REYNOLDS NUMBER INDEPENDENCE OF THE WIND-TUNNEL SIMULATION OF TRANSPORT AND DISPERSION ABOUT BUILDINGS

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ABSTRACT

Fluid modeling of atmospheric scale flows in wind tunnels requires the presumption that properly scaled fluid motions, loads and mixing rates are independent of the ratio of inertial to viscous forces characterized by the Reynolds number, $Re = UH/\nu$. Such Reynolds number independence has been identified as a precondition for realistic simulation of transport and dispersion about buildings. Wind tunnel experiments were performed of dispersion in a boundary-layer meteorological wind tunnel over a range of wind speeds (12 conditions), building orientations ($0^\circ$ - $30^\circ$), model scales (1:25 & 1:50), and source release locations (3 sites). Measurements suggest that, given the complexities of roof source location, orientation, and intermittent reattachment of streamlines on the building surface, flow conditions should be set such that $Re > 15,000$ to assure Reynolds number independence of dimensionless concentration values on the building surface and in the building wake region.

INTRODUCTION

The flow field and concentration distribution over a model building in a wind tunnel will depend upon turbulence intensities, turbulent scales, and the separation and reattachment of streamlines from the building surfaces. These properties are known to vary with the dimensionless parameter called the Reynolds number, $Re = UH/\nu$. The Reynolds number is said by Snyder (1981) to be “the most abused criterion in models of atmospheric flows.” Scale reductions commonly result in model values three to four orders of magnitude smaller than their field counterpart values. As a consequence viscous forces must be relatively more important in a laboratory physical model study than they are in the atmosphere. There can be no doubt that Reynolds number inequality results in differences between the model and prototype flows. The critical questions must be “What are the differences?” and “Are they significant?”

Consideration of the concepts of similitude focus attention on the importance of the Reynolds number criteria. Perturbation theories suggest that approximate or partial similitude is sufficient to provide concentration predictions sufficiently accurate for engineering decisions. The search for Reynolds number independent flow regimes have identified several different performance envelopes for operation of meteorological wind tunnels which might guarantee realistic concentration data. The intent of this paper is to
report new laboratory measurements which specify wind tunnel operating conditions which assure confidence in predictions of air pollution aerodynamics around buildings.

**FLUID MODELING AND SIMILITUDE**

The concept of similitude is basically simple. Two systems at different geometric scales will exhibit similitude if a one to one correspondence exists in space and time between fluid particle kinematics (locations, velocities, accelerations and rotation) caused by fluid particle dynamics (pressures, gravity, Coriolis forces, viscous forces, etc.), when properly scaled by characteristic scales of fluid properties, force, length and time. To achieve this similarity, however, is not trivial. The specification of dimensionless parameters which guarantee similarity has been the subject of much discussion and debate.

In the nineteenth century a number of workers (most notably Lord Rayleigh) commonly solved problems by direct use of the similarity principle with the intuitive identification of relevant force ratios. During the early twentieth century, the force ratio methods lost favor and were replaced almost entirely by dimensional analysis, as represented by the Buckingham Pi theorem. The most systematic and reliable method currently used to identify relevant scaling parameters in the “normalization” of the governing partial differential equations of motion. Normalization makes the equations and boundary conditions nondimensional in terms of scaling variables of standard magnitude (Kline, 1965).

Scientists who examine the equations of motion including continuity, momentum, energy and species relations identify at least eight important dimensionless groups:

