NUMERICAL TURBULENT MODEL
FOR DOUBLE DIFFUSIVE CONVECTION

by

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ABSTRACT

A numerical investigation of the transient behavior of a doubly-diffusive flow under the influence of combined heating and shear flow is presented. The analysis is based on a one-dimensional algebraic stress turbulent model (ASM), which produces a set of nine partial differential equations and six algebraic equations. The partial differential equations were solved by an implicit finite difference technique. The numerically produced temperature and density profiles as well as the entrainment rates are compared with their correspondent experimental data. The numerical simulations agree qualitatively with experimental findings and show a similar trend. It even predicts the layering phenomenon as it occurs in experimental results

NOMENCLATURE

Symbol	Definition
A_1, A_2, A_3	
C_D, C_H, C_{E1}, C_{E2}	Constants for ASM, See Table (1)
C_{Et1}, C_{Et2}, F	
C_{Ec2}, C_{p1}, C_{p2}, C_{p3}	
C	Concentration
C_p	Thermal capacity
d	Diffuser height above the interface
D	Mass molecular diffusivity
g	Gravitational constant
H	Upper interface height
k	Turbulent kinetic energy $\overline{(u_i' u_j')}/2$
k_t	Turbulent temperature fluctuation $\overline{(T')^2}/2$
k_c	Turbulent concentration fluctuation $\overline{(C')^2}/2$
p	Pressure
Pr	Prandtl number (ν/α)
Q	Flow rate
Q_H	Heat flux

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Re	Reynolds number ($u_{max} H/\nu$)
Ri	Bulk Richardson number $(g\ \Delta\rho_{max}\ H)/(\rho\ U^2)$
Ri_C	Conc. Richardson number $(g\beta_C\ \Delta C_{max}\ H)/(u_{max}{}^2)$
Ri_T	Temp. Richardson number $(g\beta_T\ \Delta T_{max}\ H)/(u_{max}{}^2)$
Sc	Schmidt number (ν/D)
t	Time
T	Temperature
u,v,w	Velocity components
x,y,z	Coordinate directions

Greek Symbols

α	Thermal molecular diffusivity
α_t	Thermal eddy diffusivity
α_c	Mass eddy diffusivity
β_T	Thermal expansion coefficient
β_C	Saline expansion coefficient
ε	Dissipation of turbulent kinetic energy
ε_t	Dissipation of turbulent temp. fluctuations
ε_c	Dissipation of turbulent conc. fluctuations
ν_t	Eddy viscosity
ρ	Density
$\sigma_{k,kt,kc,\varepsilon,\varepsilon t,\varepsilon c}$	Effective Prandtl number for $k,k_t,k_c,\varepsilon,\varepsilon_t,\varepsilon_c$
ϕ	Gravitational potential

superscripts

'	Fluctuation quantities

1.0 INTRODUCTION

Double diffusion occurs in a fluid in which the density is a function of two components of different diffusivities (e.g. salt and temperature). This phenomenon occurs in different modes, but of great importance is the diffusive mode when the higher diffusive component (i.e. heat) is destabilizing and the other component (i.e. salt) is stabilizing. This kind of flow can be seen in many natural and engineering systems. For example; in nature this may include - but not limited to- the deepening of the surface mixed layer into the stably stratified layer below as a result of turbulence induced by surface wind and cooling. While in engineering applications, this type of flows can be noticed in ocean discharge of power plants cooling water or ocean thermal conversion systems, in energy extraction from salt gradient solar ponds, in the energy storage in molten salt tanks, in the disposal of pollutants and waste water in lakes, and in the processing of binary or multicomponent solutions, to name only a few.

Experimental work is costly and time consuming. Continuous calibration of various instruments is unavoidable, and accurate measurements of the velocity in saline water are difficult to make. So a better understanding of the problem can be attained through a numerical approach rather than from the experiment. A reliable numerical technique provides a flexible tool which allows for changing the parameters (e.g., thermal and solute Richardson numbers, Schmidt number, and Prandtl number) and may lead to a better understanding of the behavior of the problem for a wide range of initial and boundary conditions. Besides, it may be used for prediction of the transient processes in large environmental systems where continuous measurements are prohibitively costly.

Under consideration in this study is the problem of transient double diffusive flow where a stably stratified salt layer is being eroded from below due to continuous heating and from the top due to heat removal and shearing. The behavior of the temperature and concentration profiles as they evolved with time is investigated, and a close attention is paid to the mixing process at the interface that takes place in the presence of combined heat and shear flow. The specific objective of the study is to develop a one-dimensional turbulent model (Algebraic stress, mass flux, and heat flux model (ASM)), which can predict the temperature and salinity profiles as they evolve with time and to use available experimental data for model verification.

2.0 PREVIOUS NUMERICAL MODELS

There have been relatively few numerical models which consider double-diffusive systems. Atkinson and Harleman (1983) developed a one-dimensional (vertical) turbulent kinetic energy (TKE) model to simulate the temporal evolution of vertical temperature and salinity profiles in a salt gradient solar pond. From their model, a general entrainment relation was derived to predict the growth of the upper convecting layer in solar ponds. However, in their model, they neglected the side-wall induced instabilities and used a fixed 10-cm grid spacing (which is several times greater than the observed vertical-length scale of instability). This led to a quantitatively unreliable model. A better version of the model which accounts for the role of sidewalls was developed by Schladow (1984,1988). Earlier Cha et al. (1981) developed a one-dimensional numerical model to predict the diurnal variation of vertical temperature and concentration profiles in solar ponds. In their model, the convection term in the equations was neglected, and the fluid motion was retained through the use of augmented diffusivities.

Murty (1988) used a numerical model based on finite elements to investigate a laminar and steady double diffusive convection in a cell. His results agree well with experimental data, but the cell size was limited by the unit cell and the model needed to be modified to account for infinite fluid. Kazmierczak (1988) solved numerically the laminar Naiver-Stokes equations for double diffusion in a horizontal fluid layer subjected to a constant bottom heat flux. His numerical simulation predicted the time varying temperature and salinity fields, but there were no comparisons made to experiments he performed because the flow in the experiment was mainly turbulent and the simulation was based on laminar governing equations.

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Murota and Michioku (1989) developed a one dimensional numerical simulation based on the work developed originally by Mellor and Yamada (1974), using a turbulent closure model to examine the mixing phenomena in single diffusive flows produced by the combined action of mechanical and thermal stirring. Their numerical simulation agrees well with the experiment they performed, and it reproduced the mixing rate, temperature profiles and turbulent profiles with remarkable accuracy. However, the model was developed for a single diffusive flow (temperature stratification only) and extension to double diffusive flow is required. Ohtsuka and Yamakawa (1989) analyzed a stably stratified shear flow generated by two streams of water at different densities and velocities, by solving the Naiver-Stokes equations without any turbulence average modelling. Instead they considered the turbulence transport mechanics analytically. They claimed that their results agree with experimental data within 10%.

In an attempt to examine the stability of double diffusive flow, Murota and Michioku (1989) also used perturbation theory to develop a stability criterion for both the direct mode and oscillatory mode. Their results compared satisfactorily with experimental data. Another effort to study the effect of variable stratification on linear diffusive stability was performed by Zangrando and Bertram (1985), where they developed a numerical solution based on Fourier series expansion. Their solution indicated that the overstable case approaches Walton's perturbation solution at large values of solute Rayleigh number and deviates significantly at low values of solute Rayleigh number.

Some calculations were also performed to predict numerically the heat and mass fluxes and/or the entrainment across a density interface. Linden and Shirtcliffe (1978) developed a model to estimate the fluxes of heat and salt across an interface. When compared with experiments, the model provides a reasonable description of the observed transport over an intermediate range of density ratios, but fails at low and high density ratios. In the more recent work by Mory (1991), three entrainment laws were established theoretically for entrainment across an interface in the absence of mean shear. His proposed laws are in good agreement with experimental data provided by various researchers.

In turbulent flows usually the number of unknowns is greater than the number of equations and a turbulent closure is required to solve the system. The algebraic stress and heat flux model (ASM) approach avoids the assumption of local turbulence equilibrium between production and dissipation required by the popular κ-ε turbulence closure approach. ASM replaces the partial differential equations for velocity-velocity, velocity-temperature, and velocity-concentration correlations by algebraic expressions. Therefore, the ASM model reduces the computational complexity by reducing the number of partial differential equations, yet ASM retains all second-order correlations, permits non-equilibrium values to exist, and retains most of the physical properties of the problem. The ASM approach model was suggested originally by Rodi (1972), where he noticed that the velocity correlations are differential and nonlinear only due to the presence of convection and diffusion terms. So to eliminate these terms, he correlated them with the convection and diffusion terms of the kinetic energy transport equation.

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Meroney (1974,1976) used the ASM model to study the behavior of stratified turbulent shear flows in the atmosphere. He found that for a clear-air turbulence test case the ASM model compared very well with a more-comprehensive but numerically more complicated mean Reynold stress model (MRS). Rodi (1980) showed that the results obtained from the ASM model for a plane vertical buoyant jet in stagnant un-stratified surroundings are in excellent agreement with experimental data. Hossain and Rodi (1982) applied the ASM model to a vertical buoyant turbulent jet in a stably stratified environment. They found that the model results are in good agreement with the experimental measurements. Uno and Ueda (1989), proposed an improved dissipation rate equation and discussed some fundamental aspects of the ASM in thermally stratified flow.

No publications were found which discussed the application of the ASM model to double-diffusive flows where the effects of solute and thermal stratifications coexist. In this contribution, the ASM model is extended to account for solute stratification; this will automatically introduce new equations as well as new terms to the governing equations.

3.0 GOVERNING EQUATIONS

The turbulent equations governing the distribution of the flow quantities in a doubly diffusive flow can be obtained from the conservation laws of mass, momentum, thermal energy and species concentration along with an equation of state. Al-Ghamdi (1993) provided a detailed derivation of the 3-D equations. This study is concerned with 1-D turbulent model and all the equations will be presented in this context. The 1-D turbulent governing equation when scaled by the scaling parameters shown in appendix (I), are:

1) MOMENTUM EQUATION:

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\frac{\partial^2 u}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z}.$$ (1)

2) ENERGY EQUATION:

$$\frac{\partial T}{\partial t} = \frac{1}{Re\ Pr}\frac{\partial^2 T}{\partial z^2} - \frac{\partial(\overline{w'T'})}{\partial z}.$$ (2)

3) CONSERVATION OF CONCENTRATION:

$$\frac{\partial C}{\partial t} = \frac{1}{Re\ Sc}\frac{\partial^2 C}{\partial z^2} - \frac{\partial(\overline{w'C'})}{\partial z}.$$ (3)

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4) TURBULENT KINETIC ENERGY:

The transport equation for the turbulent kinetic energy (k) in 1-D flows is:

$$\frac{\partial k}{\partial t} = [\frac{1}{Re}\frac{\partial^2 k}{\partial z^2} - \frac{\partial}{\partial z}(\overline{k'w'} + \overline{p'w'})] - \overline{u'w'}\frac{\partial u}{\partial z} - \epsilon$$
$$+ [Ri_T\overline{w'T'} - Ri_C\overline{w'C'}],$$

(4)

where turbulent kinetic energy is defined as $k = 1/2\,(\overline{u_i'u_i'})$. The terms on the right hand side of equation (4) represent diffusion, production, dissipation, and stratification terms, respectively. If the k is assumed to diffuse down its gradient, then we may write:

$$\frac{\partial}{\partial z}\overline{(k'w' + p'w)} = \frac{\partial}{\partial z}(\frac{\nu_t}{\sigma_K}\frac{\partial k}{\partial z}).$$

(5)

5) TURBULENT FLUCTUATIONS IN TEMPERATURE:

The turbulent temperature fluctuations equation, in the 1-D case is given by:

$$\frac{\partial k_t}{\partial t} = [\frac{1}{Re\ Pr}\frac{\partial^2 k_t}{\partial z^2} - \frac{\partial}{\partial z}(\overline{k_t'w'})] - \overline{T'w'}\frac{\partial T}{\partial z} - \epsilon_t.$$

(6)

