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NUMERICAL AND PHYSICAL SIMULATION OF A STRATIFIED AIRFLOW OVER A SERIES OF HEATED ISLANDS

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ABSTRACT

Perturbations of a stratified shear flow by two identical heated boundaries are investigated both numerically and experimentally. These heated islands may represent a simplified two-dimensional urban complex. A numerical model was constructed by solving a set of two-dimensional, time dependent, and nonlinear governing equations.

The results obtained by wind tunnel and numerical simulation agree. They both simulated the mutual interaction between the islands: the height of the upstream "thermal mountain" was reduced by half as a result of a strong approach wind induced by the downstream heat island, or the downstream one was intensified because of the existence of the upstream one. Commonly noted modifications of meteorological factors by urbanization are also reproduced - such as a temperature cross-over and a downward acceleration of vertical velocity in the surface layer over the upper half of a city. Maximum streamline displacement observed was about 5 cm both in the numerical and in the wind tunnel models. A linearized theory predicted as high as 15 cm.

All computer outputs are demonstrated in a 16 mm movie film. They include the contour line plottings of stream function, vorticity, and temperature. Wind tunnel flow is presented through streamline tracers of smoke.

INTRODUCTION

The climate over cities is quite different from that over the surrounding rural area. Measurements of diurnal variation of temperature in Vienna, Austria [Mitchell[1]*] and that in Frankfurt, Germany [Georgii[2]] are available. The urban station remained warmer most of the time ("urban heat island"). The largest temperature differences were observed at night both in summer and in winter. Maximum and minimum temperature in the cities occurred one or two hours after those in the suburbs. Many other climatic elements such as wind, radiation, humidity, cloudiness, and pollution are also changed by urbanization. Landsberg[3] organized such climatic data into a table to provide a quick understanding of average differences in climatic factors of urban and nonurban regions.

That certain cities have warmer temperature than their surroundings has been known since as early as the beginning of the eighteenth century, "but it was not until the relationships between the cities' heat island and the pathogenic and pernicious effects of air pollution were made evident that the study of this urban phenomenon was stimulated and accelerated" [Kopec[4], p. 602]. A comprehensive review of recent works on the matter are available in Peterson[5] and in a W.M.O. technical note[6].

Since urban heat island effects are most pronounced at night almost all past observers described the nocturnal heat island. Daytime temperature differences have also been observed [Ludwig and Kealoha [7]; Preston-Whyte [8]], but their magnitudes are generally small. Furthermore, measurement difficulties arise since [Kopec[4]]; "daytime attempts to record temperature patterns were frustrated by constant sun-shade changes along the roads traveled, caused by trees, buildings and other roadside obstructions".

Most prominent field experiments were conducted by Duckworth and Sandberg[9], DeMarrais[10], Bornstein [11], and Ludwig and Kealoha[7]. They examined wind and temperature fields over San Francisco, California; Louisville, Kentucky; New York, N.Y.; and Dallas, Texas, respectively. Commonly observed heat island characteristics are listed as follows:

1. Very regular variation of daily temperature over flat unpopulated areas, whereas no generalizations of variation are obtained over urban region;
2. Less frequent occurrence of nocturnal inversion over a city;
3. One or more elevated inversion layers are formed over cities, whereas less frequently over rural regions;
4. Stronger nocturnal urban heat islands are observed in a calm, clear atmosphere;
5. Daytime urban heat islands are less intense than the night counterpart;

* Superior numbers refer to similarly-numbered references at the end of this paper.

6. Intensity of urban heat islands depends on meteorological (wind, stability) and physical (city size) factors;
7. Formation of "cross over" phenomena over cities;
8. Displacement of heat island center windward and
9. Upper limit of a direct effect of urban heat islands extends occasionally up to 1000 m but average height ranges 50 - 400 m.

Rather extensive efforts have been added recently to quantitatively explain the phenomena above [Myrup[12], Tag[13], Olfe and Lee[14], Vukovich[15], Meroney and Yamada[16]].

Similar phenomena to that of urban heat islands have been observed in oceanographic fields. Malkus and Bunker[17] observed periodically-spaced rows of small cumuli leeward of small islands on sunny summer days. This phenomenon is now known as a "heated island" phenomenon [Malkus and Stern[18]]. Wavy air motion at the lee side of an island in a strongly stable stratified airflow is the result of unbalanced buoyancy forces as a result of the temperature difference between the island and over the surrounding ocean. This is a "lee wave" phenomenon as described previously by other authors. Malkus and Stern [18] noted the similarity between the heated island convection and airflow over a physical mountain. The heated island was replaced by an "equivalent mountain" whose shape is a function of the temperature excess of the island over the ocean, stability of the air, wind speed, and eddy diffusivity.

Two-dimensional [Smith[19], Tanouye[20], Estoque and Bhumralker[21], Spelman[22], Neuman and Mahrer[23]], and three-dimensional [Estoque and Bhumralker[24]] numerical models have been developed to study "heated island" effects. Their models reproduced certain characteristics of the phenomena observed in the atmosphere [Malkus and Bunker[17], Garstang et al[25]]. While some contradictory results were reported in the different models: Spelman[22] obtained larger maximum upward vertical velocities by increasing the speed of the basic current. But Tanouye's[20] results were completely opposite, i.e., the maximum vertical velocities decreased with increasing the basic wind speed.