\[
\begin{align*}
Ro &= \frac{U_R}{L \Omega_R} \text{ is the Rossby number,} \\
Eu &= \frac{P_R}{L(\rho_R U_R^2)} \text{ is the Euler number,} \\
Re &= \frac{\rho_R}{\mu_R} \frac{U_R L}{\mu_R} \text{ is the Reynolds number,} \\
Ri &= \frac{g_R \Delta T_R}{L(T_R U_R^2)} \text{ is the Richardson number,} \\
Pe &= \frac{\rho_R C_p R}{L} \frac{U_R L}{\mu_R} \text{ is the Peclet number,} \\
Pr &= \frac{\mu_R}{C_p R} \frac{U_R L}{\mu_R} \text{ is the Prandtl number,} \\
Sc &= \frac{\mu_R}{\rho_R D_R} \frac{L(\rho_R D_R)}{\mu_R} \text{ is the Schmidt number,} \\
Ec &= \frac{U_R^2}{L(C_p R \Delta T_R)} \text{ is the Eckert number.}
\end{align*}
\]

where subscript \( R \) refers to some characteristic reference condition. “Exact” similarity requires equality of the nondimensional coefficients listed above for the physical model and the prototype situations. If separate length scales are chosen for the different coordinate directions additional parameters are generated (e.g. Meroney and Melbourne, 1992)

Furthermore, boundary conditions governing the flow domain of interest must also be similar for the model and prototype. Surface boundary conditions would require
similarity of topographical relief, surface roughness distributions, surface temperature distributions, and reproduction of associated obstacles, buildings, fences, source areas, etc. Similarity of the approach-flow characteristics requires similarity of distributions of mean and turbulent velocities, distributions of mean and fluctuating temperatures and humidities, and distributions of turbulent scales and energies. Similarity of the boundary conditions aloft would require similarity of the location of the upper stream line and near zero longitudinal pressure gradients.

The stipulation of some seven equations and boundary conditions which contain some seven unknowns, U, V, W, T, p, ρ and χ, indicate (in principle) their solutions can be determined. Any prototype and model flow which is constrained by the same scaled initial and boundary conditions, and for which all the dimensionless coefficients identified above are invariant, must have a unique solution in terms of the dimensionless variables. It is not necessary to actually solve the differential equations if one uses a laboratory facility as an analog computer. If all the foregoing requirements could be met simultaneously, all scales of motion ranging from micro to mesoscale, i.e. $10^3$ to $10^5$ m, could be simulated within the modeled flow field.

Unfortunately, all similarity requirements cannot be satisfied simultaneously and modelers must use partial or approximate similitude. Hence model conditions must be chosen which are designed to simulate most accurately those scales of motion which are of greatest significance for the application (Cermak, 1975). Fortunately, several of the dimensionless parameter can be neglected due to their low relative importance when simulating transport and dispersion about buildings. For example, the Rossby number which reflects the inertial effects of transport in a rotating coordinate system (the Earth) has minor influence unless motions persist over distances long enough for the associated spatial deviations to become significant. The Euler number is automatically simulated in air filled wind tunnels; the Richardson number may be neglected when atmospheric stratification effects are small or pertinent plumes are neutrally buoyant; the model and field values of Prandtl and Schmidt numbers for wind tunnel modeling of atmospheric flows are very close; and the Eckert number is generally small compared to unity for subsonic flows.

The magnitude of the Reynolds number indicates the relative importance of inertial forces and viscous or frictional forces. It imposes very strong limitations on rigorous simulation, since scale reductions of 1:100 to 1:1000 commonly result in model Reynolds numbers two to three orders of magnitude smaller than those found in the atmosphere. Thus the viscous forces are relatively more important in the model than in the prototype. If strict Reynolds number equality is required, no atmospheric phenomena could be modeled. Various arguments have been proposed to justify the use of smaller Reynolds numbers in model studies. Snyder (1972) reviewed suggested concepts of the laminar flow analogy, dissipation scaling, and Reynolds number independence. He concluded that only Reynolds number independence is a viable scaling possibility.

The thermal or mass species Peclet numbers can also be expressed as the product
of a Reynolds number and the Prandtl number of the Schmidt number, i.e. RePr or ReSc. The parameter is a measure of the ability of the fluid to advect heat or mass compared with its ability to disperse heat or mass by molecular transport. The Peclet numbers become important when Reynolds number independence does not exist (Meroney, 1986, 1988).