This equation determines the temperature scale of the fluctuations as equation (4) determines the velocity scale. The two equations have similar form with exceptions that the latter relation does not include pressure strain nor stratification terms. Again, k_t is assumed to diffuse down its gradient; thus:

$$\frac{\partial}{\partial z}\overline{(k_t'w')} = \frac{\partial}{\partial z}(\frac{\alpha_t}{\sigma_{kt}}\frac{\partial k_t}{\partial z}).$$

(7)

6) TURBULENT FLUCTUATIONS IN CONCENTRATION:

Likewise the 1-D transport equation for turbulent concentration fluctuations is given by:

$$\frac{\partial k_c}{\partial t} = [\frac{1}{Re\ Sc}\frac{\partial^2 k_c}{\partial z^2} - \frac{\partial}{\partial z}(\overline{k_c'w'})] - \overline{C'w'}\frac{\partial C}{\partial z} - \epsilon_c.$$

(8)

This equation determines the concentration scale and is identical to equation (6). Also k_c is assumed to diffuse down its gradient; thus:

$$\frac{\partial}{\partial z} \overline{(k_c'w')} = \frac{\partial}{\partial z} \left(\frac{\alpha_c}{\sigma_{kc}} \frac{\partial k_c}{\partial z} \right). \tag{9}$$

7) EDDY DISSIPATION OF KINETIC ENERGY:

The transport equation for the dissipation of kinetic energy can be developed from the main flow equation as:

$$\frac{\partial \epsilon}{\partial t} = \left[\frac{1}{Re} \frac{\partial^2 \epsilon}{\partial z^2} - \frac{\partial}{\partial z} \left(\overline{\epsilon'w'} + \frac{1}{Re} \overline{\frac{\partial p'}{\partial z} \frac{\partial w'}{\partial z}} \right) \right] - \frac{2}{Re} \left[\frac{\partial u}{\partial z} \left(\overline{\frac{\partial u'}{\partial z} \frac{\partial w'}{\partial z}} \right) + \frac{\partial^2 u}{\partial z^2} \left(\overline{w' \frac{\partial u'}{\partial z}} \right) \right]$$
$$- \frac{2}{Re} \left[\overline{\left(\frac{\partial u'}{\partial z} \right)^2 \frac{\partial w'}{\partial z}} + \overline{\left(\frac{\partial v'}{\partial z} \right)^2 \left(\frac{\partial w'}{\partial z} \right)} + \overline{\left(\frac{\partial w'}{\partial z} \right)^3} \right] - \frac{2}{Re} \left[\overline{\left(\frac{\partial^2 u'}{\partial z^2} \right)^2} + \overline{\left(\frac{\partial^2 v'}{\partial z^2} \right)^2} + \overline{\left(\frac{\partial^2 w'}{\partial z^2} \right)^2} \right]$$
$$+ \frac{2}{Re} \left[Ri_T \frac{\partial T'}{\partial z} - Ri_c \frac{\partial C'}{\partial z} \right] \overline{\frac{\partial w'}{\partial z}}. \tag{10}$$

The first bracket on the right is the diffusion term, the second bracket represents the production, the fourth and fifth terms represent the destruction terms, and the sixth is the stratification term. This equation contains high order correlations and therefore requires model assumptions.

As in the case of the turbulent kinetic energy the second part of the diffusion term can be modeled as:

$$\frac{\partial}{\partial z} \left(\overline{\epsilon'w'} + \frac{1}{Re} \overline{\frac{\partial p'}{\partial z} \frac{\partial w'}{\partial z}} \right) = \frac{\nu_T}{\sigma_\epsilon} \frac{\partial \epsilon}{\partial z}. \tag{11}$$

Rodi (1972) argued that the production term can be written as:

$$Production \ of \ \epsilon = C_{E1} \frac{\epsilon}{k} \overline{u'w'} \frac{\partial u}{\partial z}. \tag{12}$$

Meroney (1974) concluded that the destruction term is controlled by the dynamics of the energy cascade process and is independent of viscosity, and can be modeled as:

$$Destruction \ of \ \epsilon = C_{E2} \frac{\epsilon^2}{k}. \tag{13}$$

Finally Meroney (1974) suggested that the stratification term can be modeled as:

$$\frac{1}{Re} \left[2Ri_T \overline{\left(\frac{\partial T'}{\partial z} \frac{\partial w'}{\partial z} \right)} - 2Ri_c \overline{\left(\frac{\partial C'}{\partial z} \frac{w'}{z} \right)} \right] = F\left(\frac{\epsilon}{k} \right) \left[Ri_T \overline{w'T'} - Ri_c \overline{w'C'} \right]. \tag{14}$$

8) ENERGY DISSIPATION:

The transport equation for the turbulent thermal energy dissipation is given by:

$$\frac{\partial \epsilon_t}{\partial t} = [\frac{1}{Re\ Pr}\frac{\partial^2 \epsilon_t}{\partial z^2} - \frac{\partial}{\partial z}(\overline{\epsilon'_t w'})] - \frac{2}{Pr}[\frac{\partial u}{\partial z}\overline{(\frac{\partial T'}{\partial z})^2} + \frac{\partial T}{\partial z}\overline{(\frac{\partial T'}{\partial z}\frac{\partial u'}{\partial z})} + \frac{\partial^2 T}{\partial z^2}\overline{(w'\frac{\partial T'}{\partial z})}$$
$$- \frac{2}{Pr}[\overline{(\frac{\partial T'}{\partial z})^2\frac{\partial w'}{\partial z}}] - \frac{2}{Re\ Pr}\overline{(\frac{\partial^2 T'}{\partial z^2})^2}.$$

(15)

As in the kinetic energy dissipation equation, the high order terms need to be modeled. Direct analogy suggests that the second part of the diffusion term -the first term in the right side of Equation (15)- can be modeled as before, as:

$$\frac{\partial}{\partial z}(\overline{\epsilon'_t w'}) = \frac{\alpha_T}{\sigma_{\epsilon t}}\frac{\partial \epsilon_t}{\partial z}.$$

(16)

The second term represent the production term and can be written as:

$$Production\ of\ \epsilon_t = C_{E1}\frac{\epsilon}{k}\overline{T'w'}\frac{\partial T}{\partial z},$$

(17)

and the last two terms are the destruction term which can be modeled as:

$$Destruction\ of\ \epsilon_t = C_{E2}\frac{\epsilon \epsilon_t}{k}.$$

(18)

9) CONCENTRATION DISSIPATION:

In a similar way, the transport equation for the turbulent concentration dissipation is given by:

$$\frac{\partial \epsilon_c}{\partial t} = \frac{1}{Re\ Sc}\frac{\partial^2 \epsilon_c}{\partial z^2} - \frac{\partial}{\partial z}(\overline{\epsilon'_c w'}) - \frac{2}{Sc}[\frac{\partial u}{\partial z}\overline{(\frac{\partial C'}{\partial z})^2} + \frac{\partial C}{\partial z}\overline{(\frac{\partial C'}{\partial z}\frac{\partial u'}{\partial z})} + \frac{\partial^2 C}{\partial z^2}\overline{(w'\frac{\partial C'}{\partial z})}]$$
$$- \frac{2}{Sc}[\overline{(\frac{\partial C'}{\partial z})^2\frac{\partial w'}{\partial z}}] - \frac{2}{Re\ Sc}\overline{(\frac{\partial^2 C'}{\partial z^2})^2}],$$

(19)

where: the terms on the right represent diffusion, production and destruction (the last two terms),

$$\frac{\partial}{\partial z}(\overline{\epsilon'_c w'}) = \frac{\alpha_c}{\sigma_{\epsilon c}}\frac{\partial \epsilon_c}{\partial z},$$

(20)

respectively. As in the energy dissipation equation, the various terms can be modeled in a parallel way as:

$$Production\ of\ \epsilon_c = C_{E1}\frac{\epsilon}{k}\overline{C'w'}\frac{\partial C}{\partial z}, \qquad (21)$$

$$Destruction\ of\ \epsilon_c = C_{E2}\frac{\epsilon\epsilon_c}{k}. \qquad (22)$$

10) VELOCITY CORRELATIONS:

In the 1-D case, two velocity correlations equations are produced, namely:

$$\frac{\overline{\partial u'w'}}{\partial t} = [\frac{1}{Re}\frac{\overline{\partial^2 u'w'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{u'w^{2'}} + \overline{u'p'})] - \overline{w^{2'}}\frac{\partial u}{\partial z} - \overline{p'(\frac{\partial u'}{\partial z})}$$
$$- \frac{2}{Re}\overline{(\frac{\partial u'}{\partial z}\frac{\partial w'}{\partial z})} + [Ri_T\overline{u'T'} - Ri_c\overline{u'C'}], \qquad (23)$$

and;

$$\frac{\overline{\partial w^{2'}}}{\partial t} = [\frac{1}{Re}\frac{\overline{\partial^2 w^{2'}}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{w'w^{2'}} + \overline{2w'p'})] + 2\overline{p'(\frac{\partial w'}{\partial z})} - \frac{2}{Re}\overline{(\frac{\partial w'}{\partial z})^2} \qquad (24)$$
$$+ [2Ri_T\overline{w'T'} - 2Ri_c\overline{w'C'}].$$

where the terms on the right side of the equations represent diffusion, production, pressure strain, dissipation and stratification terms, respectively.

Notice that the terms in the $\overline{(w')^2}$ equation are similar to those in $\overline{u'w'}$ equation, and they have the same physical meaning except that the production term is missing from the latter equation.

The pressure-strain term is normally an unmeasured quantity in a laboratory flow, yet it is an important term that cannot be neglected. It has been suggested by a number of researchers that the pressure-strain term can be modeled as:

$$PS_{\overline{u_iu_j}} = [Pressure\ Strain]_{\overline{u_iu_j}}$$
$$= -C_{p1}\frac{\epsilon}{k}(\overline{u_i'u_j'}-\delta_{ij}\frac{2}{3}k) - A_1(P_{ij}-\delta_{ij}\frac{2}{3}P) \qquad (25)$$

where: P_{ij} is the production term of $\overline{u_i'u_j'}$.

Meroney (1974) and Hossain and Rodi (1982) both recommend that for high Reynolds number flows, the turbulence is locally isotropic so that the energy is dissipated equally in each component of u'_i. Based on this assumption the dissipation terms in the above two equations become:

$$2\nu \overline{\frac{\partial u_i}{\partial x_l} \frac{\partial u_j}{\partial x_l}} = \frac{2}{3} \delta_{ij} \epsilon. \qquad (26)$$

11) VELOCITY TEMPERATURE CORRELATIONS:

Two velocity temperature correlations resulted for the 1-D approximation these equations are:

$$\frac{\overline{\partial w'T'}}{\partial t} = [\frac{1}{Re\ Pr} \frac{\overline{\partial^2 T'w'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{T'w'^2} + \overline{T'p'})] - \overline{w'^2}\frac{\partial T}{\partial z} + (\overline{p'\frac{\partial T'}{\partial z}})$$
$$- \frac{2}{Re\ Pr}\overline{(\frac{\partial T'}{\partial z}\frac{\partial w'}{\partial z})} + [\ 2Ri_T\overline{k'_t} - Ri_c\overline{T'C'}]. \qquad (27)$$

Physically the terms on the right represent the diffusion, production, pressure strain, dissipation and stratification, respectively; and;

$$\frac{\overline{\partial u'T'}}{\partial t} = [\frac{1}{Re\ Pr}\frac{\overline{\partial^2 u'T'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{u'w'T'})] - \overline{w'u'}\frac{\partial T}{\partial z} - \overline{w'T'}\frac{\partial u}{\partial z}$$
$$- \frac{2}{Re\ Pr}(\overline{\frac{\partial u'}{\partial z}\frac{\partial T'}{\partial z}}). \qquad (28)$$

Terms in Equation (28) have the same physical meaning as those in Equation (27), except that the pressure strain and stratification terms do not exist in the $\overline{u'T'}$ equation.

In an analogy to Equation (25), the pressure scrambling term can be assumed to be:

$$PS_{\overline{u'_iT'}} = Pressure\ Strain = -\ C_{p2}\frac{\epsilon}{k}(\overline{u'_iT'}) - A_2 P_{\overline{u'_iT'}} \qquad (29)$$

where:

$$P_{\overline{u'_iT'}} = Production = \frac{1}{2}P_{ij} = -\ \overline{u_iu_j}\frac{\partial T}{\partial x_k} - \overline{u'_kT'}\frac{\partial u_i}{\partial x_k} \qquad (30)$$

The turbulence is again assumed to behave isotropically at the local scale; thus, the dissipation terms are assumed to be zero in the above two equations.

