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VELOCITY AND SHEAR DISTRIBUTIONS IN A TRANSPIRED TURBULENT BOUNDARY LAYER

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A system of equations is derived to describe the velocity and shear stress distributions in a constant property turbulent boundary layer with transpiration. The velocity near the wall was expressed in the form of a Taylor series. This was combined with a semi-logarithmic relation (including a transpiration parameter) for the turbulent core region. Matching is accomplished by requiring agreement between the velocities and velocity gradients and is thus independent of the concept of a laminar sub-layer thickness. These two equations provide a smooth and continuous curve from the wall to the free stream which shows good agreement with existing data.

Derivation of an expression for the shear stress through the boundary layer was accomplished by integration of the momentum equation. The result compares well with available data in the turbulent core region. Deviations near the wall are attributed to inaccuracies of measurements in this region.

INTRODUCTION

The fundamental equations which describe the fluid dynamics of the turbulent boundary layer--the continuity equation, the momentum equation, and the energy equation developed from the Navier-Stokes equations--have not been solved for even the simplest case because of their complexity. Our knowledge of the mechanism of the turbulence and its transport processes in inadequate as a basis for a theory that would be sound and complete. Present concepts of time-mean velocity and scalar quantity distributions and the corresponding resistance coefficients are still semi-empirical in nature. The few experimental investigations of the distribution of scalar quantities during wall turbulence over an impervious surface are restricted to the transport of heat and the measurement of mean temperatures. No data are available on turbulent quantities of temperature fluctuations; thus any insight into this transport can only be based on some assumed analogy between the transport of a scalar quantity and the transport of momentum. This lack of definition is directly related to the current understanding of the transfer mechanisms for mass infection into a turbulent boundary layer flow. Much of the pervious work on the transpired turbulent boundary layer has been concentrated on extending to the case of transpiration the semi-empirical laws which have been found for the transport in the boundary layer on the impervious surface.

A review of recent studies of the transfer mechanisms which govern mass injection into turbulent flow emphasizes the need for a firmer foundation to undergird efforts to analyze transport rates during mass transfer to a turbulent boundary layer. This paper summarizes relations which successfully describe the velocity and shear distributions in a turbulent boundary layer in the presence of mass transfer to the gas stream.

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NOMENCLATURE

a, b	constants - see equation (13)
$Cf/2$	skin friction coefficient
g	constant - see equation (16)
Re	Reynolds number, $Re = u_e x/\nu$
u', v', w',	velocity fluctuations parallel, perpendicular, and across plat
u, v	mean velocity parallel and perpendicular to plate
u_τ	scale velocity based on wall shear stress, $u_\tau = \sqrt{\tau_w/\rho}$
u_τ^*	scale velocity based on maximum shear stress, $u_\tau^* = \sqrt{\tau_{max}}$
u^+	dimensionless velocity, $u^+ = u/u_\tau$
x	distance along plate
y	distance normal to plate
y^+	dimensionless distance, $y^+ = u_\tau y/\nu$

Greek Symbols

δ	boundary layer thickness
μ	absolute viscosity
ν	kinematic viscosity
ξ	y/δ
π	constant - see equation (21)
ρ	density
τ	shear stress

Subscripts

w	wall value
e	free stream value
L	sublayer thickness value

VELOCITY DISTRIBUTION IN A TRANSPIRED TURBULENT BOUNDARY LAYER FLOW

In fully developed turbulent channel flow, a theoretical expression may be developed which gives a continuous and smooth transition for the mean velocity distribution in the wall region to the logarithmic distribution in the fully developed turbulent core.[2] The present paper establishes a theoretical expression for the mean velocity distribution in the boundary layer along an infinite flat plate when transpiration occurs. The expression in the wall region satisfies equations of motion near the wall.

In uniform turbulent flow over an infinite two-dimensional flat plate, changes of the flow variables in the x-direction are negligible near the wall compared to changes in the y-direction. The equations of mean motion for constant property incompressible turbulent flow may then be written as:

$$v_y = 0 \tag{1}$$

$$vu_y = \nu u_{yy} - (\overline{u'v'})_y \tag{2}$$

The fluctuating velocity components must also satisfy the continuity equation:

$$u'_x + v'_y + w'_z = 0 \tag{3}$$

Near the wall the velocities can be expressed in a Taylor series as:

$$S = \Sigma S_n y^n \tag{4}$$

where

$$S_n = \frac{1}{n!}\left(\frac{d^n S}{dy^n}\right)_{y=0} \qquad (S = u',\ v',\ w',\ \text{or } u) \tag{5}$$

From the no-slip condition at the wall and from equation (3): $v'_1 = u'_o = v'_o = w'_o = U_o = 0$ at $y = 0$, and from the mean velocity continuity equation it appears that $v = V_w = \text{const.}$

Substituting equation (4) into equation (2) and collecting coefficients in the corresponding powers of y, one obtains the following equations:

$$y^o: \quad v_w u_1 - 2\nu u_2 = 0 \tag{6a}$$

$$y^1: \quad 2v_w u_2 - 3\cdot 2\cdot \nu u_3 = 0 \tag{6b}$$

$$y^2: \quad 3v_w u_3 - 4\cdot 3\cdot \nu u_4 + 3\overline{u'_1 v'_2} = 0 \tag{6c}$$

$$y^3: \quad 4v_w u_4 - 5\cdot 4\cdot \nu u_5 + 4\overline{u'_2 v'_2} = 0 \tag{6d}$$

Evaluation of equations (6a), (6b), and (4) yields

$$u = u_1 y + \frac{v_w u_1}{2\nu} y^2 + \frac{1}{3!} u_1 \left(\frac{v_w}{\nu}\right)^2 y^3 + u_4 y^4 + \dots \tag{7}$$

and the turbulent shear stress near the wall becomes

$$\overline{u'v'} = \left[4\nu u_4 - \frac{\nu}{3!}\left(\frac{v_w}{\nu}\right)^3 u_1\right] y^3 + \left[5\nu u_5 - v_w u_4\right] y^4 + \ldots \quad (8)$$