None of the studies mentioned above discussed the mutual interaction between the heated islands.* As shown from observations, the horizontal temperature distribution over urban complexities exhibited several "hot spots" corresponding to dense business areas. Parks, schools, airports, and residential areas gave cooler temperature distribution.

In this paper two-dimensional airflow over two heated islands in a series is investigated numerically and experimentally. Attention is focused on the mutual interaction between the islands.

* Leahey and Friend[26] included implicitly the effect of the first heat island onto the second one.

FORMULATION OF PROBLEM

The formulation of the problem is initially based on the assumptions of two-dimensionality and incompressibility of the fluid. x and z coordinates are to be taken in the direction perpendicular to the islands and vertical, respectively. Then the equation of continuity is written as

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 , \tag{1}$$

where u and w are velocity components in x and z direction, respectively. Vorticity transport equation is as follows

$$\frac{D\zeta}{Dt} = \nu\nabla^2\zeta + \frac{g}{T}\frac{\partial T}{\partial x} , \tag{2}$$

where ζ is a vorticity component in the y direction (which is parallel to the islands)

$$\zeta = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} . \tag{3}$$

T is absolute temperature, and ν is the kinematic viscosity. D/Dt is the Eulerian operator which is expressed as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + w\frac{\partial}{\partial z} .$$

∇^2 is the Laplacian operator in two dimensional space $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial z^2$.

The continuity equation of incompressible fluid (1) permits the existence of a stream function ψ such that

$$u = -\frac{\partial\psi}{\partial z} \text{ and } w = \frac{\partial\psi}{\partial x} . \tag{4}$$

Introduction of a stream function ψ guarantees that the continuity equation (1) is always satisfied. Substituting the stream function into Eq. (3) we obtain a relation between the vorticity ζ and the stream function ψ as

$$\nabla^2\psi = \zeta . \tag{5}$$

The equation of energy in this case is given as

$$\frac{DT}{Dt} = k\nabla^2 T \tag{6}$$

where k is the heat diffusivity.

The set of equations (2), (5) and (6) with the definition of the stream function (4) is to be integrated numerically with appropriate boundary and initial conditions.

NUMERICAL MODEL

The primary difficulty associated with the approximation of a partial differential equation by a finite difference equation is due to the existence of nonlinear inertial terms such $u\,\partial\zeta/\partial x$ or $w\,\partial\zeta/\partial z$. If one uses a forward difference for a time derivative and a centered difference for a space derivative then the difference equation for a differential equation $\partial\zeta/\partial t + u\,\partial\zeta/\partial x = 0$ is unconditionally unstable [Richtmyer and Morton(27), p. 292]. Hence, no matter how small a time step is chosen, small errors introduced in the computation grow without limit.

A solution to this instability has been provided by the upstream finite difference method which replaces convection terms, for example $u\,\partial\zeta/\partial x$, by

$$\left(u\frac{\partial\zeta}{\partial x}\right)^n_{j,\ell} = u^n_{j,\ell}\,\frac{\zeta^n_{j,\ell} - \zeta^n_{j-1,\ell}}{\delta x} \quad \text{when } u^n_{j,\ell} \geq 0 \ ,$$

$$= u^n_{j,\ell}\,\frac{\zeta^n_{j+1,\ell} - \zeta^n_{j,\ell}}{\delta x} \quad \text{when } u^n_{j,\ell} < 0 \ ,$$

where δx is the space increment in x direction. This relation states that when the velocity $u^n_{j,\ell}$ is positive then the space derivative is approximated by a backward difference, and when $u^n_{j,\ell}$ is negative a forward difference is used. Subscript j and ℓ are j^{th} and ℓ^{th} grid points in x and z direction, respectively. In the same manner, the superscript n stands for the n^{th} time step of integration. $n = 1$ is an initial time. The final form of a finite difference approximation of the vorticity transport equation (2) is obtained by replacing the diffusion terms by centered differences while the nonlinear terms are approximated by the upstream ones.

The magnitude of the stream function is obtained by solving the Poisson equation with known vorticity values. Herein a successive over-relaxation (S.O.R.) method was utilized. The convergence criterion of the iteration procedure was

$$|\psi^{r+1} - \psi^r|_{max} < \delta \ .$$

δ should be determined by numerical experiments and here we adjusted δ from 0.01 to 0.10 depending on the magnitude of the stream function at the top boundary. The finite difference expression for the energy equation has a very similar appearance to that of the vorticity transport equation.

The condition which should be satisfied in order to maintain stability in the finite difference expression of the vorticity equation (2) is

$$\delta t \leq \frac{C}{\dfrac{|u|_{max}}{\delta x} + \dfrac{|w|_{max}}{\delta z} + \dfrac{2\nu}{\delta x^2} + \dfrac{2\nu}{\delta z^2}} \ , \qquad (7)$$

where δt is a time increment in the finite-difference equation, and the constant value C is $0<C<1$, and $|u|_{max}$ and $|w|_{max}$ are the magnitudes of the maximum velocity components u and w, respectively, in the entire computation region. The stability criterion is thus a variable depending upon the magnitude of each set of newly calculated velocity components. In practice $|u|_{max}$ and $|w|_{max}$ were calculated at each time step and δt was selected such that it satisfied the stability condition (7).

