Mistiming Performance Analysis of the Energy Detection Based ToA Estimator for MB-OFDM

Huilin Xu, Liuqing Yang (*contact author*), Y. T. Jade Morton and Mikel M. Miller

***Abstract*— In this letter, we apply energy detection based time-of-arrival (ToA) estimation to multi-band orthogonal frequency-division multiplexing signals. We analyze the mistiming performance of the ToA estimator in the Nakagami-*m* channel. Analysis shows that the slope of the probability of mistiming curve increases with the number of subbands and the Nakagami-*m* parameter. This is known in communications as diversity. Simulations are carried out in various channels to corroborate our theoretical analysis.**

I. Introduction

In recent years, multi-band orthogonal frequency-division multiplexing (MB-OFDM) has attracted wide interest since it was proposed as one of the candidate technologies for the ultra-wideband (UWB) system [1], [2]. With the large bandwidth, MB-OFDM can achieve a high data rate. In addition, due to the huge bandwidth, MB-OFDM signals can provide orders of magnitude improvement on the timing precision compared to conventional narrowband signals. In this letter, we will investigate the time-of-arrival (ToA) estimation with MB-OFDM signals.

Usually, ToA estimation is carried out by a channel estimator. This means that the channel impulse response (CIR) is first recovered and then the delay of the first path automatically becomes the ToA estimate. As the MB-OFDM signal consists of several subbands each only containing part of the channel information, these subband signals need to be combined at the receiver to obtain full knowledge of the channel. Generally, two strategies can be taken to combine the multi-band channel information. Under the first strategy, the channel information from all subbands is jointly utilized for channel and ToA estimation, which is often termed as the *coherent combining* (see, e.g., [3], [4]). With the second strategy, the CIR is first estimated for each subband and then channel estimates from all subbands are averaged for a better final ToA estimate [5], which is called the *noncoherent combining*.

The coherent combining can usually provide a better timing resolution than the noncoherent combining at the price of a higher computational complexity (see, e.g., [3], [4]). However, there are scenarios where one has to adopt the noncoherent combining. For example, the channel information can not be coherently combined when random phase rotations are present after carrier demodulation. Even worse is that the channel itself may be independent across subbands due to the frequency dependent fading as the channel fading and dispersion statistics vary with the frequency [6][1]. For these reasons, we apply the noncoherent combining or the *energy detection* method for ToA estimation with MB-OFDM signals.

As the analysis of such a problem is very complicated for general channel environments, we will confine our discussion to the Nakagami-*m* fading channel. A series of channel campaigns have shown that the Nakagami-*m* distribution can fit the channel measurement results very well [7]. In addition, the Nakagami-*m* distribution is a general fading model, parameters of which can be adjusted to incorporate several fading environments, such as the Rayleigh distribution ($m = 1$) and the one-sided Gaussian distribution ($m = 0.5$). Therefore, the analysis and results of this letter are applicable to a wide range of channel fading environments.

Based on the analysis of the pairwise mistiming probability, we will prove that the timing performance can be improved with MB signals by exploiting the diversity across subbands. Although [5] touches upon a similar issue, it assumes a Rayleigh fading channel with independent multipath components and a uniform power delay profile (PDP). Instead of these limiting constraints, we consider in this letter the Nakagami-*m* fading channel with arbitrary multipath correlation and arbitrary PDP.

Following the method in [8, Ch. 12], we calculate the pairwise mistiming probability based on the moment generating functions (MGF) of the probability density function (PDF) of the instantaneous signal to noise ratio (SNR). With the union bound analysis, we can derive the diversity order of the mistiming probability curve.

II. Energy Detection Based ToA Estimator

In this section, we will first introduce the MB-OFDM system that is investigated in this letter. Architectures of the MB-OFDM transmitter and receiver are illustrated in Fig. 1 (a) and (b) as specified in [2]. Different from the basic OFDM system, for MB-OFDM, the baseband signal is carrier modulated on one of the B frequency bands at the transmitter according to the frequency hopping pattern, and the received waveform is carrier demodulated accordingly at the receiver.

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[1]Note that the independent channel across subbands can also be induced by the dense multipath. However, this type of independency will not prevent one from coherently combining subband channel information.