The above relations may be placed in a more convenient non-dimensional form by introducing the following definitions:

$$L^+ = (L\, u_{\tau_w}/\nu) \qquad (L = x,\ y,\ \text{or}\ z) \qquad (9a)$$

$$M^+ = (M/u_{\tau_w}) \qquad (M = u,\ v,\ u',\ v',\ w') \qquad (9b)$$

where

$$u_{\tau_w} = \sqrt{\nu u_1}$$

Equations (7) and (8) thus becomes

$$u^+ = y^+ + \frac{1}{2}(v_w^+)y^{+2} + \frac{1}{3!}(v_w^{+2})y^{+3} + U_4^+ y^{+4} + U_5^+ y^{+5} + \ldots \quad (10)$$

$$\overline{u'v'^+} = \left[4U_4^+ - v_w^{+3}/3!\right]y^{+3} + \left[5U_5^+ - v_w^+ U_4^+\right]y^{+4} + \ldots \quad (11)$$

The expansion of u^+ will be truncated after the terms of the fourth order in y^+. This truncation appears reasonable since higher order derivatives of mean velocity are extremely smal' in the region close to the wall. Equations (10) and (11) reduce to expressions found in refer· ence 2 when v_w^+ approaches zero.

In order to calculate coefficient U_4^+ and the value of y^+ at which a smooth and continuous transition to an expression for the turbulent core velocity distribution occurs, the value of u^+ and its first derivative, u_y^+, from equation (10) must be matched with corresponding values given by a turbulent core relation.

Recent theoretical proposals by Stevenson[3], Cornish[4], and Smith[5] for the correlation of the velocity profiles in a transpired air into air turbulent boundary layer appear to shed some light on the importance of specific parameters and the structure of the transpired boundary layer in the turbulent core region. Smith suggested that the well known velocity defect law and the law of the wake proposed by Coles[6] is applicable to both transpired and non-transpired boundary layer flows if the scale velocity employed is the friction velocity based on the maximum shear. This would indicate that the outer region of a turbulent boundary layer remains a relatively simple region which "floats" on a complicated substrate even with blowing. Smith reasoned that the force affecting the outer flow should be the force at its inner edge. For the transpired boundary layer, this force can be represented by the broad maximum shear stress which exists in the region of $y/\delta = 0.1$. Smith proposed the following velocity defect law:

$$\frac{u_e - u}{u_\tau^*} = -\frac{1}{K}\ln\frac{y}{\delta} + \frac{\pi(x)}{K}\left[2 - \omega(y/\delta)\right] = F(y/\delta) \quad (12)$$

where $F(y/\delta)$ is equivalent to Coles' velocity defect expression. Fraser suggests the following correlation to determine the value of the required scale velocity,[7,18]

$$\frac{u_\tau^*}{u_\tau} = a + b\left(1 + \frac{v_w u_e}{u_\tau^2}\right)^{1/2} \quad (13)$$

Stevenson, Cornish, and this author have extended Prandtl's mixing length theory to the case of transpiration. All derivations are essentially similar, and the differences depend upon the choice of integration constants and the final algebraic form chosen to display the results. Stevenson chose to express the equation for the outer turbulent boundary layer region with transpiration as

$$\frac{2u_\tau}{v_w}\left\{\left(1+\frac{v_w u_e}{u_\tau^2}\right)^{1/2} - \left(1+\frac{v_w u}{u_\tau^2}\right)^{1/2}\right\} = F(y/\delta) \tag{14}$$

Fraser correlated his velocity profile data by the methods of both Smith and Stevenson and indicated both gave satisfactory representation.[7] Some authors have been unable to correlate suction data with the relation suggested by Smith.[3] The expressions available to calculate the maximum shear stress necessary for Smith's equation (12) are approximate and are based on only a small amount of experimental information.

The final relation suggested by the present author for representing the velocity defect profile is derived as follows. According to the concepts of Prandtl, the momentum equation in the turbulent core region of flow where the comparative effect of microscopic transports is small may be expressed from equation (2) as [8]

$$vu_y = (\ell^2 u_y^2)_y \tag{15}$$

where the mixing length for higher Reynolds numbers is of the form[9]

$$\ell = g(v_w^+)y \tag{16}$$

Introduction of the nondimensional groupings of equation (9a and b) and integration produces from equation (15):

$$g^2 y^{+2}(u^+)^2_{y+} + u^+_{y+}/(u^+_{y+})_w - 1$$

$$v_w^+ u^+ = g^2 y^{+2}(u^+)_y^2 + u_y^+/(u_y^+)_w - 1 \tag{17}$$

In the turbulent core region the third term of equation (17) is negligible. Integration of equation (17) with respect to y^+ and introduction of boundary conditions at the edge of the boundary layer reveals:

$$u^+ = \frac{v_w^+}{4g^2}\ln^2\frac{y^+}{\delta^+} + \frac{1}{g}\left(1 + v_w^+ u_e^+\right)^{1/2}\ln\frac{y^+}{\delta^+} + u_e^+ \tag{18}$$

when v_w^+ approaches zero equation (18) becomes

$$u^+ = \frac{1}{g}\ln\frac{y^+}{\delta^+} + u_e^+ \tag{19}$$

which is a form of von Karman's logarithmic velocity distribution.[10]

Equation (18) is similar to the expression found by Clarke, Menkes, and Libby[11] which was of the form

$$u^+ = A + B\ln y^+ + \frac{v_w^+}{4g^2}\ln^2 y^+ \tag{20}$$

where A, B, and g are determined to be functions of v_w^+ by dimensional reasoning. The parameters A, B, and g were chosen to conform with typical values for non-transpiration theory. In reference 12 an experimental study was made of the effect of the blowing parameter v_w^+ on the parameter g. The author found that for blowing rates less than $v_w/u_e \leq 0.005$ the parameter g appeared to be independent of v_w^+. For this reason the value $g = K = 0.4$ will be accepted here.