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12) VELOCITY CONCENTRATION CORRELATIONS:

In a similar fashion, the two equations produced for velocity-concentration correlations are:

$$\frac{\overline{\partial w'C'}}{\partial t} = [\frac{1}{Re\ Sc}\frac{\overline{\partial^2 C'w'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{C'w^2'}+\overline{C'p'})] - \overline{w^2'}\frac{\partial C}{\partial z} + \overline{(p'\frac{\partial C'}{\partial z})}$$
$$- \frac{2}{Re\ Sc}\overline{(\frac{\partial C'}{\partial z}\frac{\partial w'}{\partial z})} + [-2Ri_c\overline{k_c'} + Ri_T\overline{T'C'}], \tag{31}$$

and;

$$\frac{\overline{\partial u'C'}}{\partial t} = \frac{1}{Re\ Sc}\frac{\overline{\partial^2 u'C'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{u'w'C'}) - \overline{w'u'}\frac{\partial C}{\partial z} - \overline{w'C'}\frac{\partial u}{\partial z}$$
$$- \frac{2}{Re\ Sc}\overline{(\frac{\partial u'}{\partial z}\frac{\partial C'}{\partial z})}. \tag{32}$$

The terms in these equations carry the same physical meanings as the corresponding terms in the velocity temperature correlations. Also the dissipation terms are assumed to vanish under the assumption of turbulent local isotropy. The pressure-strain term will behave in a similar way and can be written as:

$$PS_{\overline{u_iC'}} = Pressure\ Strain = -\ C_{p2}\frac{\epsilon}{k}(\overline{u_i'C'}) - A_2 P_{\overline{u_iC'}}, \tag{33}$$

where:

$$P_{\overline{u_iC'}} = Production = \frac{1}{2}P_{ij} = -\ \overline{u_iu_j}\frac{\partial C}{\partial x_k} - \overline{u_k'C'}\frac{\partial u_i}{\partial x_k}. \tag{34}$$

13) TEMPERATURE CONCENTRATION CORRELATIONS:

The correlation between temperature and concentration fluctuations is given by;

$$\frac{\overline{\partial T'C'}}{\partial t} = [(\frac{1}{Re\ Pr} + \frac{1}{Re\ Sc})\frac{\overline{\partial^2 T'C'}}{\partial z^2} - \frac{\partial}{\partial z}(\overline{w'T'C'})] - [\overline{T'w'}\frac{\partial C}{\partial z} + \overline{w'C'}\frac{\partial T}{\partial z}] \tag{35}$$
$$- 2[(\frac{1}{Re\ Pr} + \frac{1}{Re\ Sc})(\overline{\frac{\partial T'}{\partial z}\frac{\partial C'}{\partial z}})]$$

The terms on the right side represent the diffusion, production and dissipation terms, respectively. To be consistence with previous analysis, the dissipation term vanishes due to the assumed turbulent local isotropy.

The Boussinesq approximation, which assumes the density is constant except in the buoyancy term of the momentum equation has been utilized in developing the above equations. The density of the fluid is a function of both salinity and temperature and is given by:

$$\rho = \rho_o[1 - \beta_T(T-T_o) + \beta_C(C-C_o)]. \tag{36}$$

In the above equations, u is the velocity in the x direction, P is the pressure, T is the temperature and C is the species concentration. υ, α and D are the molecular viscosity, thermal diffusivity, and solute diffusivity respectively. β_T is the thermal expansion coefficient, β_C is the saline expansion coefficient, and T_o and C_o are reference temperature and concentration, respectively. The various turbulent terms are:

$$k = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}), \quad \epsilon = \frac{1}{Re}[(\overline{\frac{\partial u'}{\partial z}})^2 + (\overline{\frac{\partial v'}{\partial z}})^2 + (\overline{\frac{\partial w'}{\partial z}})^2], \tag{37}$$

$$k_t = \frac{1}{2}(\overline{T'^2}), \quad \epsilon_t = \frac{1}{Re\ Pr}(\overline{\frac{\partial T'}{\partial z}})^2, \tag{38}$$

$$k_c = \frac{1}{2}(\overline{C'^2}), \quad \epsilon_c = \frac{1}{Re\ Sc}(\overline{\frac{\partial C'}{\partial z}})^2, \tag{39}$$

and the dimensionless numbers resulting from the scaling process are defined as Re, Reynolds number; Pr, Prandtl number; Sc, Schmidt number; Ri_T, temperature Richardson number; and Ri_C, concentration Richardson number. They are defined mathematically as:

$$Re = \frac{U_{max}H}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D},$$
$$Ri_T = \frac{g\beta_T\Delta T_{max}H}{\Delta U_{max}^2}, \quad Ri_C = \frac{g\beta_C\Delta C_{max}H}{\Delta U_{max}^2}. \tag{40}$$

By using Prandtl's assumption, which relates shear stress to local velocity gradients through the eddy viscosity, and incorporating the length scale proposed by Harlow and Nakayama in 1967, one gets:

$$\nu_t = C_D\frac{k^2}{\epsilon}. \tag{41}$$

Following a similar argument, the thermal and salinity eddy viscosities can be written as:

$$\alpha_t = C_H k\frac{k_t}{\epsilon_t}, \quad \alpha_c = C_m k\frac{k_c}{\epsilon_c}. \tag{42}$$

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4.0 ALGEBRAIC STRESS MODEL DEVELOPMENT

A model of turbulent closure would be considered useful if it possess width of applicability, accuracy, simplicity, and economy in computational time and storage. The ASM has the advantage of reducing the number of partial differential equations , thus, resulting in reduction in computer time and storage, and yet it retains the applicability to a wide range of problems with a reasonable accuracy. The ASM model requires algebraic expressions for the velocity, velocity-temperature, velocity-concentration, and temperature-concentration correlations to replace the partial differential correlations. These new expressions are dependent upon an accurate representation of the production-dissipation terms in their respective exact relationships. These algebraic equations can be derived as follows:

4.1 Reynolds Stress Equations

By examining the governing equations one finds that the convection and diffusion terms in the Reynolds stresses equations are responsible for the differential form of the relations. If these terms are eliminated, one ends up with more manageable algebraic relations to replace the (computer-time consuming) differential forms. One possibility is to neglect the rate of change, convection and diffusion terms in the $\overline{u_i'u_j'}$, $\overline{u_i'T'}$, $\overline{u_i'C'}$, $\overline{T'C'}$ equations to produce a simplified algebraic equations for these quantities. A more reasonable and physically appealing idea is the one introduced by Rodi (1972). He postulated that the convection and diffusion terms of the Reynolds stress are related to their corresponding terms in the turbulent kinetic equation, and the later terms are related to the production, dissipation, and stratification terms of the turbulent kinetic energy; thus:

$$(Convection\text{-}Diffusion)_{of\ \overline{u_i'u_j'}} = \frac{\overline{u_i'u_j'}}{k}(Convection\text{-}Diffusion)_{of\ k} \qquad (43)$$

$$= \frac{\overline{u_i'u_j'}}{k}(P - \epsilon + S)_{of\ k} = (P + PS - \epsilon + S)_{of\ \overline{(u_i'u_j')}}, \qquad (44)$$

or:

$$\overline{u_i'u_j'} = \frac{k(P + PS - \epsilon + S)_{of\ \overline{u_i'u_j'}}}{(P - \epsilon + S)_{of\ k}} \qquad (45)$$

where: the production, dissipation, and stratification terms of the turbulent kinetic energy and $\overline{u_i'u_j'}$ are as defined previously in Equations (4) and (23)-(24). However, to avoid numerical singularity Equation (45) is rearranged to take the form:

and the diffusion, convection, dissipation, and stratification terms of the turbulent kinetic energy

$$\overline{u_i' u_j'} = \frac{(P - \epsilon + S)_{of\ \overline{u_i u_j}}}{(\frac{PS}{\overline{u_i u_j}})_{of\ \overline{u_i u_j}} + \frac{1}{k}(P - \epsilon + S)_{of\ k}}. \tag{46}$$

4.2 Heat Flux Equations

Following Rodi's reasoning, Meroney (1974,1976) extended Rodi's postulate to the heat flux equation, and he argued that the diffusion and convection terms of the heat flux equation are related to the convection and diffusion terms of the turbulent kinetic energy. Gibson and Launder (1976) proposed that the turbulent heat flux is related to both the turbulent kinetic energy and temperature fluctuations as:

$$(Convection - Diffusion)_{of\overline{u_i' T'}} = \frac{\overline{u_i' T'}}{\sqrt{k\ k_t}}(Convection - Diffusion)_{of\sqrt{k\ k_t}}$$

$$= \frac{\overline{u_i' T'}}{2k}[\frac{Dk}{Dt} - D(k)] + \frac{\overline{u_i' T'}}{2k_t}[\frac{Dk_t}{Dt} - D(k_t)]$$

$$= \frac{\overline{u_i' T'}}{2k}[P + S - \epsilon]_{of\ k} + \frac{\overline{u_i' T'}}{2k_t}[P - \epsilon]_{of\ k_t}. \tag{47}$$

In their analysis they assumed that the dissipation and production of k_t are in balance so that the second half of the above equation vanishes. In the current analysis the assumption of balance between temperature fluctuations production and dissipation is not imposed, so that the turbulent heat flux equation takes the form:

$$\overline{u_i' T'} = \frac{(P + PS - \epsilon + S)_{of\ \overline{u_i' T'}}}{\frac{1}{2k}(P - \epsilon + S)_{of\ k} + \frac{1}{2k_t}(P - \epsilon)_{of\ k_t}}, \tag{48}$$

and the diffusion, convection, dissipation, and stratification terms of the turbulent kinetic energy and temperature fluctuations are as defined in Equation (4) and (6) and (27)-(28), respectively.

4.3 Mass Flux Equations

Since the mass is a scalar quantity, one would expect by analogy that Gibson and Launder's postulations for heat flux can be further applied to the mass flux terms. Similarly, one can postulate that:

$$\overline{u_i' C'} = \frac{(P + PS - \epsilon + S)_{of\ \overline{u_i' C'}}}{\frac{1}{2k}(P - \epsilon + S)_{of\ k} + \frac{1}{2k_c}(P - \epsilon)_{of\ k_c}}, \tag{49}$$

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and concentration fluctuations are as defined in Equation (4) and (8) and (31)-(32), respectively.

4.4 Temperature Concentration Fluctuations

The temperature concentration fluctuation is expected to be related to both k_t and k_c according to the following correlation:

$$(Convection - Diffusion)_{of\ \overline{T'C'}} = \frac{\overline{T'C'}}{\sqrt{k_t\ k_c}}(Convection - Diffusion)_{of\ \sqrt{k_t}}$$

$$= \frac{\overline{T'C'}}{2k_t}[\frac{Dk}{Dt} - D(k_t)] + \frac{\overline{T'C'}}{2k_c}[\frac{Dk_t}{Dt} - D(k_c)]$$

$$= \frac{\overline{T'C'}}{2k_t}[P + S - \epsilon]_{of\ k_t} + \frac{\overline{T'C'}}{2k_c}[P - \epsilon]_{of\ k_c}.$$

$$(50)$$

Rearrangement of this equation, yields an equation for $\overline{T'C'}$ of the form:

$$\overline{T'C'} = \frac{(P - \epsilon)_{of\ \overline{T'C'}}}{\frac{1}{2k_t}(P - \epsilon)_{of\ k_t} + \frac{1}{2k_c}(P - \epsilon)_{of\ k_c}},$$

$$(51)$$

When proper terms are substituted in Equations (46), (48), (49) and (51), one obtains the algebraic equations for the turbulent fluctuations terms.