It is necessary to specify both boundary and initial conditions to obtain a set of unique solutions. Boundary conditions for the vorticity transport equation cannot be given directly. However, they are closely related to interior values of vorticity and stream functions by means of a Taylor s series expansion. For rigid boundaries this relation is very simple and may be derived analytically from the known boundary conditions of velocities and stream functions.

In this study both the upper and the lower boundaries are rigid and a no-slip velocity condition is used; i.e.

$$u = w = 0 \text{ at } z = 0 \text{ and}$$

$$u = \text{const},\ w = 0 \text{ at } z = H \ .$$

where H is the height of the top boundary. The stream function is assigned a zero value along the bottom boundary and a constant value is maintained along the top boundary. The final expression for the boundary values of vorticities along rigid boundaries is

$$\zeta_{bound} = \frac{3}{(\delta z)^2}\,(\psi_{int} - \psi_{bound}) - \tfrac{1}{2}\,\zeta_{int} \ ,$$

where ζ_{bound} and ψ_{bound} are the boundary values of vorticity and stream function, respectively. Subscript "int" indicates the values at one grid inside from the boundary.

Boundary conditions at the up-and down-stream boundaries are more difficult and must be determined more or less empirically. Lateral boundary conditions were sought which imposed the least severe restrictions on the solutions in the interior region, i.e., such that no distorted values at the boundaries propagate into the interior area. The following boundary conditions have been determined from numerical experiments to give the least apparent restrictions and the least distortions. Linear extrapolation formulae were used for all dependent variables ψ, ζ, and T. For example, the upstream boundary values of stream functions, $\psi_{1,\ell}$ are computed from

$$\psi_{1,\ell} = 2\psi_{2,\ell} - \psi_{3,\ell} \ ^{*}$$

where $\psi_{2,\ell}$ and $\psi_{3,\ell}$ are the values one and two grid interior the region, respectively. The physical implication of the above condition is as follows: Streamlines are assumed to change linearly, i.e., maintain constant slopes at the lateral boundaries. Thus,

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^3\psi}{\partial x^3} = \frac{\partial^4\psi}{\partial x^4} = \ldots\ldots = 0 \ .$$

* Linear extrapolation formula was shown to be one of the desirable boundary conditions which did not give appreciable computational instability [Nitta (28)].

In terms of velocity component w this requires $\partial w/\partial x = 0$. However, since this numerical model is formulated in terms of the vorticity equation one needs boundary conditions for vorticities at the lateral boundaries. With the assumption that the stream function varies linearly at the lateral system boundaries one may conclude from

$$\frac{\partial^2 \zeta}{\partial x^2} = \frac{\partial^4 \psi}{\partial x^4} + \frac{\partial^2}{\partial z^2} \left(\frac{\partial^2 \psi}{\partial x^2}\right)$$

that

$$\frac{\partial^2 \zeta}{\partial x^2} = 0 \ .$$

Boundary conditions for the energy equation at both streamwise boundaries are similarly

$$T_{1,\ell} = 2\ T_{2,\ell} - T_{3,\ell},$$

and temperature at the top and the bottom boundaries are specified and kept constant.

To integrate the set of equations described above, initial values must be specified to initialize the numerical integration. Hence, initial velocity components u and w are originally given, to obey continuity and the vorticities and stream functions are initialized by their respective definitions.

Numerical integrations and laboratory experiments have been conducted in such a manner that they may be directly compared. The numerical area was divided by a 81 x 16 square mesh whose dimension is 1 x 1 cm. Therefore, a 15 cm height x 80 cm length area is the computational region.

EXPERIMENTAL FACILITY

A wind tunnel facility was used to augment the numerical model proposed. Dimensions of the test section of the wind tunnel are 2 ft height x 2 ft width x 15 ft length.

To provide a conditioned stratification in the test section, a series of heaters and cooling plates were added to the entrance, ceiling and floor of the tunnel. Sixteen electric heaters 6 x 24 in. were arranged in a grid across the entrance section. Four larger heaters 2 x 3 ft. were adhered to the adjustable ceiling. The floor was constructed from a series of water cooled aluminum ducts. Final tunnel provided thermal gradients as large as 2.5°C/cm and wind speeds from 1 cm/sec to 200 cm/sec.

Copper-constantan thermocouples of 30 gage were utilized to monitor temperature variations. Sixteen thermocouples were mounted on the entrance heaters, four were on the ceiling heaters, and three were along the floor. Nine thermocouples mounted on a rake were used for vertical temperature distribution measurements.

A smoke wire method has been utilized to investigate flow field during thermal stratification. It has been perfected for a practical use at Engineering Research Center, Colorado State University. (See Yamada and Meroney[29] for the details of the experimental facilities.)

RESULTS AND DISCUSSIONS

Experimental Results

The experimental results will be discussed first. The upstream heated island was placed at 200 cm downstream from the beginning of the wind tunnel test section. The second one was located 27 cm downstream from the first one. Both islands had 8 cm width. The origin of the coordinate was taken at the leading edge of the first heated island: x coordinate is in the direction of the basic flow and z coordinate is in the vertical direction. The basic flow characteristics were determined from measurements when no heated islands were placed in the wind tunnel. A Froude number of 0.01 was obtained by means of a basic flow speed of 1.5 cm/sec, and a temperature gradient of 2.2 °C/cm. Wind tunnel height (60 cm) was used as a characteristic height to compute the Froude number.