**REYNOLDS NUMBER INDEPENDENCE**

Fortunately, it is possible to circumvent the effect of Reynolds number equality in many cases. Townsend (1956) suggested that, for a flow system in which thermal and Coriolis effects were absent and whose boundary conditions were similar when normalized by the appropriate characteristic length $L$ and velocity $U_R$, the turbulent flow structure would be similar at all sufficiently high Reynolds numbers. Townsend called this hypothesis of Reynolds number independence "Reynolds number similarity." Exceptions must be (a) the very small scale turbulent structures involved in the dissipation of turbulence into heat and (b) flow fields very close to boundary surfaces where the no-slip boundary conditions results in locally very small Reynolds numbers. Consequently, viscosity has very little effect on the bulk of the fluid motions.

But how does such similarity come about in the face of orders of magnitude of Reynolds number inequality? First, for simple flat plate and pipe flow situations it has long been observed that the surface drag coefficient which reflects momentum mixing rates becomes invariant with respect to Reynolds number when the surface is "sufficiently rough" and "sufficiently long." Since the surface friction coefficient is invariant, then the normalized mean velocity profiles which advect and shear plumes will also be invariant.

Second, the gross structure of the turbulence can be geometrically similar over a very wide range of Reynolds numbers. Consequently, flow mixing rates will be similar when unmatched turbulent scales do not contribute significantly to dilution. Since we are limiting our observations to flow and dispersion near buildings and structures we need not consider mixing induced by very large scales associated with plume meandering, weather or the diurnal cycle. It is the smaller scales of motion below the Van der Hoven spectral gap which are simulated in fluid modeling facilities. Thus, the equivalent upper limit for averaging time in laboratory facilities must be about 10 to 20 minutes.

Consideration of turbulent spectral plots show that a reduced Reynolds number changes only the higher frequency portion of an Eulerian spectral energy distribution. To determine whether such deviation from field scale behavior is critical, one must consider the contribution of individual scales to dispersion. Examination of the Taylor diffusion equation reveals that at very small travel times all scales of turbulence contribute to dispersion with the same weight, but for longer and intermediate travel times the larger scales of turbulence progressively dominate the dispersion process. Thus, eddies with scales less than one-tenth of the plume diameter or depth do not contribute significantly to the spread of the plume. If the fluid model replicates all eddy scales down to a small fraction of the building dimension, then one expects realistic plume dispersion a short distance away from a finite dimension source.
PREVIOUS FLUID MODELING EXPERIENCE

The most convincing evidence for the presence of adequate Reynolds number magnitude is obtained when laboratory evidence over a range of Reynolds numbers show that no strong deviations in concentration decay rate or plume growth occur. A number of authors have discussed flow studies about simple cubical or rectangular sharp-edged obstacles. An extensive review about such flow fields and the subsequent character of diffusion near obstacles was provided by Hosker (1984). Unfortunately, though data for many fluid modeling studies exist in the literature, few experiments have focused on the limiting role of the Reynolds number on transport and dispersion near buildings.

Golden (1961) measured the concentration patterns above the roof of model cubes in a wind tunnel. Two sizes of cubes were used to vary the Reynolds number from 1,000 to 94,000. The concentration isopleths in the fluid above the cube roof showed only slight variations over the entire range of Reynolds numbers studied. The maximum concentration on the roof itself was found to vary strongly with Reynolds numbers less than 11,000, but to be invariant with Reynolds numbers between 11,000 and 94,000. Frequently modelers quote Golden’s experiments as justification for presuming dispersion invariance when obstacle Reynolds numbers exceed 11,000. However, Golden’s “11,000 rule” is limited to the measurement of concentrations at only one point on the roof of smooth-walled cubes placed in an uniform approach flow of very low turbulent intensity. Halitsky (1968) observed that for dispersion in the wake region no change in isoconcentration isopleths from passive gas releases was found to occur for values of Reynolds number as low as 3,000.