5.0 ALGEBRAIC STRESS MODEL EQUATIONS

Substitution of the terms outlined in the previous two sections into the one-dimensional turbulent governing equations yields, after some additional mathematical manipulation, a simplified form of the governing equations:

$$\frac{\partial u}{\partial t} = \frac{1}{Re}\frac{\partial^2 u}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} \qquad (52)$$

$$\frac{\partial T}{\partial t} = \frac{1}{Re\ Pr}\frac{\partial^2 T}{\partial z^2} - \frac{\partial(\overline{w'T'})}{\partial z} \qquad (53)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Re\ Sc}\frac{\partial^2 C}{\partial z^2} - \frac{\partial(\overline{w'C'})}{\partial z} \qquad (54)$$

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$$\frac{\partial k}{\partial t} = \frac{1}{Re}\frac{\partial^2 k}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_D k^2}{\sigma_k \epsilon}\frac{\partial k}{\partial z}\right) - (\overline{u'w'})\frac{\partial u}{\partial z} - \epsilon \qquad (55)$$
$$+ [Ri_T\overline{w'T'} - Ri_c\overline{w'C'}]$$

$$\frac{\partial k_t}{\partial t} = \frac{1}{Re\,Pr}\frac{\partial^2 k_t}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_H k k_t}{\sigma_{kt}\epsilon_t}\frac{\partial k_t}{\partial z}\right) - (\overline{w'T'})\frac{\partial T}{\partial z} - \epsilon_t \qquad (56)$$

$$\frac{\partial k_c}{\partial t} = \frac{1}{Re\,Sc}\frac{\partial^2 k_c}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_m k k_c}{\sigma_{kc}\epsilon_c}\frac{\partial k_c}{\partial z}\right) - (\overline{w'C'})\frac{\partial C}{\partial z} - \epsilon_c \qquad (57)$$

$$\frac{\partial \epsilon}{\partial t} = \frac{1}{Re}\frac{\partial^2 \epsilon}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_D k^2}{\sigma_k \epsilon}\frac{\partial \epsilon}{\partial z}\right)$$
$$- C_{E1}(\overline{u'w'})\left(\frac{\epsilon}{k}\right)\frac{\partial u}{\partial z} - C_{E2}\frac{\epsilon^2}{k} + F\frac{\epsilon}{k}[Ri_T\overline{w'T'} - Ri_s\overline{w'C'}] \qquad (58)$$

$$\frac{\partial \epsilon_t}{\partial t} = \frac{1}{Re\,Pr}\frac{\partial^2 \epsilon_t}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_H k k_t}{\sigma_{\epsilon t}\epsilon_t}\frac{\partial \epsilon_t}{\partial z}\right)$$
$$- C_{Et1}(\overline{w'T'})\left(\frac{\epsilon}{k}\right)\frac{\partial T}{\partial z} - C_{Et2}\left(\frac{\epsilon\epsilon_t}{k}\right) \qquad (59)$$

$$\frac{\partial \epsilon_c}{\partial t} = \frac{1}{Re\,Sc}\frac{\partial^2 \epsilon_c}{\partial z^2} + \frac{\partial}{\partial z}\left(\frac{C_m k k_c}{\sigma_{\epsilon c}\epsilon_c}\frac{\partial \epsilon_c}{\partial z}\right)$$
$$- C_{Ec1}(\overline{w'C'})\left(\frac{\epsilon}{k}\right)\frac{\partial C}{\partial z} - C_{Ec2}\left(\frac{\epsilon\epsilon_c}{k}\right) \qquad (60)$$

$$\overline{w^2} = \frac{\frac{2}{3}k\left[A_1\overline{(-u'w')}\frac{\partial u}{\partial z} + \epsilon(C_{p1}-1) + 3Ri_T\overline{w'T'} - 3Ri_c\overline{w'C'}\right]}{[C_{p1}\epsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'}]} \qquad (61)$$

$$\overline{u'w'} = \frac{[-(1-A_1)\overline{(w')^2}\frac{\partial u}{\partial z} + Ri_T\overline{u'T'} - Ri_c\overline{u'C'}]k}{[C_{p1}\epsilon - \overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'}]} \qquad (62)$$

$$\overline{w'T'} = \frac{[-(1-A_2)\overline{(w')^2}\frac{\partial T}{\partial z} + 2Ri_T k_t - Ri_c\overline{T'C'}]\,kk_t}{k_t[C_{p2}\epsilon + \frac{1}{2}(-\overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'})] + \frac{1}{2}k[-\overline{w'T'}\frac{\partial T}{\partial z} - \epsilon_t]} \qquad (63)$$

$$\overline{u'T'} = \frac{[(1 - A_3)(-\overline{u'w'}\frac{\partial T}{\partial z} - \overline{w'T'}\frac{\partial u}{\partial z})]\, kk_t}{k_t[C_{p3}\epsilon + \frac{1}{2}(-\overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'})] + \frac{1}{2}k[-\overline{w'T'}\frac{\partial T}{\partial z} - \epsilon_t]} \tag{64}$$

$$\overline{w'C'} = \frac{[-(1-A_2)\overline{(w')^2}\frac{\partial C}{\partial z} + Ri_T\overline{T'C'} - 2Ri_c k_c]\, kk_c}{k_c[C_{p2}\epsilon + \frac{1}{2}(-\overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'})] + \frac{1}{2}k[-\overline{w'C'}\frac{\partial C}{\partial z} - \epsilon_c]} \tag{65}$$

$$\overline{u'C'} = \frac{[(1 - A_3)(-\overline{u'w'}\frac{\partial C}{\partial z} - \overline{w'C'}\frac{\partial u}{\partial z})]\, kk_c}{k_c[C_{p3}\epsilon + \frac{1}{2}(-\overline{u'w'}\frac{\partial u}{\partial z} - \epsilon + Ri_T\overline{w'T'} - Ri_c\overline{w'C'})] + \frac{1}{2}k[-\overline{w'C'}\frac{\partial C}{\partial z} - \epsilon_c]} \tag{66}$$

$$\overline{T'C'} = \frac{(-\overline{T'w'}\frac{\partial C}{\partial z} - \overline{C'w'}\frac{\partial T}{\partial z})\, k_t k_c}{\frac{1}{2}k_c[-\overline{w'T'}\frac{\partial T}{\partial z} - \epsilon_t] + \frac{1}{2}k_t[-\overline{w'C'}\frac{\partial C}{\partial z} - \epsilon_c]} \tag{67}$$

The set of constants incorporated in the above equations has been determined experimentally by different researchers. The average values used in the current model are listed in table (1).

Table (1): Coefficients used in ASM model.

C	C_H	C_{E1}	C_{E2}	C_{Et1}	C_{Et2}	F	σ_k	σ_ϵ	σ_{kt}
0.09	0.09	1.44	1.92	1.44	1.92	1.5	1.0	1.3	0.7

$\sigma_{\epsilon t}$	σ_{kc}	$\sigma_{\epsilon c}$	A_1	A_2	A_3	C_{p1}	C_{p2}	C_{p3}	
1.0	0.7	1.0	0.55	0.0	0.0	2.5	2.5	2.5	

6.0 NUMERICAL SOLUTION

For a one-dimensional problem, nine partial differential equations and seven algebraic equations (Equations (52) through (67)) need to be solved numerically. The partial differential equations are parabolic, because the time derivative is first order and present in every equation while there is, at most, a second derivative in a spatial dimension. A finite difference approach is used to solve this set of equations. For the main variable in each equation, the method uses central-difference formulae in space evaluated at forward time steps, while for the other variables, information from previous time step were used. The resulting algebraic equations were solved by use of Thomas algorithm in the z-direction. This method requires a double sweep of the z profile and is actually a straightforward Gaussian elimination procedure.

It was realized (Meroney (1974)) that it would be physically meaningless to allow positive definite quantities (such as velocity, temperature, concentration, turbulent energy, turbulent energy fluctuation, or turbulent concentration fluctuations) become negative. One can avoid the anomalous behavior by evaluating all the sink terms as if they were $(sink)^n f^{n+1} / f^n$, where n is the time step, and f is the dependent variable.

A computer code in Fortran language, was written to perform the numerical calculations needed to solve the set of equations. The logic of the solution is as follows:

1) The physical properties of the fluid and the ASM constants are specified.
2) The initial values of all the variables are stated at time t =0.
3) The coefficients for the tri-diagonal matrix for each partial differential equation is calculated using the information of the previous time step. Then the tri-diagonal matrix is solved for each equation to evaluate the values of u, T, C, k, k_t, k_c, ε, $ε_t$, and $ε_c$ at time t + Δt.
4) Using the result of step 3 above, the Reynolds stresses, the heat fluxes and mass fluxes are calculated by their algebraic equations.
5) Steps 3-4 are repeated for the next time step, and the solution is continued until the desired number of time steps is achieved.
6) Output is printed at selected time steps.

The solution requires the initial profiles for temperature, velocity and concentration. To initiate the turbulence, the initial conditions for the other variables are assigned the following values:

$$k(z,0) = ε(z,0) = k_t(z,0) = k_c(z,0) = ε_t(z,0) = ε_c(z,0) = 1.0 \times 10^{-3}$$
$$\overline{u'T'}(z,0) = \overline{w'T'}(z,0) = \overline{u'C'}(z,0) = \overline{w'C'}(z,0) = 1.0 \times 10^{-3}$$

In order to satisfy the continuity equation for this 1-D problem the velocity in the upper mixed layer is assumed to be uniform, and its value decreases with time due to the thickening of the layer. By applying the conservation of mass equation one can calculate the value of the uniform velocity at each time step as:

$$\rho Q = \int_0^H \rho(t) u(t) \, dz = \overline{\rho} \, \overline{u} \Delta z + \int_0^{(H-\Delta z)} \rho(t) u(t) \, dz.$$

By rearranging this equation, the velocity of the upper layer becomes:

$$\overline{u} = \frac{Q}{\Delta z} + \frac{1}{\overline{\rho} \Delta z} \int_0^{(H-\Delta z)} \rho(t) u(t) \, dz.$$

For constant heat flux cases the temperature at z = 0, 1 can be calculated from the energy balance in the cell adjacent to the wall:
while for the constant temperature cases the temperature at the bottom wall can be specified by

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$$T_1^{n+1} = T_1^n + \frac{Q_H}{\rho C_v}\left(\frac{\Delta t}{\Delta z}\right) - \overline{w'T'}\left(\frac{\Delta t}{\Delta z}\right),$$

$T_1 = 1.0$, and at the free surface $T_N = 0.0$.

The rest of boundary conditions are specified as:

@ z=0: $C_1 = C_2$
$u_i(0,t) = 0$
$k(0,t) = \varepsilon(0,t) = 0, k_t(0,t) = k_c(0,t) = \varepsilon_t(0,t) = \varepsilon_c(0,t) = 0$
$\overline{u'T'}(0,t) = \overline{w'T'}(0,t) = \overline{u'C'}(0,t) = \overline{w'C'}(0,t) = \overline{T'C'}(0,t) = 0$

@ z=1: $C_N = C_{N-1}$
$k(1,t) = \varepsilon(1,t) = 0, k_t(1,t) = k_c(1,t) = \varepsilon_t(1,t) = \varepsilon_c(1,t) = 0$

$\overline{u'T'}(1,t) = \overline{w'T'}(1,t) = \overline{u'C'}(1,t) = \overline{w'C'}(1,t) = \overline{T'C'}(1,t) = 0$

7.0 NUMERICAL RESULTS AND DISCUSSION

The numerical model developed in the previous sections is used here to predict the behavior of double diffusive flows. In order to verify the performance of the model, initial and boundary conditions similar to experimental data provided by Al-Ghamdi (1993) were used to establish a base for comparison between the numerical results and the corresponding experimental data. The numerical model has the advantage of producing some variables that are not measured in the experiments, either due to absence of the appropriate instrumentation during the course of the experiments (mean velocity, velocity fluctuation, temperature fluctuation and concentration fluctuation), or to the lack of methods able to measure some variables (such as the dissipation of energy, temperature and concentration). The test case examined is for a salty fluid that consists of three layers, namely; a bottom mixed layer at high density, an upper mixed layer at low density, and a stably stratified layer in between. The system is heated from below while the temperature of the upper layer is kept constant. During the experiment the upper layer moved at controlled velocity. Numerical calculations for two different cases are presented below, and the changes of the temperature, salinity, and density with time as well as the entrainment across the interface are examined.