Streamlines were visible by titanium tetrachloride ($TiCl_4$) smoke. The result is shown in Fig. 1. It is interesting to note how different the streamline displacements are for the two identically heated islands. The upstream heated island (A) had a temperature excess of 34.4°C over the surrounding surface temperature of 294k, while that of the downstream one (B) was 36.3 °C.

A separation of the streamline was observed at about 5 cm downstream from the leading edge of the first island. This suggests a different behavior of streamlines from the prediction proposed by Stern and Malkus[30], where an "equivalent thermal mountain" starts from the leading edge of the heated island. The details will be discussed in the next section of the numerical results.

Numerical Results

A numerical simulation was conducted utilizing the model described previously. It was hoped to see how the observed mutual interactions occurred.

In the numerical model it was found to be necessary to impose slightly different boundary conditions from the observed ones. Because of the finite grid size (1 cm) it was impossible to reproduce exactly the observed temperature distribution near the surface. Temperature jumped from 294k at the surface to 308k at z = 0.6 cm, then increased gradually with an approximately constant gradient. Therefore, the surface temperature used in the numerical model was determined by extrapolating the observed temperature gradient above surface to the ground. The value thus obtained was 307.5k. The heated island's temperature of 316k was obtained in a similar manner.

Now the numerical results are discussed. All computer outputs are demonstrated in a 16 mm movie film. They include the contour line plottings of stream function, vorticity, and temperature. Some of the outputs are reproduced here (Fig. 2 to Fig.5) for convenience. The heavy lines along the lower boundary indicate the locations of the heated islands. Integration was conducted over 400 steps (t = 58 sec), but only streamlines were plotted for the entire time. Vorticity and temperature contour line plottings were terminated at t = 33 sec (250 steps). It was assumed that a uniform flow was heated suddenly from the two heated islands at t = 0. It is of interest to note how the basic flow is disturbed and what the effects are of the introduction of a second heated island. For the latter purpose the numerical simulation for single island is included (see Fig. 5).

Streamlines developed identically over the two heated islands up to t = 4 sec (Fig. 2). A separation occurred at about 2 cm downstream from the leading edge of the island (an interval between the tick marks in the Figures corresponds to 1 cm). However, a slight difference in the streamline patterns had started at t = 7 sec and it developed quickly with time. It was observed that the perturbations reached their maxima at about t = 30 sec (see Fig. 3). Then the heights of the thermal mountains decreased before they attained maximum values again. The period observed was about 6 sec.

Distinguishable differences are apparent between the streamline displacements over the upstream and downstream heated islands. The maximum height of the first streamline from the surface is 2.8 cm over the first island, while a height of 4 cm is obtained over the second island. These results agree closely with the experimentally observed values of 2.5 cm and 5 cm, respectively. It is noted that the separation points are displaced considerably downstream: the first one is located at about 6 cm from the leading edge of the first island, while the separation point on the second island is at about 5 cm. The commonly observed downward vertical velocity is also reproduced in the surface layer over the island. Acceleration of the horizontal velocity is suggested from the narrow intervals of the streamlines near the ground over the upstream half of the island. Reverse flow areas were observed in the region downstream of the islands. A maximum of -2cm/sec was computed. The maximum positive velocity was about 3 cm/sec.

Vorticity and temperature contour lines are included in the same Figure which again indicates unequal developments of the flow fields over the two islands. A vortex pair is observed over each island: a counterclockwise circulation in the upper region and a clockwise one in the lower region. It is very helpful to observe the development of vorticity in order to understand the phenomena physically. Initially (t = 0 sec) vortices are formed at both edges of the island since there exist strong temperature gradients. Mathematically this effect is introduced through the term $(g/T)\ (\partial T/\partial x)$ in the vorticity transport equation (2). The upstream vortex accelerates the surface velocity and the downstream one induces a negative flow in the surface region. This mechanism explains why less streamline displacement was observed over the first island: the upstream island decreased the magnitude of the approach flow velocity to the second island. It resulted in stronger developments of the streamline displacements over the second island than those without the upstream island. Then a strong vortex over the second island accelerated the upstream flow which reduced the thermal effects of the first island. If the height of the first thermal mountain is decreased, then that of the second one is also decreased since a reduced effect of the first island would be expected. This reduction is followed by an increase of the streamline displacements over the upstream island, followed by the same result over the downstream one. The periodical behavior was observed from the streamline outputs: maximum developments are recorded at t = 30, 36, 42, 48, and 54 sec. As seen in Fig. 4, streamlines at t = 36 and 54 sec are quite similar.

The last Figure (Fig. 5) shows the results obtained for a single island, where only the first island in Fig. 3 was retained. From the streamline output it is clearly observed that the island introduced a large low velocity area downstream the island, which increased the thermal effect of the second island as discussed above. Isotherms in the same Figure show a weak "cross-over" effect over the island, where the contour lines are displaced upwards.