Comparison of values for u^+ versus y^+ obtained from equation (18) with experimental results reveals a strong variation from measured values. Examination of the relationships involved revealed that this difference is due to the non-logarithmic variation of the empirical data in the far turbulent core where $y/\delta > 0.15$. This phenomenon has been previously observed for the non-transpiration case, and numerous authors have commented on the phenomenon and suggested empirical and semi-empirical corrections.[6,10,13]

Coles noticed the similarity between the flow in the turbulent core region and wake flow. He proposed a purely empirical correction function $\omega(\xi)$ which has a universal character for all non-blowing wall shear flows.[13] Coles suggested that the velocity distribution be written,

$$u^+ = \frac{1}{g} \ln y^+ + B + \frac{\pi}{g} \omega(\xi) \tag{21}$$

where B is the typical constant of van Karman, i.e., approximately 4.9 to 5.3, and π is a profile parameter which is related to the free stream pressure gradient. Coles suggested a value of $\pi = 0.55$ (assuming $g = 0.4$ and $B = 5.1$). The actual wake function may be approximated by the antisymmetrical function

$$\omega(\xi) = 1 + \sin \frac{(2\xi - 1)\pi}{2} \tag{22}$$

It is now proposed to apply Cole's Law of the Wake correction to equation (18) and to establish the variation of π with v_w^+ by comparison with experimental data. Hence, in the form of a velocity defect law, equation (18) becomes

$$u_e^+ - u^+ = -\frac{v_w^+}{4g^2} \ln^2 \frac{y^+}{\delta^+} - \frac{1}{g}(1 + v_w^+ u_e^+)^{1/2} \ln \frac{y^+}{\delta^+}$$

$$+ \frac{\pi^*(v_w^+)}{g} \left[2 - \omega(y^+/\delta^+)\right] \tag{23}$$

where $\pi^*(v_w^+)$ varies with the blowing parameter.

As previously noted, several authors have developed expressions for the transpired turbulent boundary layer velocity profile.[5,7,14-17] Most of these expressions are an extension of Prandtl's momentum transfer theory with various corrections and assumptions for the integration constants. The theories of Goodwin[15] and Rubesin[14] depend on empirical knowledge of the laminar sublayer thickness. No satisfactory correlation for this quantity, which varies with x and v_w, has been found which will fit the data of all investigators.[7] Smith proposes the use of a correlation parameter based on the maximum total shear stress.[5,17]

In connection with the development of his theory for the transpired turbulent boundary layer, Goodwin suggested the following correlation for the constant $\pi^*(v_w^+)$,

$$\frac{\pi^*}{g} = \frac{\pi}{g} (d\theta/dx)^{1/2} \sqrt{2/Cf} \tag{24}$$

After considering the momentum integral equation with injection this may be written,

$$\frac{\pi^*}{g} = \frac{\pi}{g}(1 + v_w^+ u_e^+)^{1/2} \tag{25}$$

Comparison of this expression with various experimental data reveals satisfactory agreement.[5, 7, 9, 15]

Substitution of equation (25) into equation (23) provides finally,

$$u_e^+ - u^+ = -\frac{v_w^+}{4g^2} \ln^2 \frac{y^+}{\delta^+} - \frac{1}{g}(1 + v_w^+ u_e^+)^{1/2} \ln \frac{y^+}{\delta^+}$$

$$+ \frac{\pi}{g}(1 + v_w^+ u_e^+)^{1/2} \left[2 - \omega(y^+/\delta^+)\right] \tag{26}$$

Equation (26) was derived in the manner described prior to the appearance of equation (14). By rearrangement, equation (14) can be written in terms of $u_e^+ - u^+$. The small order terms which are neglected in equation (26) do not appear to increase the accuracy of the results, and the simplified form has the important advantage of being easier to match with equation (10). Figure 1 compares equation (26) with various data from reference 9.

To match the velocity distribution in the turbulent core region, equation (26), with that close to the wall, equation (10), we divide equation (26) into two equations,

$$u^+ = \frac{v_w^+}{4g^2} (\ln^2 y^+ - 2 \ln y^+ \ln \delta^+) + \frac{1}{g}(1 + v_w^+ u_e^+)^{1/2} \ln y^+$$

$$+ \frac{\pi}{g} (1 + v_w^+ u_e^+)^{1/2} \; \omega(\xi) + B^* \tag{27}$$

and

$$u_e^+ = -\frac{v_w^+}{4g^2} \ln^2 \delta^+ + \frac{1}{g}(1 + v_w^+ u_e^+)^{1/2} \ln \delta^+$$

$$+ 2\frac{\pi}{g} (1 + v_w^+ u_e^+)^{1/2} + B^* \tag{28}$$

where B^* is a function of v_w^+.[3] At present, experimental data are not adequate to provide a clear picture of this functional relationship. Hinze remarks that for non-transpired boundaries B^* is related to the laminar sublayer thickness by[6]

$$B^* = \frac{1}{g} (\ln 4g - 1) + \delta_L^+ \tag{29}$$

The variation of the laminar sublayer with blowing has been suggested by Rubesin to be[14]

$$\delta_L^+ = \frac{1}{v_w^+} \ln (1 + Z\, v_w^+) \tag{30}$$

where Z is set equal to 6.70 herein.

To obtain the complete transpired velocity profile for a specific skin friction, blowing rate, and boundary layer thickness, the values of u^+ and its first derivative u_y^+ are calculated from equations (10) and (27). The equivalent terms from the above expressions are set equal, and the constant U_4^+ and the matching point y_m^+ are determined algebraically. These steps have been carried out for a wide range of blowing rates and wall skin friction conditions on a computer.