CASE (I): A constant bottom temperature condition is considered. The parameters for this case are similar to the conditions of experiment MLH9 of Al-Ghamdi (1993), and they are:

Bottom temperature	32 °C
Upper temperature	22 °C
Average upper layer velocity	0.0026 m/s

Salinity in bottom layer	5%
Salinity in upper layer	0%
Water level in the tank (H), mm	1533
Lower interface location (z/H)	.33
Upper interface location (z/H)	.75
Reynolds number	4.15×10^3
Thermal Richardson number	8.90×10^3
Solute Richardson number	8.88×10^4

The calculated temperature, concentration and density profiles are shown in Figures (1) to (3). One can see a trend similar to that observed experimentally. The interfacial layer thickness predicted by the numerical solution was found to be about 32 mm. It lies in the range of the experimental findings given by Al-Ghamdi (1993) of 16 mm to 40 mm. An expanded view of the upper interface is depicted in Figure (4). From this figure the location of the upper interface was calculated, and its progress with time is compared with the experimental results of in Figure (5). Figure (5) indicates that the numerical solution overpredicted the progress of the upper interface by about 5%. A Comparison of the numerically calculated temperature profiles with the experimental data are presented in Figure (6). This figure suggests that the numerical calculations and the experimental data aligned well in the upper part of the tank, but the numerical solution significantly overpredicted the entrainment at the lower interface. This overprediction of the entrainment rate is most likely due to the approximation of strongly 3-D mixing process by a 1-D numerical mixing model.

CASE (II): In this case a constant heat flux is applied to the tank bottom. These conditions represented the situation studied by experiment MHL2 of Al-Ghamdi (1993), and they are:

Heat flux supplied to the bottom	196 w/m^2
Upper temperature	22 °C
Average upper layer velocity	0.0028 m/s
Salinity in bottom layer	5%
Salinity in upper layer	0%
Water level in the tank (H), mm	1600
Lower interface location (z/H)	.35
Upper interface location (z/H)	.78
Reynolds number	4.67×10^3
Thermal Richardson number	4.80×10^3
Solute Richardson number	7.01×10^4

The numerical results for this case are presented in Figures (7) to (12). Again these figures suggest a qualitatively satisfactory solution, but the numerical model again overpredicts the entrainment rate at the interfaces. These results show that the numerical solution can predict the layering phenomenon observed experimentally, but it predicted layering to occur earlier than observed during experiments.

Once this layering occurred, a numerical instability developed, and the program was terminated. In this specific case only one layer appeared, but in some other test cases a series of layers were produced before an instability in the program occurred. A comparison of experimental and numerical entrainment rates at the upper interface are shown in Figure (11). The numerical model overpredicted the entrainment rate by about 15 %. The temperature profiles calculated numerically are compared with their corresponding experimental results in Figure (12). Good agreement between the numerical solution and the experimental data occurs in the upper part of the tank, where the flow is mainly one-dimensional, but in the lower part of the tank where the flow is fully 3-dimensional, the numerical model overestimated the mixing process.

8.0 CONCLUSIONS AND RECOMMENDATIONS

The phenomenon of double diffusive flow, where both heat and concentration coexist in the flow field, was investigated in this research numerically using a 1-D Algebraic Stress Model (ASM). When compared with experimental data, the numerical model exhibited trends consistent with the experimental results, revealed the same profile phenomena, and gave acceptable mixing rates. The numerical results permitted a wider range of boundary conditions than the one possible during the experiments. It provided a qualitative estimation of experimentally unmeasured quantities.

The model was able to predict the layering phenomenon observed in the constant heat flux experiments. But quantitatively, the model overpredicted the entrainment rate at the upper interfaces by 5 to 15 %. The numerical model agreed well with the experimental results in the upper part of the system where the flow was mainly 1-D, but in the lower part of the system where the flow was fully 3-D the numerical model overpredicted the mixing process. Finally the 1-D model turned out to be a good tool for first order prediction. Expansion of the model to a 2-D or 3-D case is expected to result in a better prediction of the mixing processes associated with doubly-diffusive flow subjected to combined heating and shear flow.

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APPENDIX

The algebraic stress model equations were made dimensionless by using the following scaling parameters:

$$u^* = u / u_{max}$$
$$z^* = z/H$$
$$t^* = (t \, u_{max})/H$$
$$T^* = (T - T_c)/(T_h - T_c)$$
$$C^* = (C - C_l)/(C_h - C_l)$$
$$k^* = k / u_{max}^2$$
$$k_t^* = k_t / \Delta T_{max}^2$$
$$k_c^* = k_c / \Delta C_{max}^2$$
$$\varepsilon^* = (\varepsilon \, H)/u_{max}^3$$
$$\varepsilon_t^* = (\varepsilon_t \, H)/(\Delta T_{max}^2 u_{max})$$
$$\varepsilon_c^* = (\varepsilon_c \, H)/(\Delta C_{max}^2 u_{max})$$
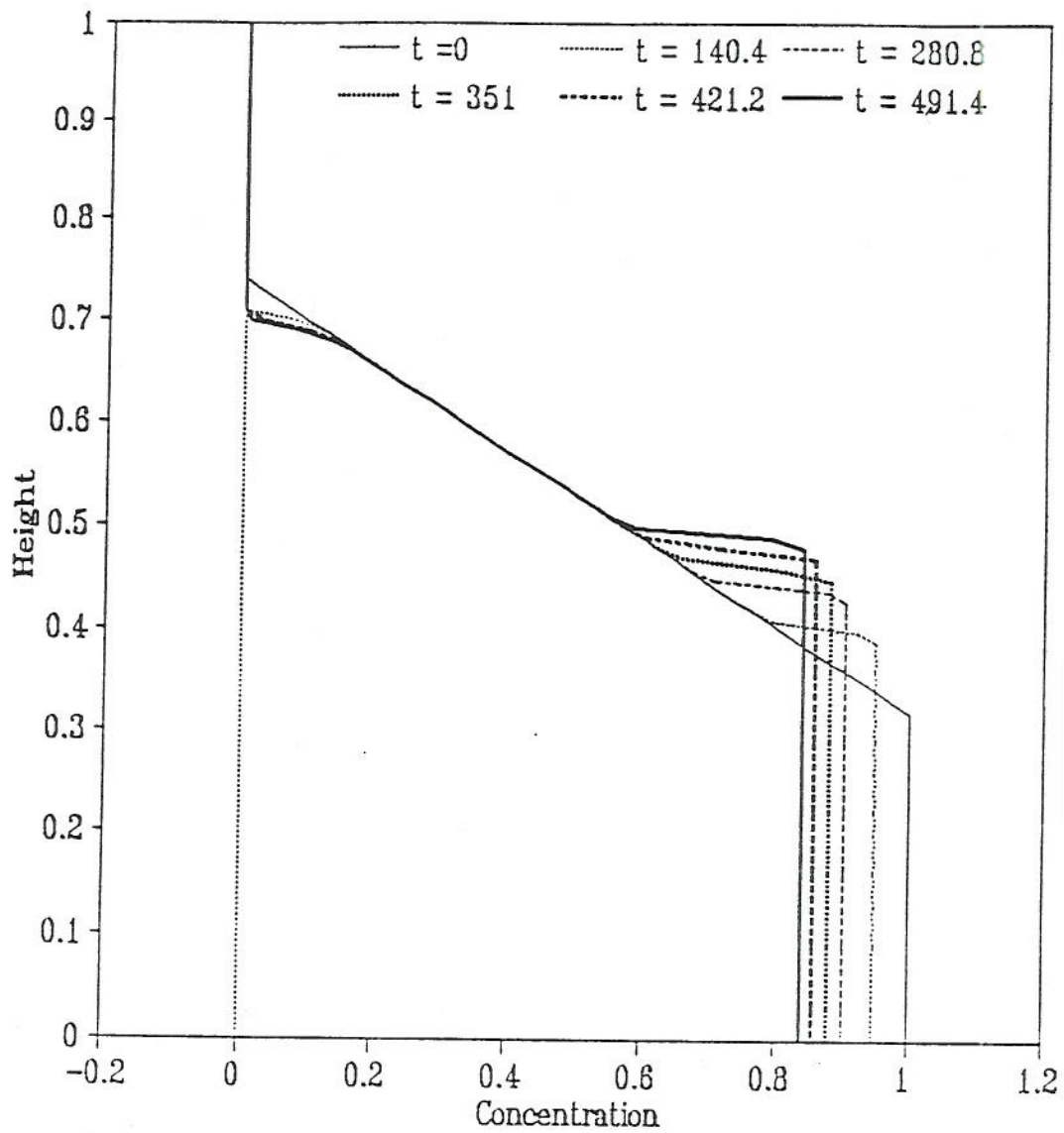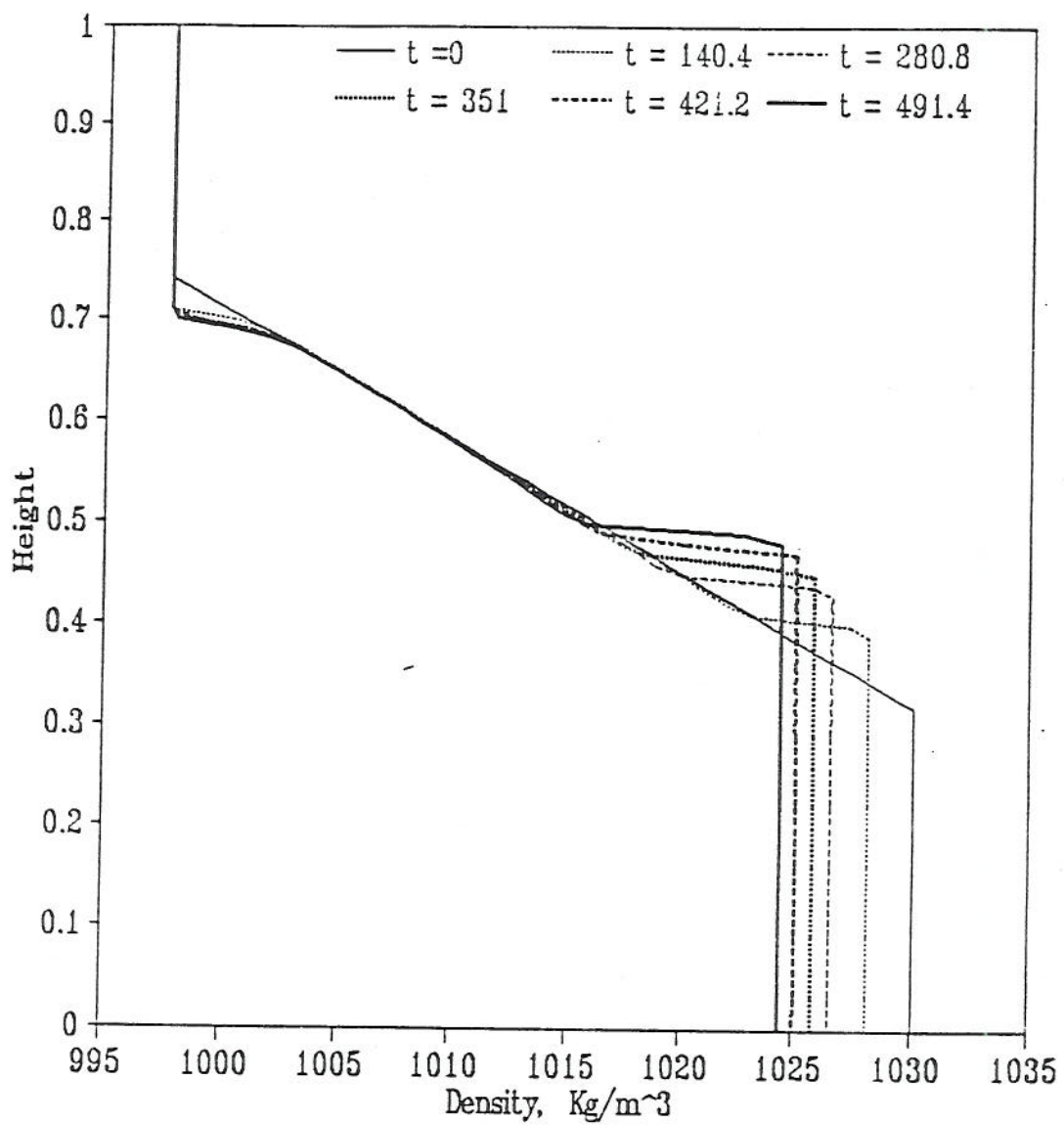$$p^* = p/(\rho u_{max}^2)$$
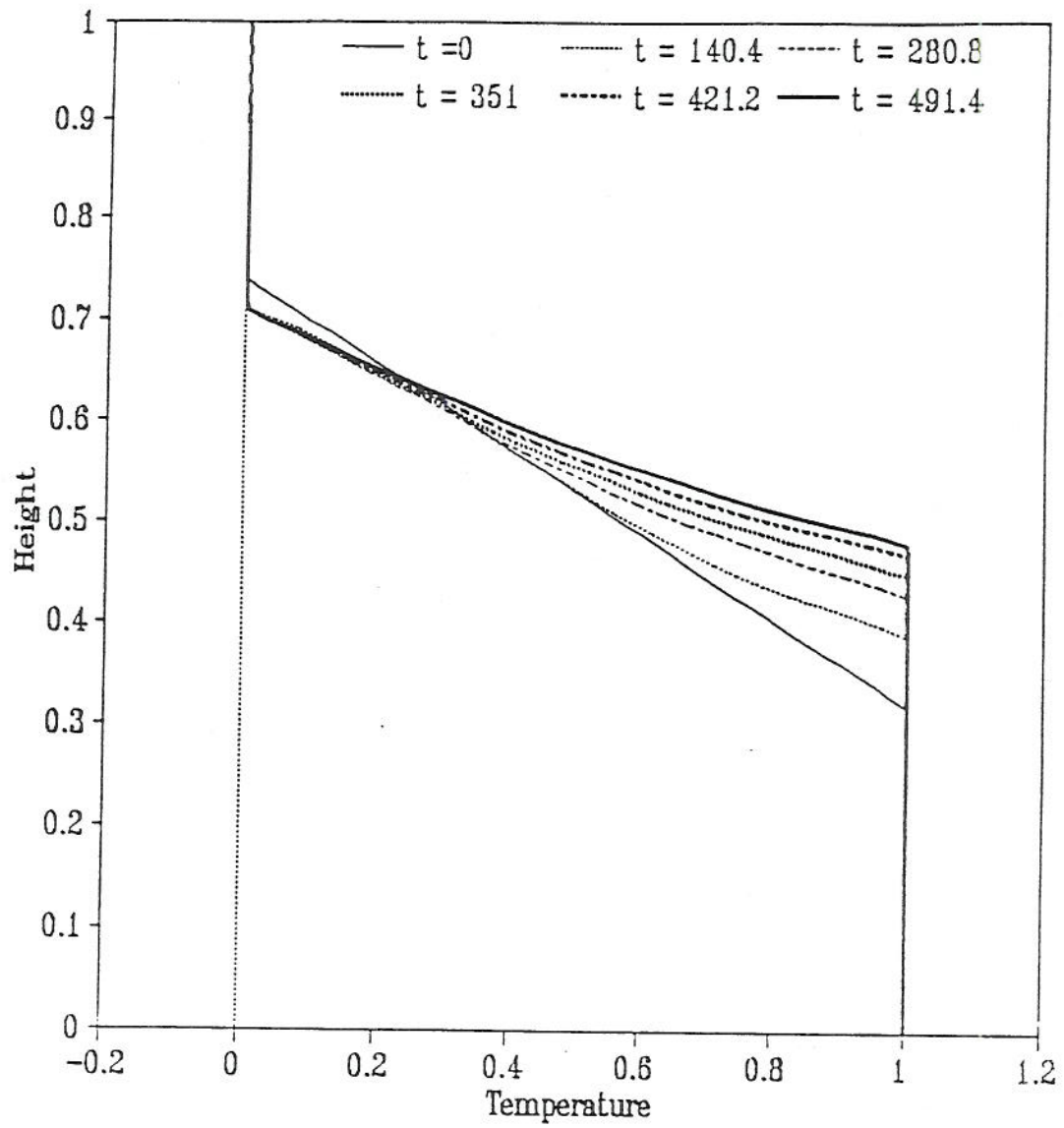
Figure 1   Numerically Calculated Temperature Profiles for Case (I).