An experimental investigation of the flow around surface-mounted cubes in both uniform, irrotational and sheared, turbulent flows was performed by Castro and Robins (1977). Measurements of body surface pressures and mean and fluctuating velocities within the obstacle wake were made with pressure transducers and pulsed-wire anemometry. In the case of the uniform upstream flow and a cube normal to the flow, no changes in the flow field occurred beyond a cube Reynolds number of about 30,000. For a cube at $45^\circ$ to the approach flow the high negative pressure measured near the top leading corner increased in magnitude by a factor of three as the Reynolds number increased from 20,000 to 100,000, which might suggest a very high critical Reynolds number for this situation. For the case of a turbulent upstream boundary-layer flow, no Reynolds number effects were discernible for situations corresponding to a Reynolds number based on cube height and the velocity at that height in the undisturbed flow of about 4,000.

Fackrell and Pearce (1981) examined wind tunnel measurements of near-wake parameters for many different building shapes in a variety of boundary layer flows. Measurements of recirculation region (cavity) characteristics at Reynolds numbers of 5,000 and upward showed no significant differences in residence time or recirculation region length. They concluded their results were independent of Reynolds number.
Parallel studies of the effects of Reynolds number on plume dispersion near cubes oriented normal to the flow field were performed in a water drag tank and a boundary layer wind tunnel by Snyder (1992). All measurements were made for a single source location in the near wake of cube. Drag tank tests (uniform approach flow) reproduced Golden's results, and Snyder established that the Reynolds number must exceed a value of approximately 11,000 in order to obtain a Reynolds-number-independent flow structure for uniform approach flows. Snyder extended this study for shear flows oriented perpendicular to a cube face for a source also released at the downwind base of the cube. He concluded that Re > 4,000 would limit perturbations to twice the minimum inherent concentration standard deviation, S, where

\[ S = \left[ \frac{1}{N} \sum_{i=1}^{N} (\log K_{ij} - \log K_{ij})^2 \right]^{1/2} \]  \hspace{1cm} [2]

where \( K_{ij} = X_{ij} U H^2 / Q \) is the dimensionless concentration coefficient measured at port \( i \) during situation \( j \), and situation 2 is selected to be a case which definitely exceeds the critical Reynolds number. Snyder chose a minimum \( S \) of 0.1 and a minimum discrimination level in \( K \) of 0.001. In the Snyder data there may be some influence of the low Reynolds number approach flow on the specified critical Reynolds number since the friction Reynolds number under that experimental condition was low, i.e. \( Re^* = 0.9 < 2.5 \) (see following section on model approach flow conditions required for simulation).

**IMPLICATIONS FOR FLUID MODELING CRITERIA:**

Based on measurements reviewed above, a fluid modeler would feel justified in performing dispersion experiments in the vicinity of model buildings when the Reynolds number based on building height and upwind wind speed at that height exceeded 4,000. However, these data are limited to flow about cubes, cubes oriented normal to the approach wind, and sources located at ground level in the near downwind wake of the obstacle, \( x = H/4 \). It was felt these conditions are not necessarily the most severe or complex situations one would encounter during a fluid modeling study; hence, additional measurements were planned for a non-cubic but rectangular prism building shape, orientations other than normal to the approach flow, and source positioned at various roof top locations.

**TRACER GAS RESULTS FOR TTU WERFL MODEL TESTS**

The building geometry selected for this study was that of the Texas Tech Wind Engineering Research Field Laboratory (WERFL) located in Lubbock, Texas. This building has been extensively studied both in the field and the laboratory as part of the Colorado State University/Texas Tech University Cooperative Program in Wind Engineering (1989-1999) organized to gather wind effects data on low rise structures (Mehta and Meroney, 1995). The building is 13.7 m long, 9.1 m wide, and 4.0 m high. Approach flow wind conditions have been documented on site using a local 49 m tall meteorological tower.
instrumented with fast response Gill UVWV type anemometers and temperature gauges.
Local surface conditions have an approach surface roughness of $z_o = 7$ to 38 mm, a
power law profile exponent $\alpha = 0.14$ to 0.17, local integral scale lengths of $\Lambda_h = 35$ to 200
m at building height, turbulence intensity $u'/U = 18$ to 22% at building height, and a
surface drag coefficient of $u^*/U_{10} = 0.04$ to 0.06.