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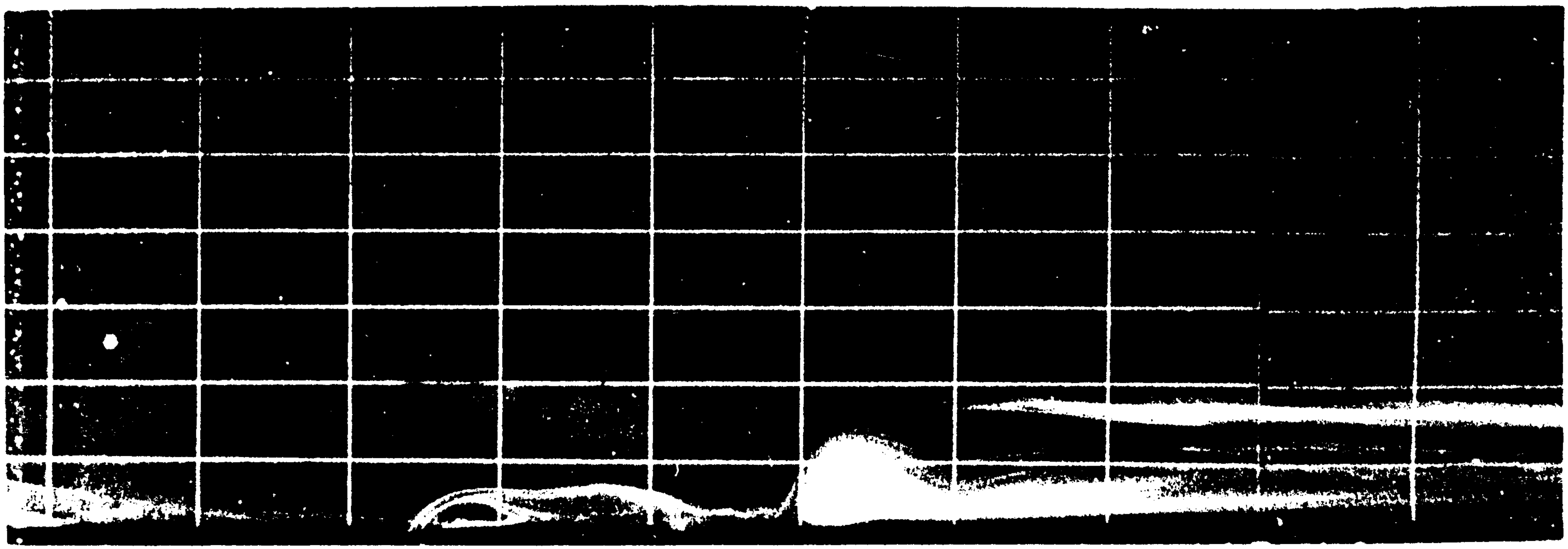

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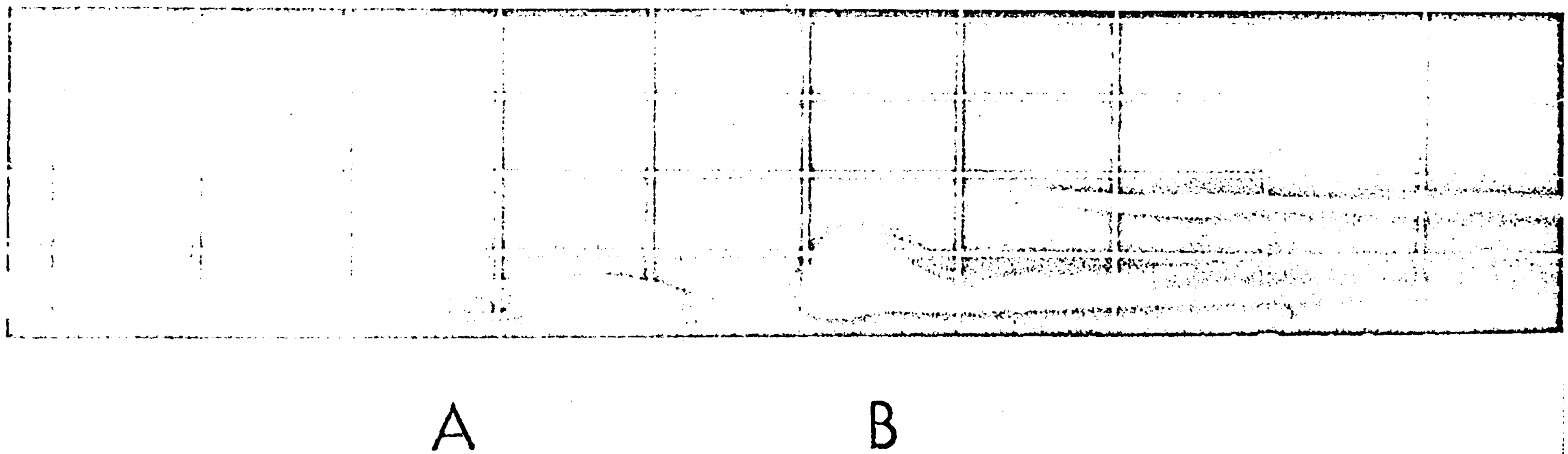

Fig. 1. Flow visualization by $TiCl_4$ smoke over the heated islands.