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Figure 2   Numerically Calculated Concentration Profiles for Case (I).

Figure 3 Numerically Calculated Density (Dimensional) Profiles for Case (I).
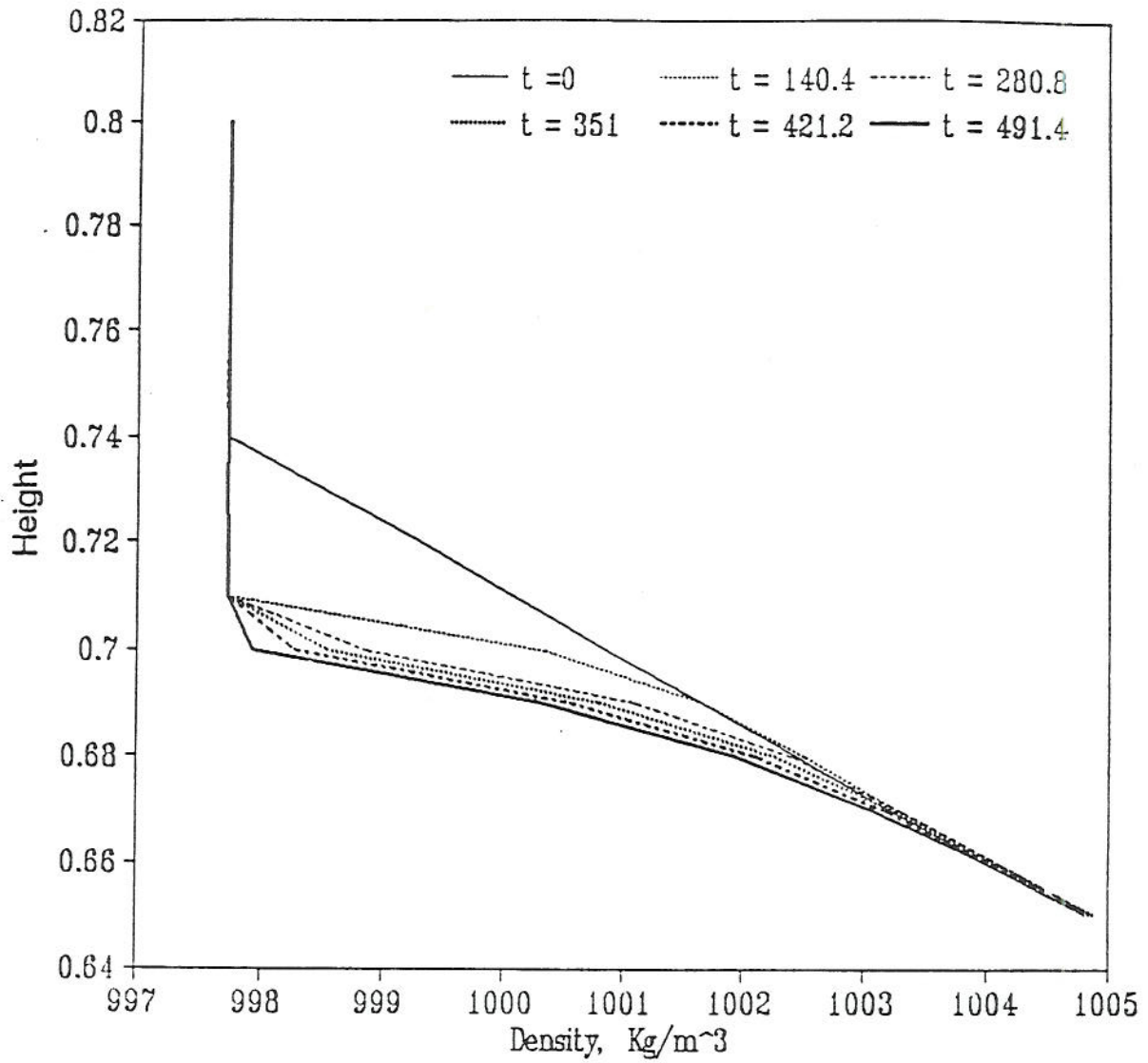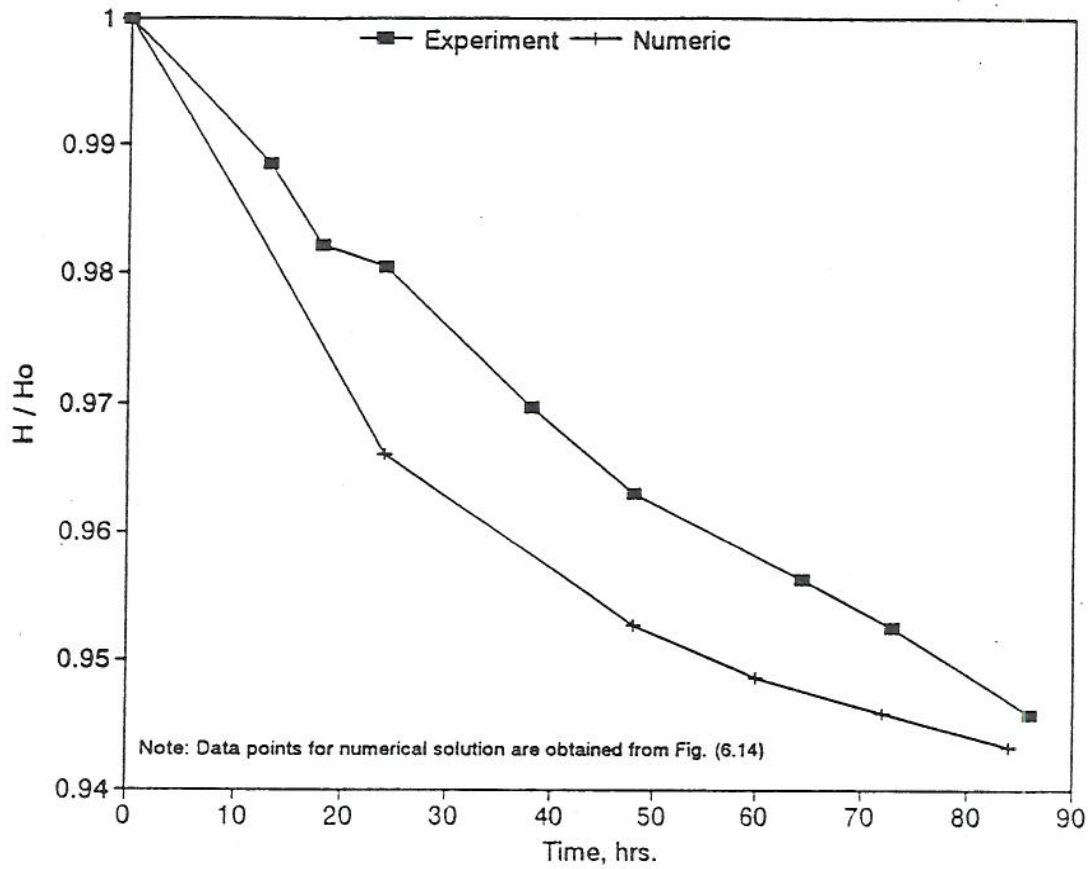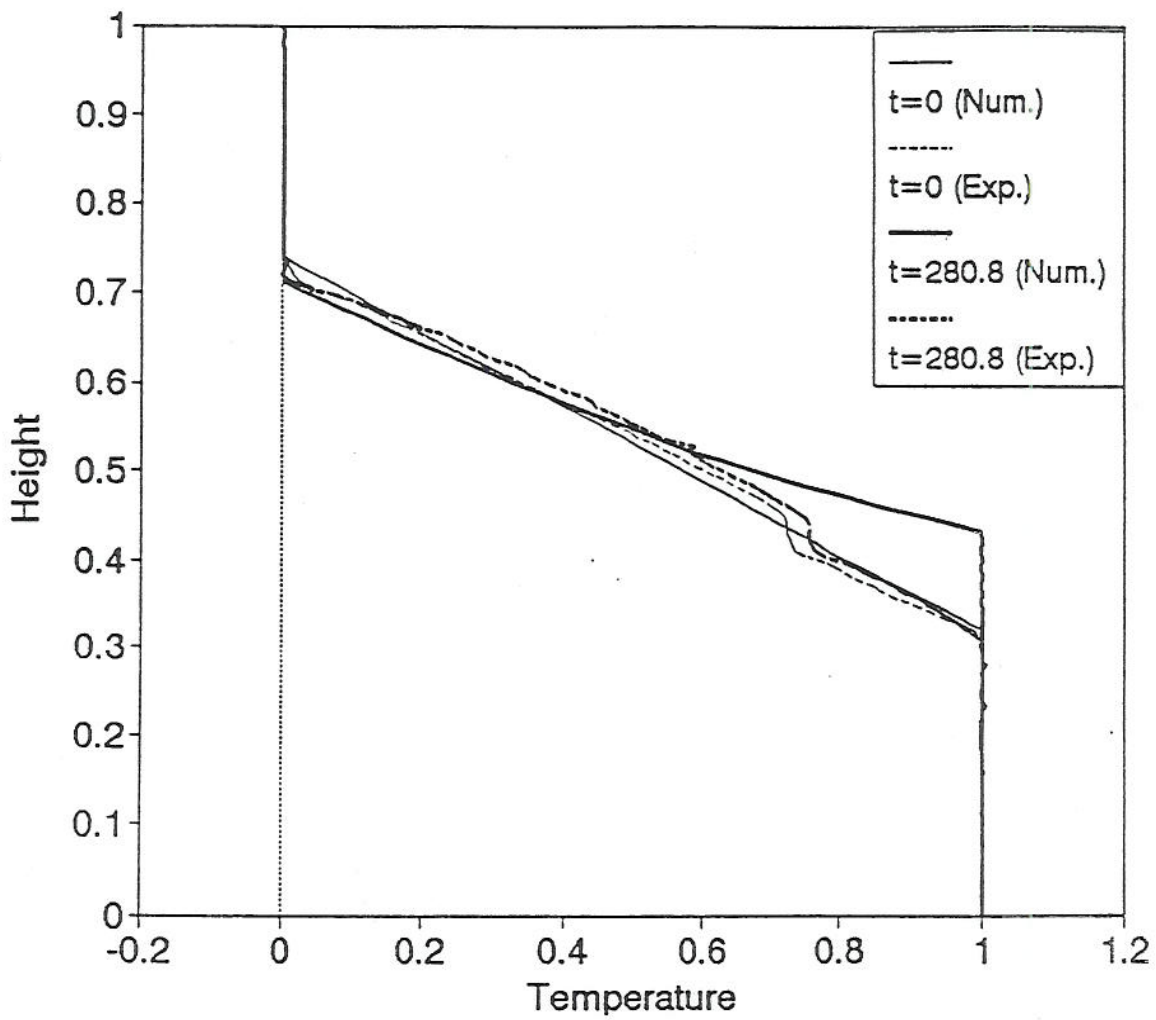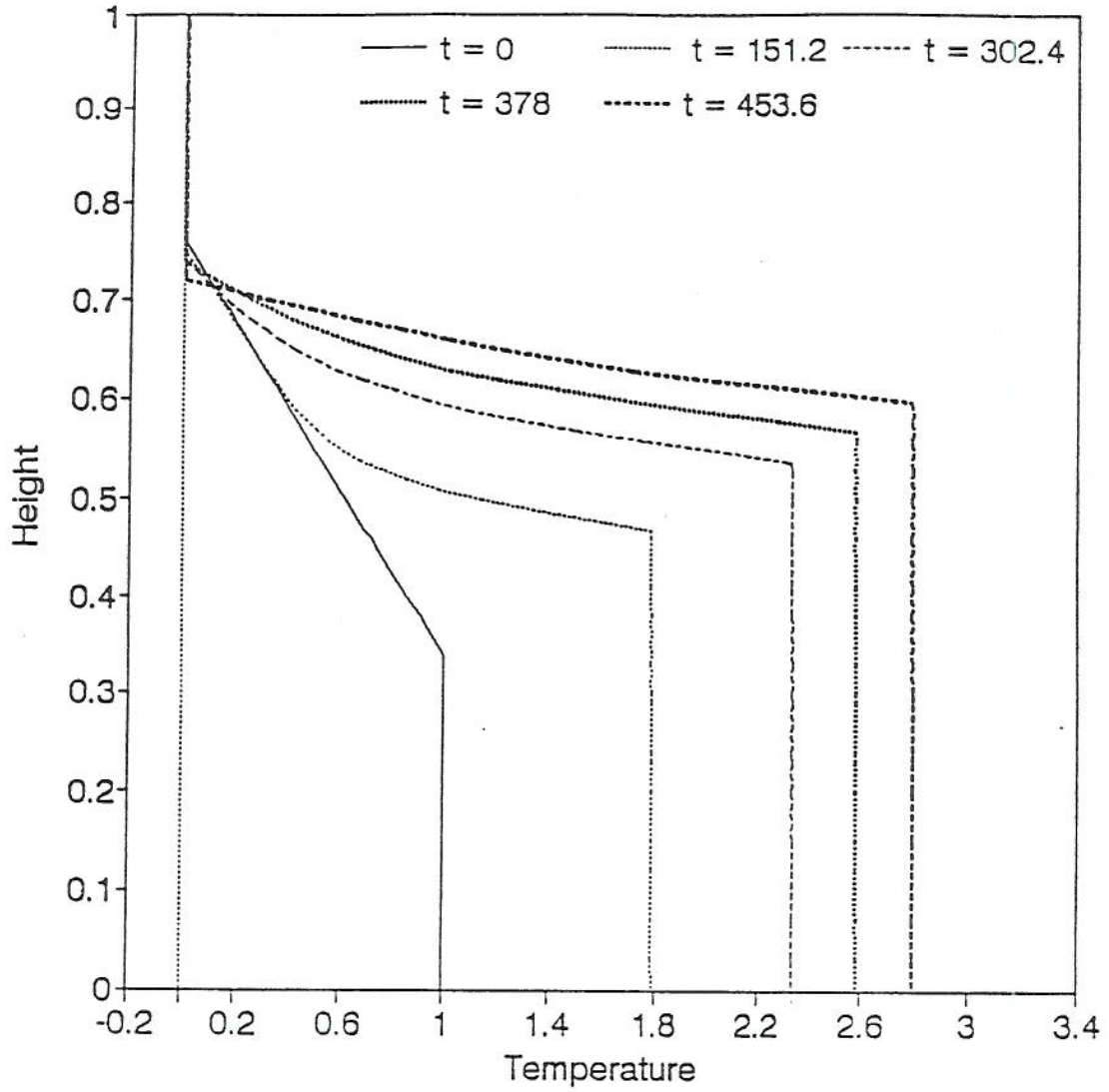
26