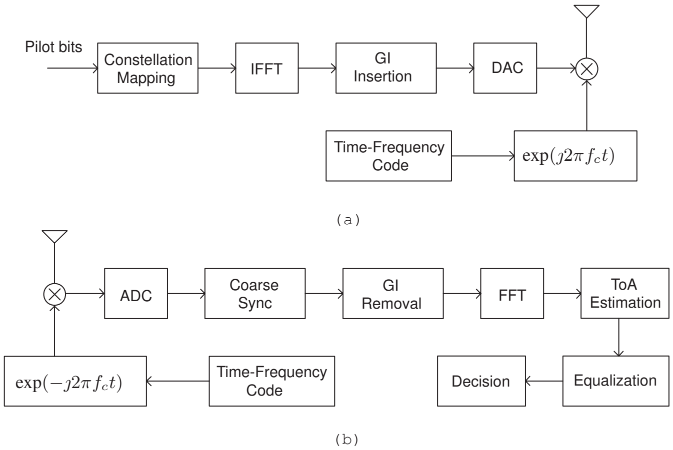

Fig. 1. MB-OFDM transceiver block diagrams (a) Transmitter; (b) Receiver.

Let $s_b(t)$ denote the baseband signal transmitted on the bth subband. The noise-free baseband received signal $r_b(t)$ can be expressed as

$$r_b(t) = s_b(t) * g_b(t) * c_b(t),\ b \in [1, B] \tag{1}$$

where $*$ denotes the convolution operation, $g_b(t)$ is a linear time invariant filter including possible transmit and receive filters as well as the bandlimiting effect of transmitting a finite bandwith signal, and $c_b(t)$ is the baseband channel for the bth subband

$$c_b(t) = \sum_{l=1}^{L_c} A_{b,l}\delta(t - \tau_{b,l}),\ b \in [1, B] \tag{2}$$

where $\{A_{b,l}\}_{l=1}^{L_c}$ and $\{\tau_{b,l}\}_{l=1}^{L_c}$ are amplitudes and delays of the L_c channel paths. Note that due to the frequency dependent fading [6], $c_b(t)$ can be independent across subbands.

The baseband received signal $r_b(t)$ is sampled at the rate of $1/T_s$ which is essentially the bandwidth of the transmitted signal $s_b(t)$. The noise-free discrete time samples $r_{b,n} = r_b(t)|_{t=nT_s}$ can be expressed as

$$r_{b,n} = s_{b,n} * h_{b,n},\ b \in [1, B] \tag{3}$$

where $s_{b,n}$ is the nth sample transmitted on the bth subband and $h_{b,n}$ is the equivalent discrete time channel

$$h_{b,n} = [c_b(\tau) * g_b(\tau)]\,|_{\tau=nT_s},\ b \in [1, B]\,, n \in [0, L-1] \tag{4}$$

with L being the number of channel taps. Representing the delay spread of the channel, L is assumed to be known at the receiver.

In order to facilitate the ToA estimation, the discrete time channel (4) is first estimated for all B subbands. The nth tap of the estimated channel for the bth subband can be expressed as follows:

$$\bar{h}_{b,n} = \begin{cases} h_{b,n-L_1} + \eta_{b,n},\ n \in [L_1, L_1 + L - 1] \\ \eta_{b,n},\ n \in [0, L_1 - 1] \\ \quad \text{and } n \in [L_1 + L, L_1 + L + L_2 - 1] \end{cases},\ b \in [1, B] \tag{5}$$

with $L_1 < L$ and $L_2 < L$. Due to the lack of timing information, the channel is estimated with a larger number of taps than L. Among the $(L_1 + L_2 + L)$ taps in (5), L consecutive taps contain the noise-contaminated channel information and the rest $(L_1 + L_2)$ taps only contain noise. Following [5], we also assume that the noise $\eta_{b,n}$ is independent zero mean circularly symmetric complex Gaussian with unit variance.

In [5], the following assumptions are made: 1) channel taps are independent within each subband; 2) the channel is uncorrelated over all B subbands; 3) channel taps are Gaussian random variables, and the channel has a uniform PDP. In this letter, we will consider the correlation among channel taps both inside each subband and over all subbands. This is more realistic since subbands always share some common channel information. In addition, the analysis is performed when the channel tap $h_{b,n}$ has a Nakagami-*m* distribution with arbitrary PDP.