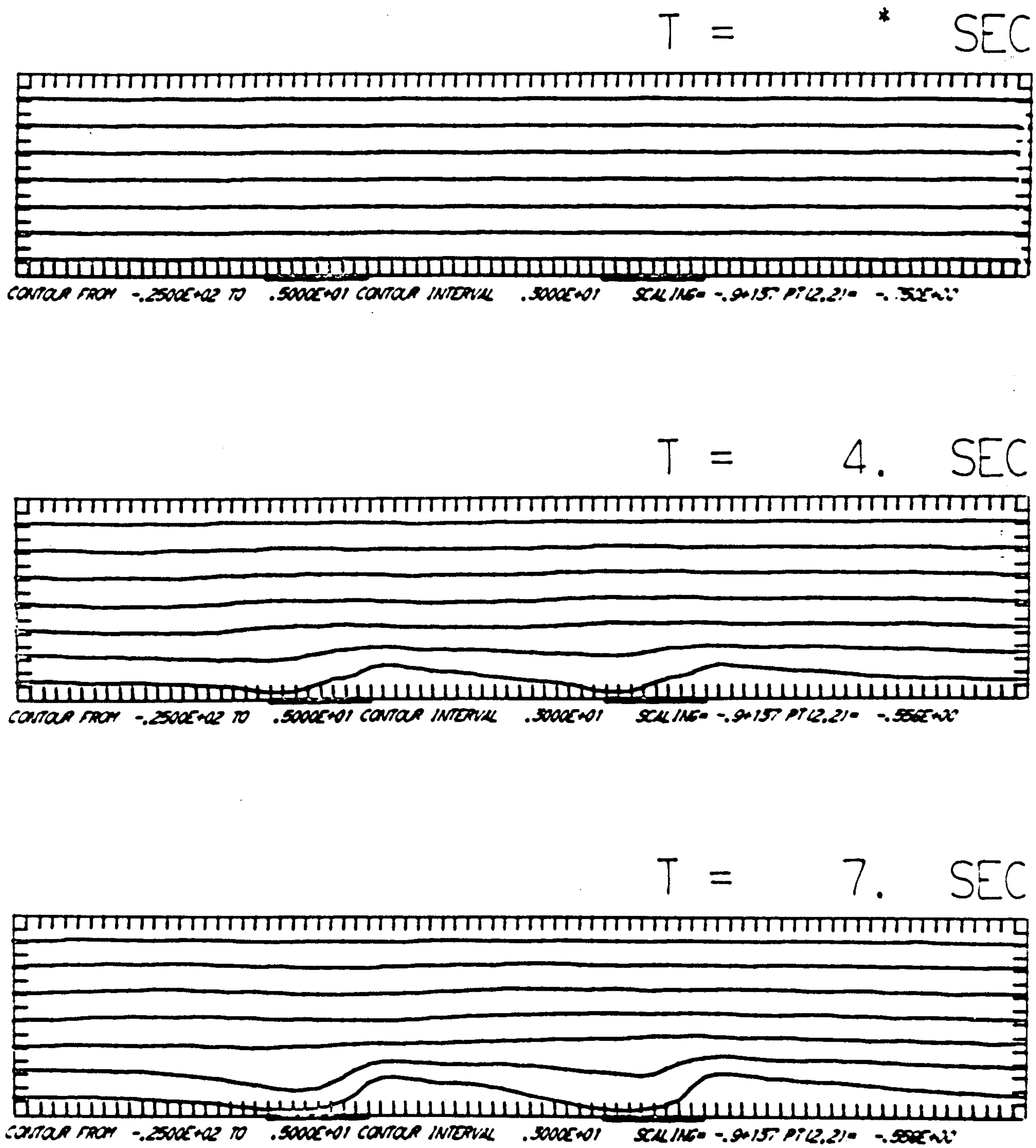

Fig. 2. Numerically obtained streamlines at t = 0, 4, and 7 sec. Heated islands are indicated by heavy lines from x = 0 to 8 cm and from x = 27 to 35 cm.