Figure 4 Density (Dimensional) Profiles Near the Upper Interface for Case (I).

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Figure 5   Comparison of Experimentally Measured and Numerically Calculated Upper Density Interface Locations for Experiment MLH9.
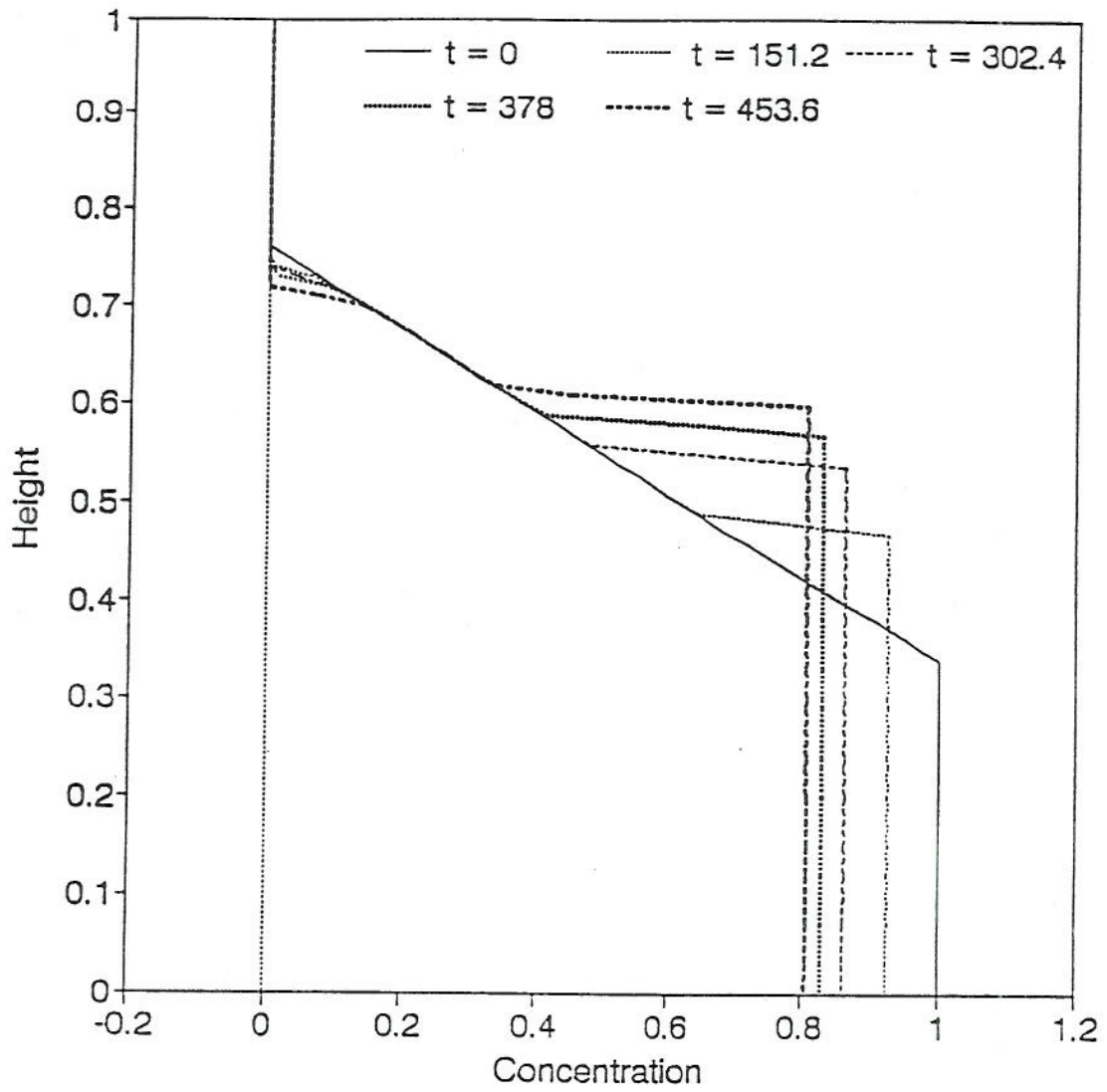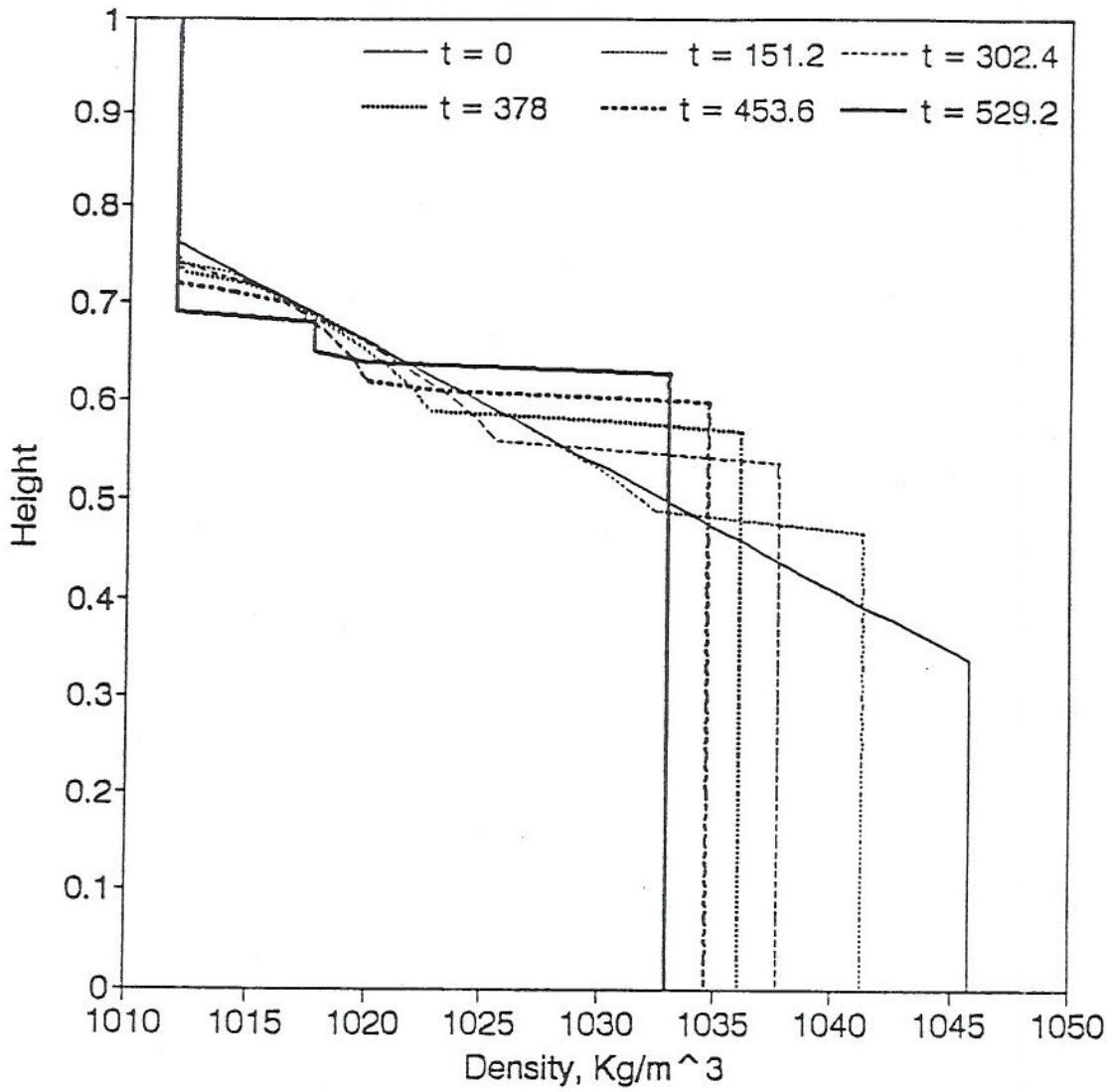
Figure 6   Comparison of the Experimentally Measured and Numerically Calculated
Temperature Profiles for Case (I).