Functional relationships which appear to fit the results well are

$$y_m^+ = 15.67 - 860\,(v_w^+)^{1/2} / (u_e^+)^2 - 11.4\,(v_w^+)^{0.45}$$

$$U_4^+ = -5.4 \times 10^{-5} - 1.3(v_w^+)^{1.7} / (u_e^+)^2 - 1.6 \times 10^{-2}\,(v_w^+)^{1.7}$$

Figure 2 is a typical example of matching equation (10) with (27) for the experimental conditions of Mickley, et al.[12]

It should be noted that the expression for the velocity defect law proposed by Stevenson, equation (14), is considered accurate. The derived formulation is considered convenient, however, for matching to the new expression for the law of the wall with injection, equation (10); and useful for shear stress calculations as noted in the following section. Together, equations (10) and (27) provide a continuous velocity distribution from the wall to the free stream. This condition has historically been found to provide more accurate results when the velocity distribution is used to calculate shear stress distributions through the boundary layer or the wall skin friction variation with local Reynolds number.

SHEAR STRESS DISTRIBUTION IN A TRANSPIRED TURBULENT BOUNDARY LAYER

One of the most productive assumptions utilized in conventional non-transpired turbulent boundary layer theory has been that the shear stress is constant across the boundary layer at its wall value. From this assertion have been generated the various Law of the Wall relations of von Karman, Van Driest, Martenelli, and others, and various equations relating skin friction and heat transfer to the local Reynolds number. Even the most casual glance at the transpired turbulent boundary layer problem reveals that the assumption of a constant shear stress is no longer satisfactory. The shear increases from its wall value to a broad maximum and finally drops to zero at the edge of the boundary layer for the transpired case.

Early investigators of the shear stress across the transpired boundary layer thought it adequate to make a simple adjustment for the injection of gas at the wall of the form:

$$\frac{\tau}{\rho} = \frac{\tau_w}{\rho} + v_w u \qquad (31)$$

This relation is only a good assumption to the edge of the laminar sublayer, i.e., approximately $y/\delta = 0.1$; beyond this point the shear stress falls rapidly to zero at the boundary layer outer edge. A more precise shear stress distribution may be generated by the following consideration of the momentum integral equation and the application of the velocity laws derived previously.

For an incompressible constant property and uniform free stream the momentum equation can be expressed as,

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} - \rho\, \overline{v'u'}\right) \qquad (32)$$

Integrating the above relation from 0 to y one obtains

$$\int_0^y 2\tilde{u}\frac{\partial \tilde{u}}{\partial x}dy - \tilde{u}\int_0^y \frac{\partial \tilde{u}}{\partial x}dy + \tilde{v}_w \tilde{u} = \left(\frac{\tau}{\rho u_e^2} - \frac{\tau_w}{\rho u_e^2}\right) \qquad (33)$$

where

$$\tilde{u} = u/u_e$$

$$\tilde{v} = v/u_e$$

$$\frac{\tau}{\rho u_e^2} = \nu \frac{\partial u}{\partial y} - \overline{u'v'}$$

$$v = -\int_0^y \frac{\partial u}{\partial x} dy + v_w$$

If one introduces a skin friction coefficient and a dimensionless coordinate system, then equation (33) becomes

$$\int_0^{y^+} 2u^+\left(\frac{1}{\sqrt{Cf/2}} \frac{\partial u^+}{\partial Re} - u^+ \frac{\partial\sqrt{2/Cf}}{\partial Re}\right) dy^+$$

$$-u^+ \int_0^{y} \left(\frac{1}{\sqrt{Cf/2}} \frac{\partial u^+}{\partial Re} - u^+ \frac{\partial\sqrt{2/Cf}}{\partial Re}\right) dy^+ + v_w^+ u^+ = \frac{\tau}{\tau_w} - 1 \tag{34}$$

where

$$u^+ = \frac{u}{u_e}\sqrt{2/Cf} \quad ; \quad \frac{Cf}{2} = \frac{\tau_w}{\rho u_e^2}$$

$$y^+ = \frac{u_e y}{\nu}\sqrt{Cf/2} \quad ; \quad \frac{1}{\sqrt{Cf/2}} \frac{\partial \tilde{u}}{\partial Re} = \frac{\partial u^+}{\partial Re} - \frac{d\sqrt{2/Cf}}{dRe}(\tilde{u})$$

Now, from experimental data and from consideration of the velocity defect law results, we may assume $\partial u^+/\partial Re = 0$. This assumption is exact near the wall for the zero and finite mass transfer cases when v_w^+ = constant. In the outer region, the assumption implies $\partial u_e^+/\partial Re = 0$, and hence, $Cf/2$ = constant; however, this is reasonable since the general effect of injection might be considered equivalent to a situation where there is such a large roughness that the flow is inertially dominated and the wall shear is constant. In any event, in turbulence analysis the incompleteness of various models or assumptions has not detracted greatly from their practical usefulness which must be judged aposteriori. Hence, equation (34) may be expressed as

$$\frac{\tau}{\tau_w} = 1 - 2\left(\frac{d\sqrt{2/Cf}}{dRe}\right)\int_0^{y^+} (u^+)^2\, dy^+ + u^+\left(\frac{d\sqrt{2/Cf}}{dR}\right)\int_0^{y^+} u^+ dy^+ + v_w^+ u^+$$

When $y^+ = \delta^+$; then $\tau = 0$; therefore,

$$\frac{d\sqrt{2/Cf}}{dRe} = \frac{1 + v_w^+ u_e^+}{2\delta^+ \int_0^1 (u^+)^2 \, d\xi - u_e^+ \delta^+ \int_0^1 u^+ \, d\xi} \tag{36}$$

Inserting equation (36) into equation (35) reveals

$$\frac{\tau}{\tau_w} = (1 + v_w u^+) - (1 + v_w^+ u_e^+) \frac{2\int_0^{y/\delta} (u^+)^2 \, d\xi - u^+ \int_0^{y/\delta} u^+ \, d\xi}{2\int_0^1 (u^+)^2 \, d\xi - u_e^+ \int_0^1 u^+ \, d\xi} \tag{37}$$

Hence, the shear stress may be expressed as a function of the blowing parameter v_w^+, the skin friction $u_e^+ = \sqrt{2/Cf}$, and a boundary layer distance parameter $\xi = y/\delta$.