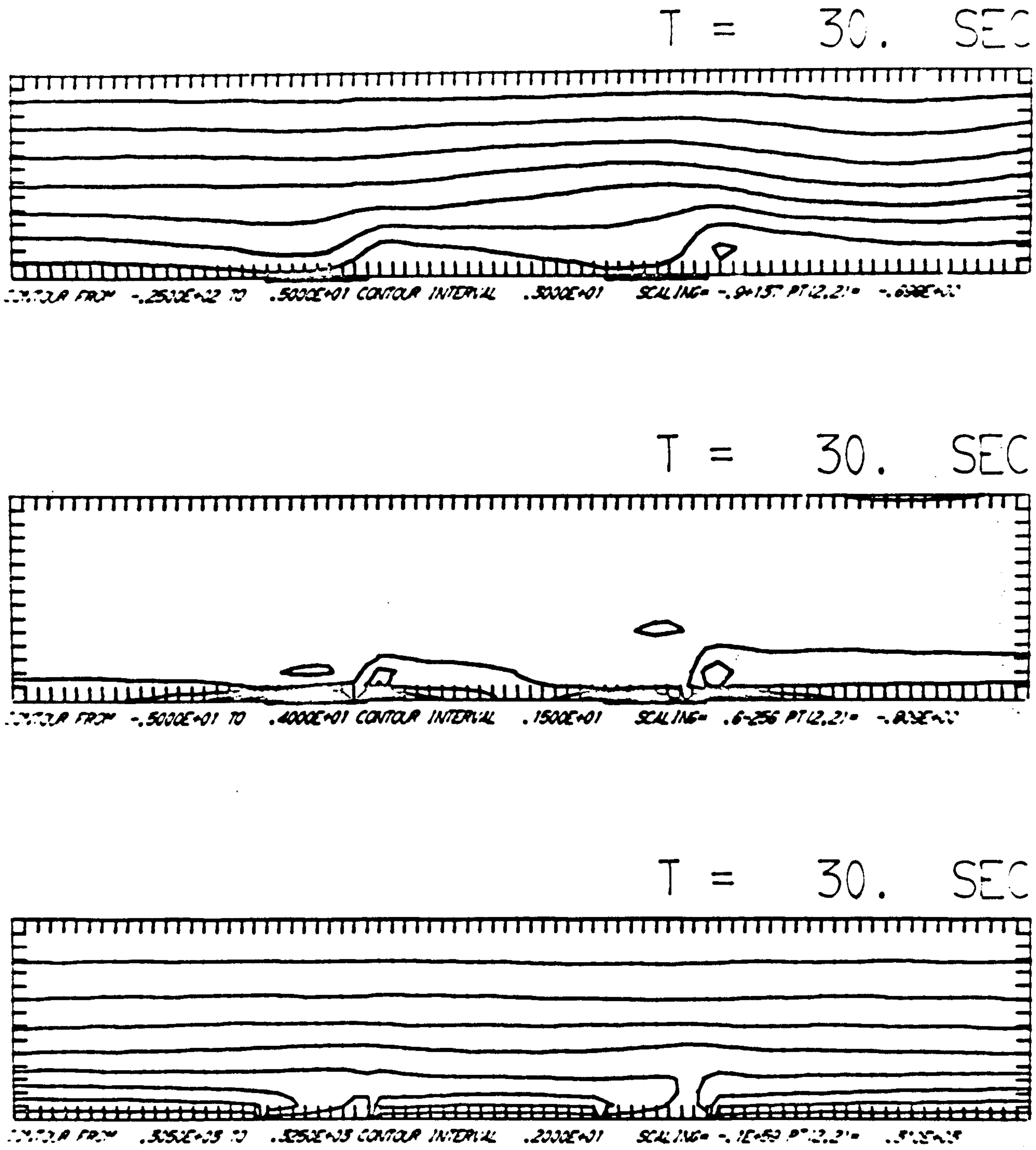

Fig. 3. Numerically obtained contour lines of stream function, vorticity, and temperature at t = 30 sec.