After the channel is estimated, a simple energy detection based synchronizer can be adopted for ToA estimation. In particular, for a single band, this estimator detects the start of the channel by seeking the maximum total energy of a length L segment in the channel estimate sequence given by (5). With the availability of multiple subbands, this ToA estimator simply combines the energy from all subbands. Then, the index of the first channel tap can be estimated by

$$\bar{k} = \arg\max_p \left(\sum_{b=1}^{B} \sum_{n=p}^{p+L-1} |\bar{h}_{b,n}|^2 \right). \tag{6}$$

According to the synchronization criterion (6), mistiming occurs when the following inequality holds for any $l \in [1, L_2]$ or $l \in [-L_1, -1]$:

$$\sum_{b=1}^{B} \sum_{n=L_1+l}^{L_1+L+l-1} |\bar{h}_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1}^{L_1+L-1} |\bar{h}_{b,n}|^2 . \tag{7}$$

After the common terms are canceled in (7), the inequality becomes

$$\sum_{b=1}^{B} \sum_{n=L_1+L}^{L_1+L+l-1} |\eta_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1}^{L_1+l-1} |h_{b,n-L_1} + \eta_{b,n}|^2, \quad \text{for } l \in [1, L_2] \tag{8}$$

and

$$\sum_{b=1}^{B} \sum_{n=L_1+l}^{L_1-1} |\eta_{b,n}|^2 > \sum_{b=1}^{B} \sum_{n=L_1+L+l}^{L_1+L-1} |h_{b,n-L_1} + \eta_{b,n}|^2, \quad \text{for } l \in [-L_1, -1]\,. \tag{9}$$

III. Analysis of Mistiming Probability

In this section, we will analyze the mistiming probability of the energy detection based ToA estimator. We will first derive the pairwise probability of mistiming by an arbitrary number of l taps. Then we use the union bound to obtain an upper bound of the mistiming probability of the ToA estimator. The high SNR approximation of the upper bound will clearly reveal the relationship between the channel parameters and the timing performance. In the following, we will focus on the $l > 0$ case. The analysis can be directly applied to the $l < 0$ case.

Let $v_l = \sum_{b=1}^{B}\sum_{n=L_1}^{L_1+l-1}|h_{b,n-L_1}+\eta_{b,n}|^2$ and $y_l = \sum_{b=1}^{B}\sum_{n=L_1+L}^{L_1+L+l-1}|\eta_{b,n}|^2$, $l \in [1, L_2]$. The pairwise probability of mistiming by l taps is

$$P_l = \Pr(y_l > v_l),\ l \in [1, L_2] \tag{10}$$

which can be further expressed as

$$P_l = \int_0^\infty f_V(v_l)\int_{v_l}^\infty f_Y(y_l)dy_l dv_l \tag{11}$$

where $f_V(v_l)$ and $f_Y(y_l)$ are PDFs of v_l and y_l, respectively. Since variance of the noise is unity, i.e., $\mathrm{E}\{\eta_{b,n}^2\} = 1$, $\forall b$ and n, the random variable y_l has a chi-square distribution with $2Bl$ degrees of freedom

$$f_Y(y_l) = \frac{1}{(Bl-1)!}y_l^{Bl-1}\exp(-y_l)\ . \tag{12}$$

Using the result in [9], we have the pairwise probability of mistiming by l taps

$$P_l = \sum_{n=0}^{Bl-1}\frac{1}{n!}\int_0^\infty f_V(v_l)v_l^n\exp(-v_l)dv_l\ . \tag{13}$$

Before deriving P_l, we first calculate the conditional pairwise probability of mistiming, given the channel fading coefficients. For certain $\lambda_l = \sum_{b=1}^{B}\sum_{n=0}^{l-1}|h_{b,n}|^2$, the random variable v_l obeys a noncentral chi-square distribution with PDF [8, Ch. 10]

$$f_V(v_l;\lambda_l) = \left(\frac{v_l}{\lambda_l}\right)^{\frac{Bl-1}{2}}\exp\left(-(v_l+\lambda_l)\right)I_{Bl-1}\left(2\sqrt{v_l\lambda_l}\right) \tag{14}$$

where $I_a(x)$ is the modified Bessel function of the first kind.