When the velocity distribution near the wall in the form of equation (10) is introduced into equation (31), the following equation is determined,

$$\frac{\tau}{\tau_w} = 1 + v_w^+ y^+ + \frac{v_w^{+2}}{2!} y^{+2} + \frac{v_w^{+3}}{3!} y^{+3} + v_w^+ U_4^+ y^{+4} + \ldots \tag{38}$$

This expression is, of course, only true near the wall surface. The insertion of equation (26) into equation (37) yields the shear stress distribution in the regions from the edge of the laminar sublayer to the outer edge of the boundary layer. These numerical steps have been carried through for one of the velocity profiles of Mickley and Davis,[9] and the results are compared with the experimental data in figure 3. The conditions of the analytical and experimental results are only approximately the same since the experimental data are for $\frac{Cf}{2} = 0.00062$ and the analytical curve is for $\frac{Cf}{2} = .00060$. Nevertheless, the data agree well in the turbulent core region from $y/\delta = 0.1$ to 1.0. The difficulty encountered in measuring the velocity profile in the laminar sublayer produces questionable experimental results in that region; hence, discrepancies which occur for $y/\delta < 0.1$ are not decisive.

Precise measurements of shear stress through a transpired turbulent boundary layer have recently have reported by Smith, Goodwin, and Fraser.[5,7,15] Their data are representative, however, of only the outer turbulent boundary due to a three-dimensional tunnel flow defect near the wall. Figure 4 compares the results of equations (37) and (26) with some data of Fraser.[7] A series of curves for a given skin friction but several different blowing rates is also displayed on figure 6.

SUMMARY

On the basis that any substantial improvement in current design formulations for heat transfer and skin friction from a turbulent boundary layer in the presence of mass transfer will depend on a better picture of the velocity and shear distributions near the wall, the structure of the transpired turbulent boundary layer has been considered. A semi-empirical expression has been developed for the velocity profile in a transpired turbulent boundary layer which possesses a smooth, continuous distribution from the wall to the free steram. In the wall region the new formulation also satisfies the equation of motion near the wall--a quality not necessarily true of many of the law of the wall expressions previously proposed for non-transpired flows.

A more precise shear stress distribution has been obtained by the introduction of the derived velocity laws into an expression determined from the momentum integral equation including mass transfer. The agreement between the curves so obtained and the sparse experimental data available on the shear stress distributions through transpired turbulent boundary layers is considered very good.