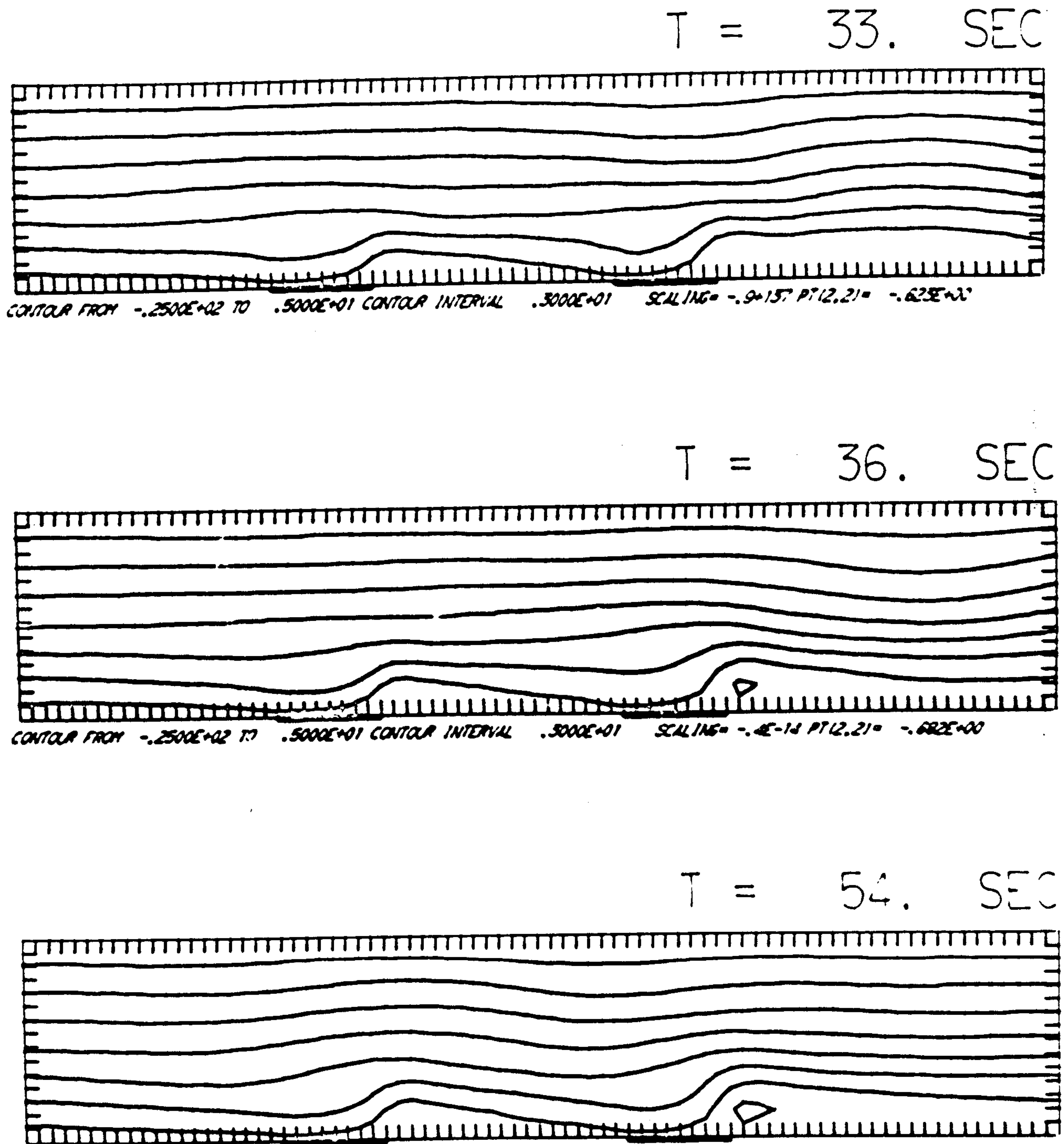

Fig. 4. Same as in Fig. 2 but at t = 33, 36, and 54 sec.

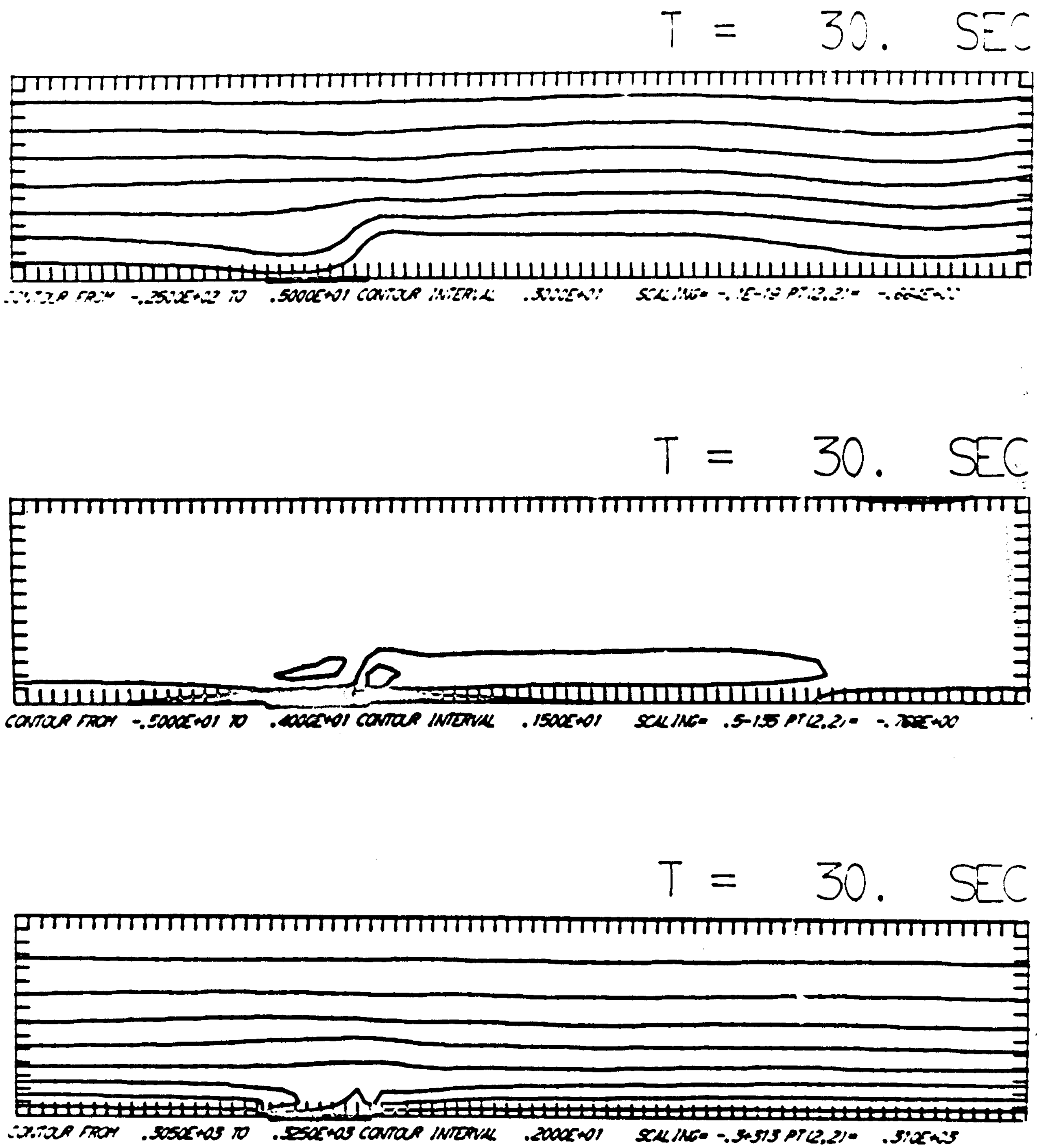

Fig. 5. Same as in Fig. 3 but for single island located from x = 0 to 8 cm.