Model Approach Flow Conditions
At this time no dispersion measurements are available from the field site, but wind
tunnel laboratory measurements have examined dispersion around and infiltration through
the modeled building (Birdsall, 1994; Birdsall and Meroney, 1995; Neff, Meroney and
Birdsall, 1995). The model building configuration is shown in Figure 1. The approach flow
wind speed and turbulence intensity conditions modeled in the Industrial Wind Tunnel at
Colorado State University were developed downstream of spires, barriers and over surface
roughness and are shown in Figure 2. Laboratory results are compared to average field
results. Modeled surface roughness, power law profile exponent, integral scale and
surface drag are $z_o = 10$ mm, $\alpha = 0.14$ to 0.17, $\Lambda_h = 140$ m, and $u^*/U_{10} = 0.05$,
respectively. The friction Reynolds number, $Re^* = u^*z_o/\nu$, ranged from 2.1 to 9.0. Values
of $Re^*$ greater than 2.5 are believed to assure modelers that surface conditions are fully
rough and the approach wind flow will be Reynolds number independent.

Source and Measurement Locations
Source gas was released from the model roof center, upwind roof center and
upwind roof corners as noted in the Figure 1 schematic drawing. Concentrations were
measured at 42 locations over the building surface and 49 additional locations in the wake
of the building as shown in Figures 3 and 4. Two model scales were examined (1:25 and
1:50). Two wind approach angles were studied (normal to the narrow end of the building,
0°, and oblique to the end of the building, 30°. Wind speeds at roof height were varied over
the range 0.5 to 5.0 m/sec which resulted in building height Reynolds numbers varying
from 2,400 to 52,000.

Concentration Measurement Instrumentation
Concentration samples were extracted from the flow stream using a 50 port
hyperdermic syringe sampling system. Samples were evaluated using a Hewlett Packard
Model 5710A gas chromatograph with a flame ionization detector system. The system
measures local concentrations to a resolution of 5 ppm within an accuracy of 3% for
concentrations significantly above background and 20% at concentrations near
background. (For a detailed description of the measurement system, its operation, and
accuracy see Appendix A of Neff and Meroney, 1994.)

Characteristic Concentration Patterns
Figures 5 through 9 provide an overview of the entire concentration field for the
1:25 scale model, two wind orientations and three source locations. The average
concentration values at each sampling port are plotted versus port number. Since the
ports were numbered in a logical sequence, we may view root top, downwind face, and
wake concentration profiles. In Grids 1 and 3 sampling ports 1-30 are arranged in six cross-building profiles from up to the downwind end of the roof. Ports 31-42 are arranged in three cross-building profiles from top to bottom of the downwind building face. Ports 43-46 are located one building height downwind at ground level. In Grids 2 and 4 there are 45 sampling ports located in a 5 x 9 vertical array perpendicular to the wind direction two building heights downwind (160 mm).

Examination of the data on Figures 5 and 6 (wind orientation of 0°) indicates that dimensionless concentration, K, asymptote to similar values as Reynolds numbers increase. On the roof centerline at low Reynolds numbers the plume concentrations display less dilution or higher concentrations and less lateral dispersion. Thus, plume concentration along the building roof edge may be much lower at Reynolds numbers below some critical value. However, the trend reverses on the downwind face and wake regions where local Reynolds number values at ground level tend to be lower. This may be explained by the fact that the less dispersive plume has not yet mixed fully to ground level. Maximum concentration errors due to lack of Reynolds number independence appear to be about +30% on the roof and about -20% in the wake regions.