ACKNOWLEDGMENTS

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LIST OF FIGURES

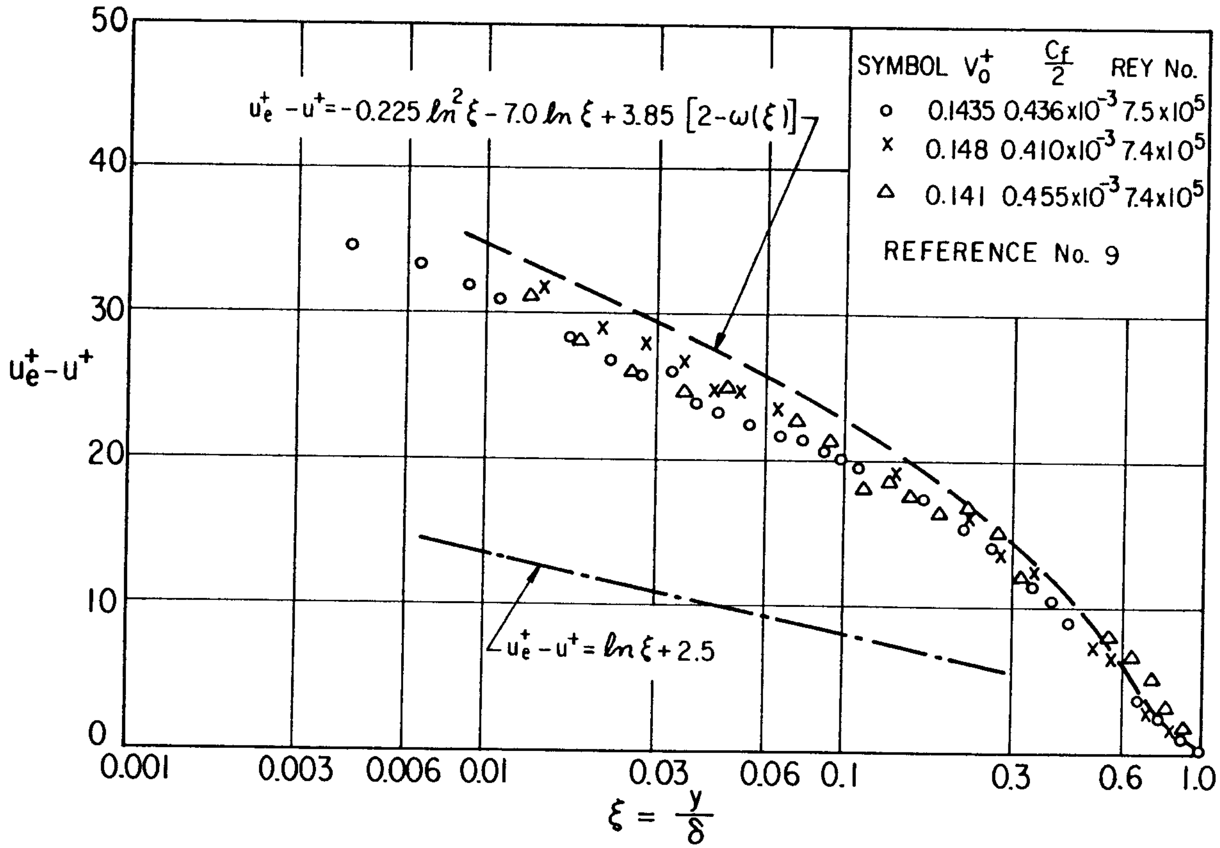


FIG. 1 VELOCITY DEFECT LAW

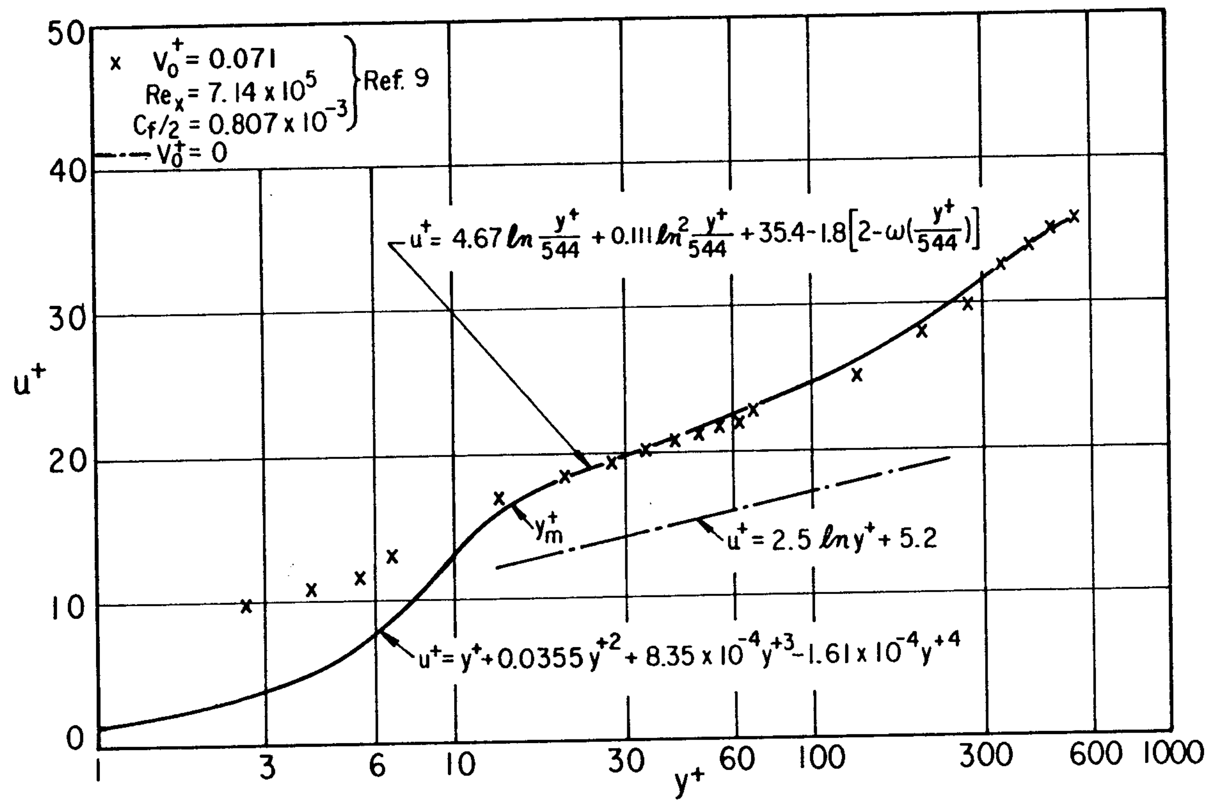
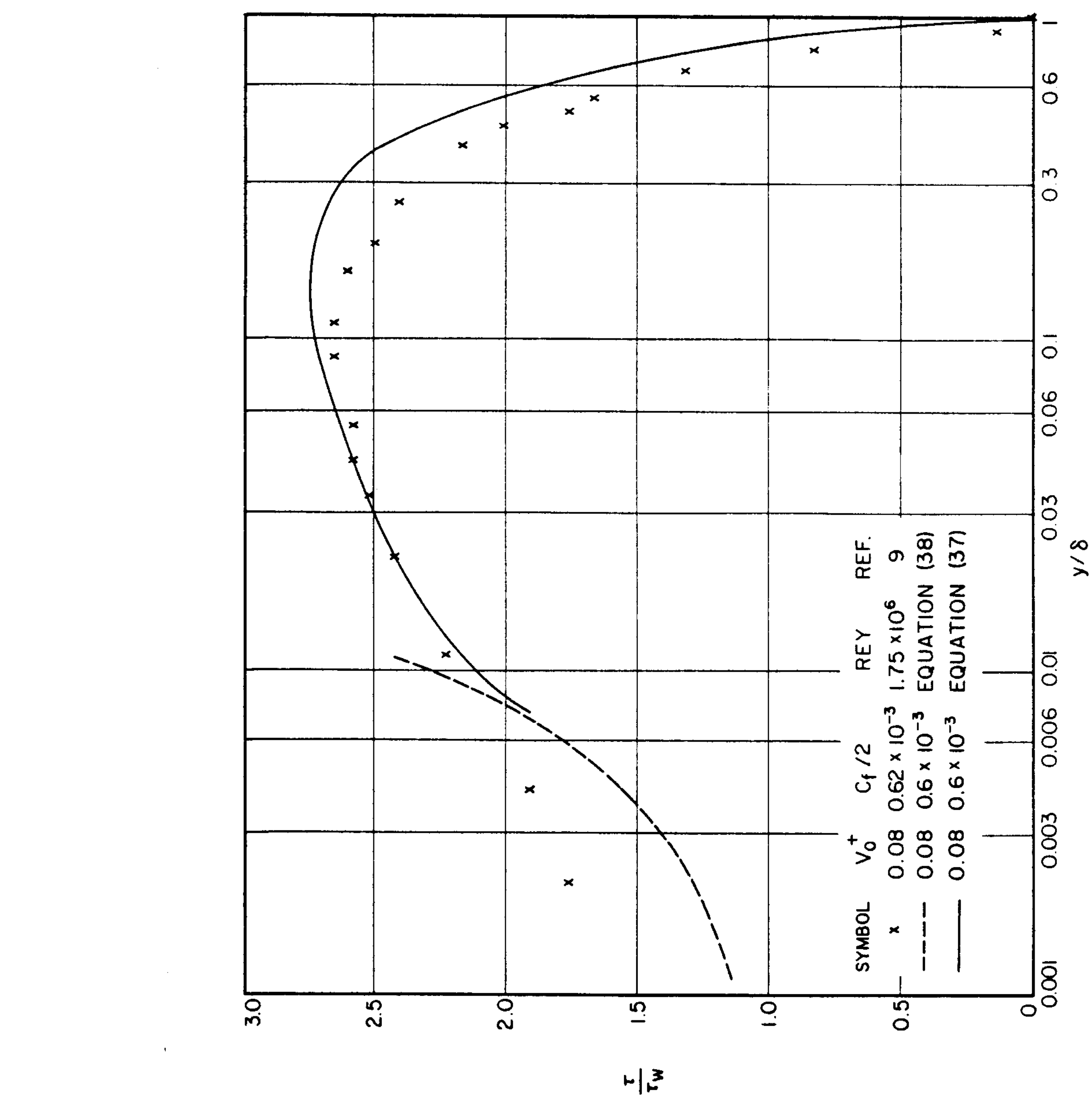