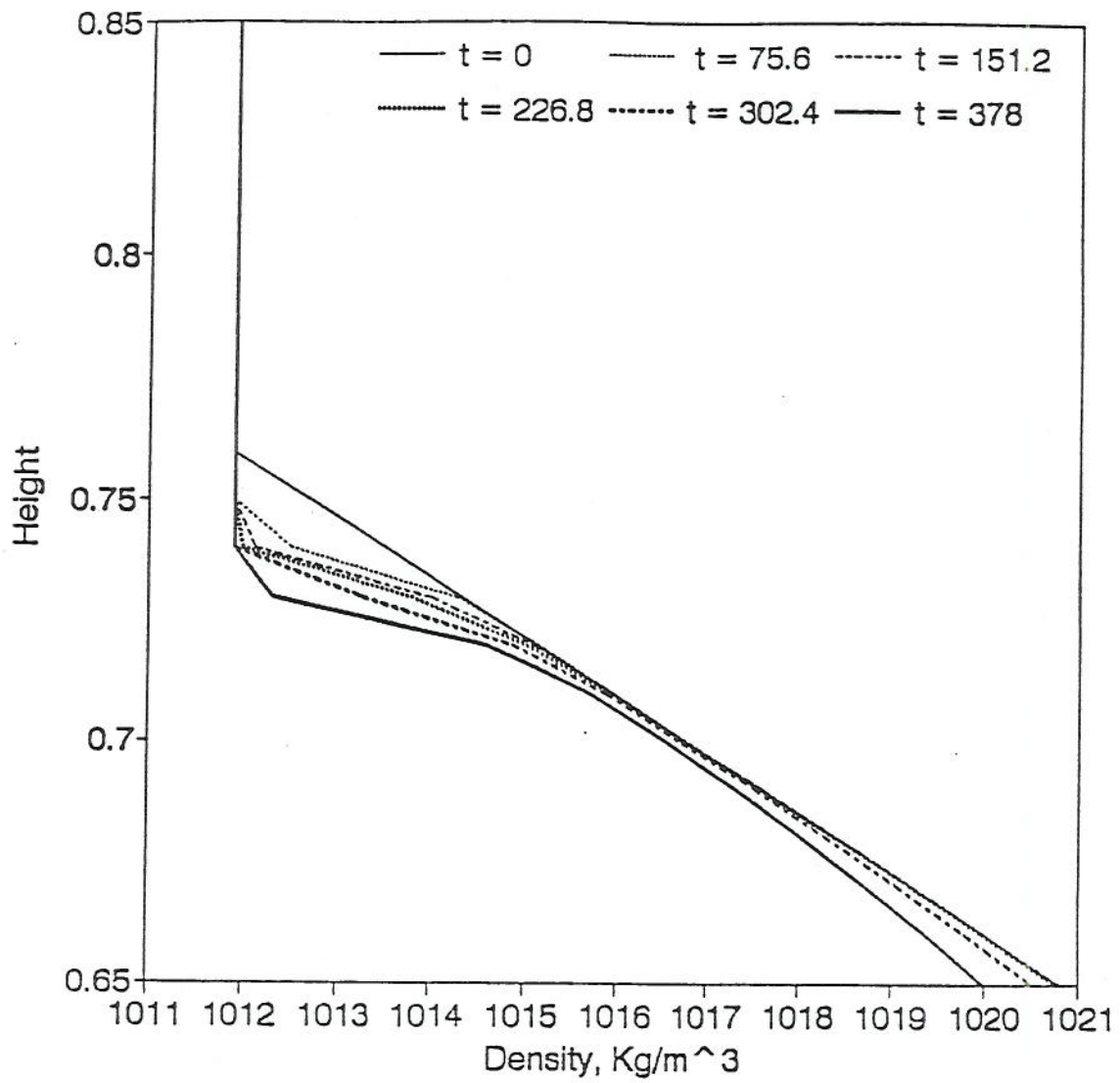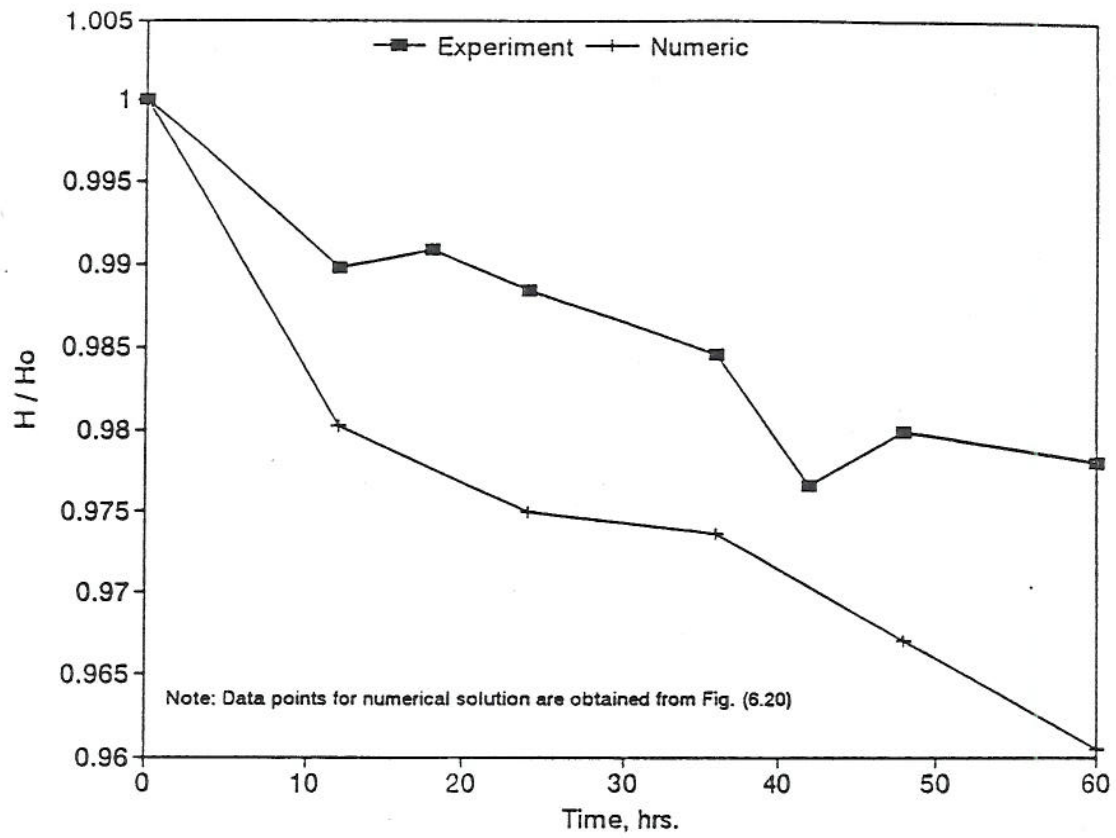**Figure 7**   Numerically Calculated Temperature Profiles for Case (II).

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Figure 8   Numerically Calculated Concentration Profiles for Case (II).
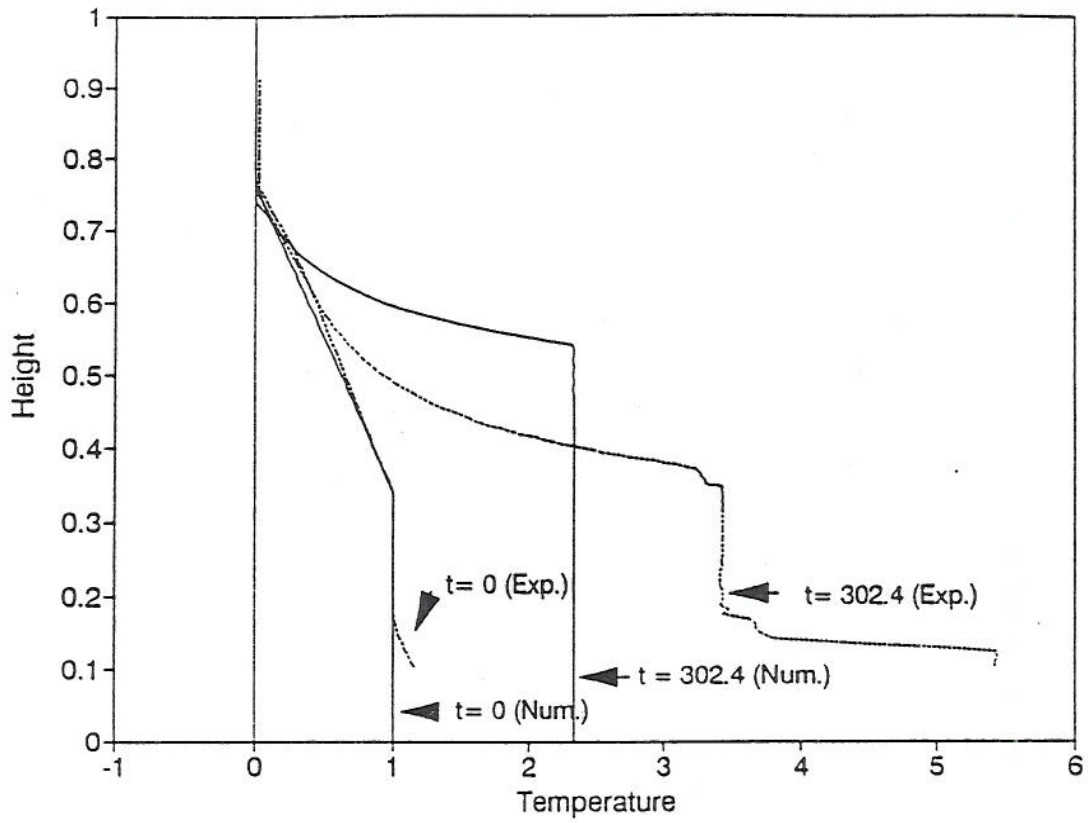
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Figure 9   Numerically Calculated Density (Dimensional) Profiles for Case (II).

Figure 10   Density (Dimensional) Profiles Near the Upper Interface for Case (II).

Figure 11   Comparison of Experimentally Measured and Numerically Calculated Upper Density Interface Locations for Experiment MHL2.

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Figure 12   Comparison of the Experimentally Measured and Numerically Calculated Temperature Profiles for Case (II).