Under oblique wind approach flow conditions (Figures 7 to 9) (wind orientation of 30°) the concentration field is more asymmetric. At low Reynolds numbers the maximum K values may be up to +50% too large; however, due to lateral plume displacement the local upwind surface concentrations may be up to 500% too large! Wake measurements are difficult to interpret, because slight orientation deviations cause the plume centerline to deviate significantly. Local Reynolds numbers may be either lower or higher, but variations are not large.

Measured results appear similar at scales of 1:25 and 1:50 within inherent measurement variability.

**Concentration Standard Deviation Parameter**

The invariance of the concentration field with Reynolds number was again quantified in terms of the concentration standard deviation, S, as defined in Equation [2]. Minimum discrimination levels in K of 0.001 were specified similar to the shear flow condition limits set by Snyder (1992). Reference concentration levels were chosen as the averages of values measured at Reynolds numbers of 30,000, 40,000, and 50,000.

Figures 10 and 11 compare results for the 1:25 scale model; source locations A, B, and C; and the 0° and 30° wind orientations for the building surface (grid 1) and downwind wake (grid 2) locations, respectively. Figures 12 and 13 display the results for measurements over the 1:50 scale models, also for the building surface (grid 3) and downwind wake (grid 4) locations, respectively. Because of the inherent variability of the turbulence fields, the minimum value of the S parameter is not zero. From Figures 10 through 13 it appears that the minimum inherent S_{min} values were 0.05-0.15 for concentrations on the building surface and 0.05-0.1 in the wake.
It is clear that as the Reynolds number increases, the S parameter approaches the minimum value $S_{\text{min}}$ irregularly. A sharp or precise "critical" Reynolds number where the concentration patterns become Reynolds number independent is not apparent. The choice of a critical Reynolds number is thus arbitrary, but following Snyder (1992) one can define a Reynolds number value where the parameter $S$ is twice its inherent minimum value.

Since each source location and building orientation generates a somewhat different value of the minimum inherent standard deviation parameter, $S_{\text{min}}$, the $2 \times S_{\text{min}}$ criteria results in a finite range of critical Reynolds numbers. Nonetheless, Reynolds number independence appears to be established once $Re > 15,000$.

Given that Snyder (1992) established a value of $Re = 4,000$ for the critical Reynolds number for independent experiments, why is the current value three times larger? It is likely that the smaller critical Reynolds number results from his source location in the cavity wake region. A source located in the cavity will spread most of the mass released downwind in the wake, whereas roof source locations will produce narrower plumes over the roof, and the plumes will penetrate the wake cavity more slowly. Smaller plumes will be dominated more by plume size eddies rather than building size eddies, which is likely to make their dilution more Reynolds number dependent.

CONCLUSIONS

Given the additional complexities of roof source location, orientation, and intermittent reattachment of streamlines on the building surface, we recommend that $Re > 15,000$ during fluid model simulations of dispersion about and downwind of sharp-edged building models.

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REFERENCES:


- Full-Scale Building Exterior Dimensions

- Building Wind Approach Angle Notation

Figure 1
Reynolds Number Test
Release Points and Sampling Locations

A, B & C are Source locations

Fig. 3
Figure 2 - 1:25 Model-Scale Wind Tunnel Velocity Profile Properties
Field Data from Chok, 1988
Reynolds Number Test
Downwind Sampling
Locations

320 mm Downwind from Rear Building Face

Crosswind Distance (mm)

Height (mm)
1:25 Scale; 00 degree wind direction; Source C (CH4)

Concentration Coefficient (K x 10^3)

GRID #2

2819    • 5023    ▲ 6988    ◹ 9501    ▲ 12286    ◹ 14576    • 17043    ▲ 20387    • 25591    ▲ 30240    ◹ 39769    ▲ 51734

Fig 6