FIG. 2 VELOCITY PROFILE WITH TRANSPIRATION



τ/τw
3.0
2.5
2.0
1.5
1.0
0.5
0
0.001
0.003
0.006
0.01
0.03
0.06
0.1
0.3
0.6
1
y/δ
SYMBOL V$_0^+$ C$_f$/2 REY REF.
x 0.08 0.62 x 10^{-3} 1.75 x 10^6 9
— — — 0.08 0.6 x 10^{-3} EQUATION (38)
——— 0.08 0.6 x 10^{-3} EQUATION (37)

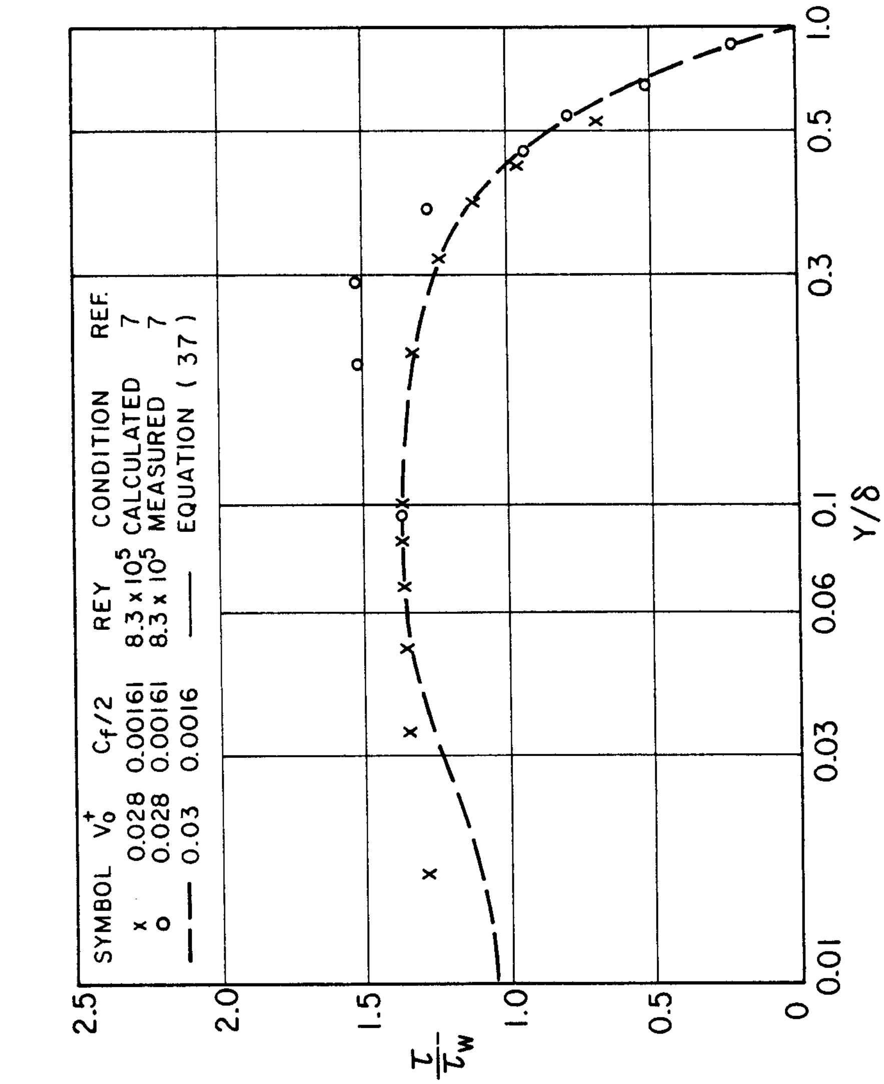


FIG. 4 SHEAR STRESS DISTRIBUTION

