on the assumption that to the left of this crossover point, two dimensional waves are amplifying faster than three dimensional vortices. To the right of this point, the vortices would grow at a faster rate than the waves. The actual situation is probably far more complex. For example, an initial arbitrary disturbance may start amplifying as a wave if the critical Rayleigh number for two dimensional waves is first exceeded. Later on however in it's downstream history, the Rayleigh number for longitudinal vortices may be exceeded and if their amplification rate is sufficiently high so as to "overtake" the two dimensional component, then eventually vortices will appear in the experiments. In fact, the experiments of Lloyd and Sparrow [2] show mixed modes of vortices and waves appearing at inclination angles of between 14° to 17°. Since the intersection point referred to earlier occurs at around 17°, it is reasonable to suggest that the essential mechanism is not badly described by the present linear analysis, even though actual quantitative results for the instability Rayleigh numbers differ by orders of magnitude.

Figure 5 also contains the experimental results of Lloyd and Sparrow [2] together with some data of other researchers in the field. The data of Lock, Gort and Pond [19] as well as that of Tritton [15] and Kierkus [14] plotted in comparison contain a large amount of uncertainty as pointed out by Lloyd and Sparrow. For this reason, although their data are in closer agreement with the present calculations than the experiments of Lloyd and Sparrow, the agreement must be considered fortuitous. By far the most reliable data at the present time is Lloyd and Sparrow's and even they obtained standard deviations of 50 per cent in their measurements. The difference between their experimental and the theoretically predicted values is about two orders of magnitude.

Some of the possible reasons for the discrepancy are:

(i) The inability of linear theory to predict eventual finite disturbance growth as opposed to initial instability. The experimental observations are of well developed secondary flows. By the time the vortices are visible, the fluctuations may have amplified a thousand times. Since we have considered perturbations that grow in the streamwise direction and the value of the Rayleigh number has a third power dependence on x, the actual point of first instability corresponds to a much lower value of the Rayleigh number.

(ii) The experimental value of the Rayleigh number is very sensitive to errors made in measuring x. The experiments of Lloyd and Sparrow seem to be consistent and reliable; therefore most of the discrepancy is probably due to (i).

(iii) The use here of boundary layer solutions for a vertical heated plate with the simple $\cos\varphi$ correction to the Grashof number, rather than the solutions for an inclined plate. To assess some idea of this effect, we performed a small number of numerical experiments in which the boundary layer solutions were perturbed slightly towards those calculated by Kierkus [14]. The indications were that this resulted in lower values of critical Rayleigh number, thus increasing the discrepancy between theory and experiment. This points to (i) as being the principal reason for the disagreement, a fact already well known.

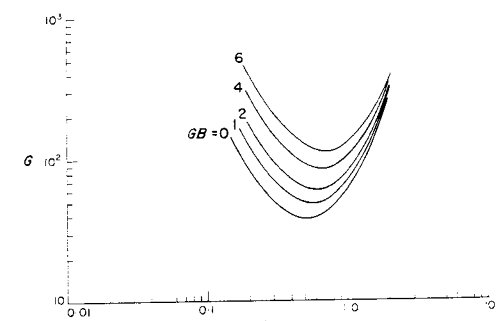

Fig. 6. Amplification curves for $\varphi = 30°$ and $Pr = 0{\cdot}72$.

The amplification curves for the case $Pr = 0{\cdot}72$ and $\varphi = 45°$ are drawn in Fig. 6. Unfortunately, sufficient data on experimentally observed wavenumbers is unavailable at this time to permit a comparison between theory and experiment.

Figure 7 is a sample of the eigenfunctions obtained, arbitrarily normalised to unity. They indicate that the fluctuations do not penetrate beyond the base flow region for that particular case, and are more or less confined to the boundary layer, chiefly because of the mean vertical velocity component directed towards the plate.

5. CONCLUSIONS

A linearized stability analysis of the natural convection flow along inclined plates has been performed, considering three dimensional spatially growing disturbances.

The results indicate that:

(a) The critical Rayleigh number of the vortex instability is lowered as plate inclination is increased.

(b) The critical wavenumbers of the vortices are unaffected by inclination angle and increase with increasing Prandtl number.

(c) The vortex instability dominates the flow beyond an inclination angle of about 17°.

(d) The disturbances appear to be confined to a region within the boundary layer, at the point of first instability.

The results are mostly reinforced by experiment. Quantitative discrepancies may be attributed to the limitations of linear theory in predicting transition, and the uncertainties involved in experimentally determining the first point of instability.

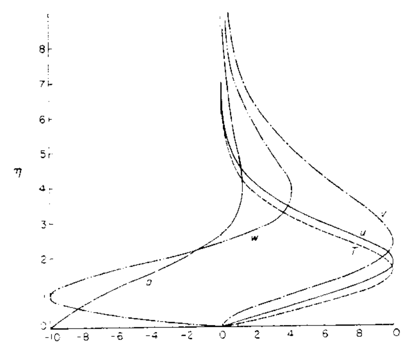

Fig. 7. Normalized eigenfunctions at critical conditions $Pr = 0{\cdot}72$, $\varphi = 45°$.

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L'INSTABILITE DE TYPE TOUBILLONNAIRE DANS LA CONVECTION NATURELLE LE LONG DE PLAQUES INCLINEES

Résumé—On a étudié la stabilité d'un écoulement de convection naturelle le long de plaques inclinées à l'aide des méthodes de perturbation linéaire. L'attention a été portée sur des perturbations tridimensionnelles croissantes dans l'espace et qui prennent la forme de rangées parallèles de tourbillons orientés dans le sens de l'écoulement. On présente les résultats concernant les nombres critiques de Rayleigh en fonction de l'angle d'inclinaison de la plaque. La théorie est en accord qualitatif avec l'expérience. L'angle pour lequel l'instabilité tridimensionnelle devient une instabilité bidimentionnelle ondulatoire est assez bien estimé par la théorie.

INSTABILE WIRBEL BEI FREIER KONVEKTION LÄNGS GENEIGTER PLATTEN

Zusammenfassung—Die Stabilität der Strömung bei freier Konvektion längs geneigter Platten wurde mit Hilfe der linearen Strömungstheorie untersucht. Besondere Aufmerksamkeit wurde auf dreidimensionale, räumlich wachsende Störungen gerichtet, die die Form von parallelen Reihen in Strömungsrichtung orientierter Wirbel annehmen. Als Ergebnis werden die kritischen Rayleigh-Zahlen über den Plattenneigungswinkel angegeben. Die Theorie stimmt qualitativ mit dem Experiment überein. Der Winkel, bei dem die dreidimensionale Instabilität übergeht in eine zweidimensionale wellenförmige Instabilität, wird von der Theorie recht gut vorausgesagt.

ВИХРЕВАЯ НЕУСТОЙЧИВОСТЬ ТЕЧЕНИЯ ПРИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ ВДОЛЬ НАКЛОННОЙ ПЛАСТИНЫ

Аннотация — С помощью методов теории линейных возмущений исследовалась устойчивость течения при естественной конвекции вдоль наклонной пластины. Рассматривались трехмерные возрастающие возмущения, которые принимают вид параллельных рядов вихрей, ориентированных по течению. Результаты представлены в виде зависимости критического числа Релея от угла наклона пластины. Теория качественно согласуется с экспериментом. Теоретически хорошо предсказывается угол, при котором трехмерная неустойчивость переходит в двумерную волновую неустойчивость.