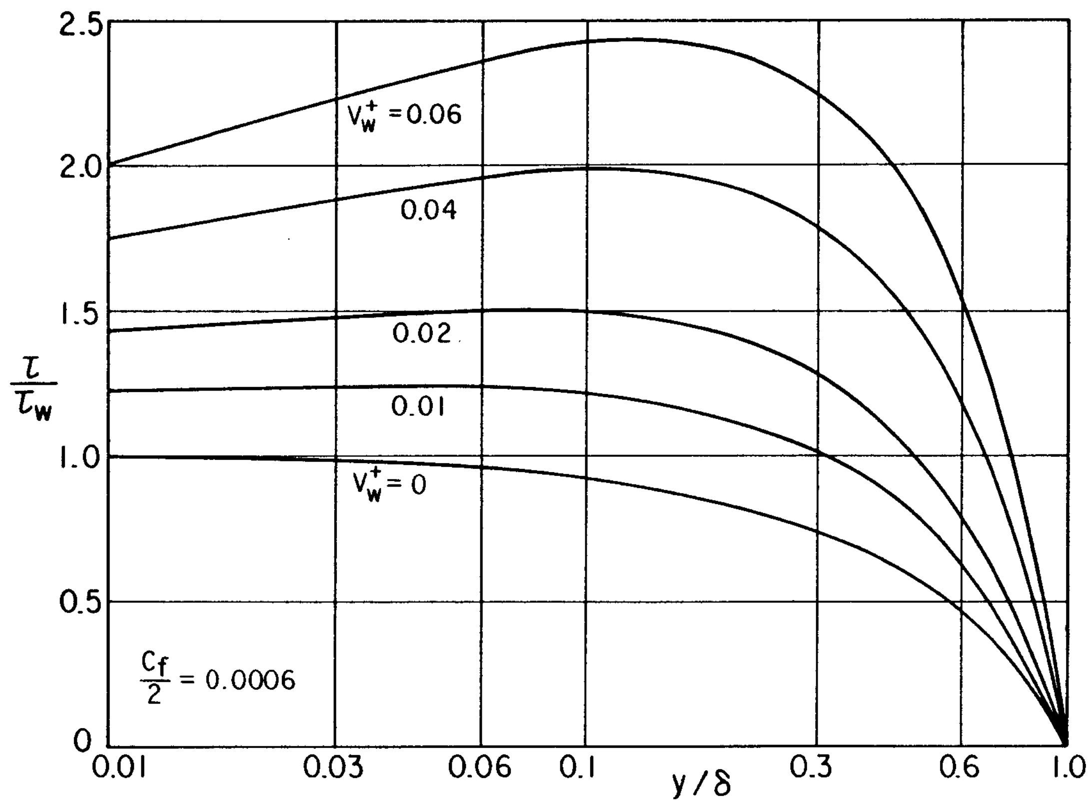
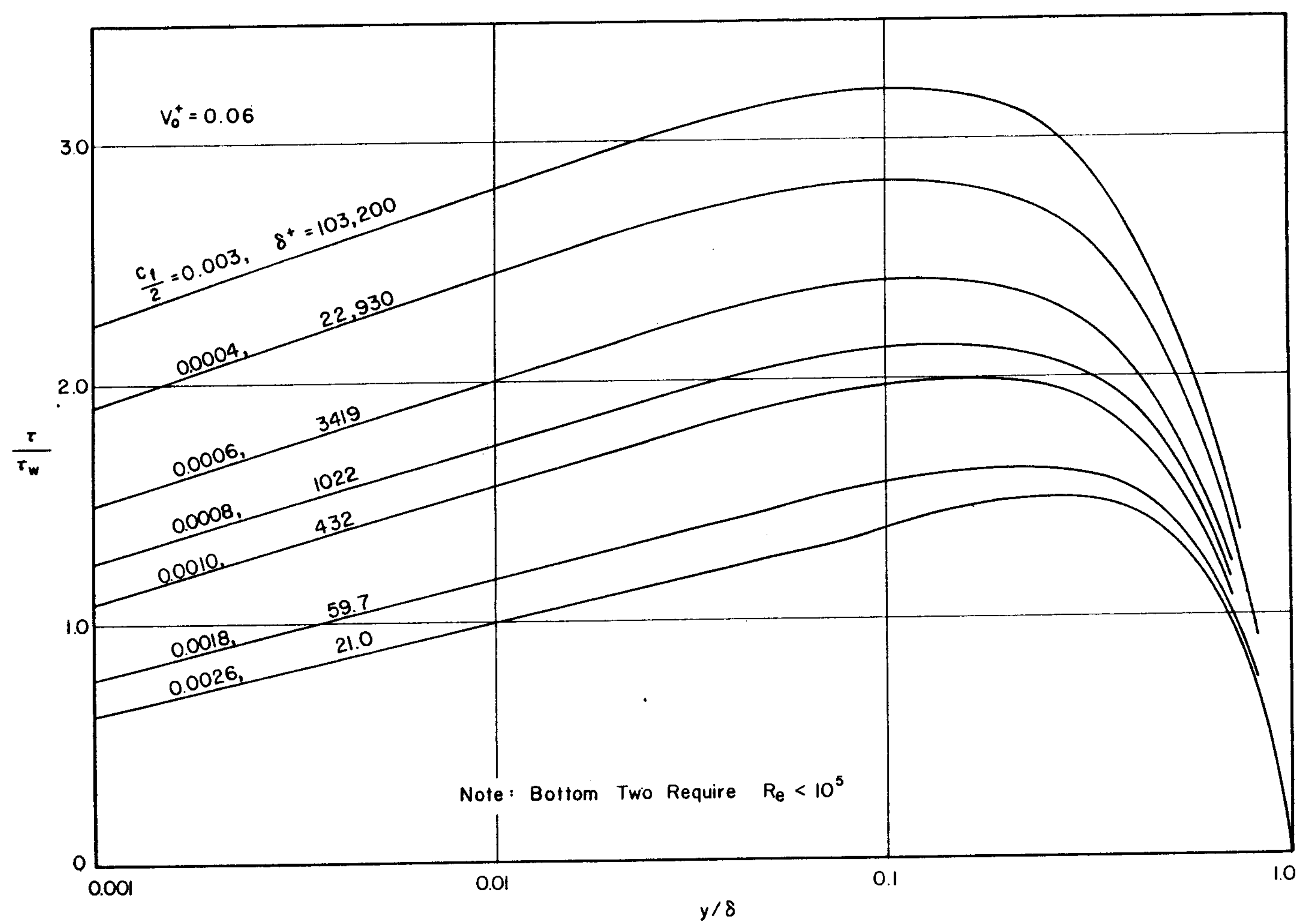


FIG. 5 SHEAR STRESS AT VARIOUS BLOWING RATES



V₀⁺ = 0.06
c_f/2 = 0.003, δ⁺ = 103,200
0.0004, 22,930
0.0006, 3419
0.0008, 1022
0.0010, 432
0.0018, 59.7
0.0026, 21.0
Note: Bottom Two Require Rₑ < 10⁵
3.0
2.0
1.0
0
τ/τw
0.001
0.01
0.1
1.0
y/δ

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