Using (13) and property of the Laplace transform: $\mathcal{L}\{v_l^n f_V(v_l;\lambda_l)\} = (-1)^n F_V^{(n)}(s;\lambda_l)$, the conditional pairwise probability of mistiming $P_l(\lambda_l)$ can be expressed as

$$\begin{aligned}P_l(\lambda_l) &= \sum_{n=0}^{Bl-1}\frac{1}{n!}\int_0^\infty f_V(v_l;\lambda_l)v_l^n\exp(-v_l)dv_l \\ &= \sum_{n=0}^{Bl-1}\frac{(-1)^n}{n!}F_V^{(n)}(1;\lambda_l)\end{aligned} \tag{15}$$

where $F_V(s;\lambda_l)$ is the Laplace transform of $f_V(v_l;\lambda_l)$. From the MGF, the Laplace transform of $f_V(v_l;\lambda_l)$ can be expressed as (see [8, Ch. 9])

$$F_V(s;\lambda_l) = \mathcal{L}\{f_V(v_l;\lambda_l)\} = \mathrm{E}\{e^{-sv_l};\lambda_l\} = \frac{\exp\left(\frac{-s\lambda_l}{1+s}\right)}{(1+s)^{Bl}}\ . \tag{16}$$

The nth order derivative of $F_V(s;\lambda_l)$ can be expressed as

$$F_V^{(n)}(s;\lambda_l) = \sum_{p=0}^{n}\binom{n}{p}f_1^{(p)}(s;\lambda_l)f_2^{(n-p)}(s) \tag{17}$$

where $f_1(s;\lambda_l) = \exp\left(\frac{-s\lambda_l}{1+s}\right)$ and $f_2(s) = (1+s)^{-Bl}$. According to the Faà di Bruno's formula [10], the pth order derivative of $f_1(s;\lambda_l)$ can be expressed as

$$\begin{aligned}f_1^{(p)}(s;\lambda_l) =& f_1(s;\lambda_l)\sum_{k=0}^{p}\lambda_l^k \\ &\times B_{p,k}\left(g'(s), g''(s), \ldots, g^{(p-k+1)}(s)\right)\end{aligned} \tag{18}$$

with $B_{p,k}$ being the Bell polynomial and $g(s) = \frac{-s}{1+s}$. Applying (18) to (17) and combining terms with the same order of λ_l, we have

$$F_V^{(n)}(s;\lambda_l) = \exp\left(\frac{-s\lambda_l}{1+s}\right)\sum_{k=0}^{n}\lambda_l^k f_{n,k}(s) \tag{19}$$

with $f_{n,k}(s)$ being a function of s

$$\begin{aligned}f_{n,k}(s) =& \sum_{p=k}^{n}\binom{n}{p}f_2^{(n-p)}(s) \\ &\times B_{p,k}\left(g'(s), g''(s), \ldots, g^{(p-k+1)}(s)\right)\ .\end{aligned} \tag{20}$$

The conditional pairwise probability of mistiming can then be simplified using (15) and (19)

$$P_l(\lambda_l) = \exp\left(-\frac{\lambda_l}{2}\right)\sum_{k=0}^{Bl-1}c_{B,l,k}\lambda_l^k \tag{21}$$

where $c_{B,l,k} = \sum_{n=k}^{Bl-1}\frac{(-1)^n}{n!}f_{n,k}(1)$. The pairwise mistiming probability P_l is the expectation of $P_l(\lambda_l)$

$$P_l = \int_0^\infty f_\Lambda(\lambda_l)P_l(\lambda_l)d\lambda_l \tag{22}$$

where $f_\Lambda(\lambda_l)$ is the PDF of the random variable λ_l. Same as Eq. (21), the integral of (22) can also be obtained with the aid of MGF.

The channel taps undergo Nakagami-m fading with PDF [8]

$$f(\gamma_{b,n};m) = \frac{m^m\gamma_{b,n}^{m-1}}{(A_{b,n}\bar{\gamma})^m\Gamma(m)}\exp\left(-\frac{m\gamma_{b,n}}{A_{b,n}\bar{\gamma}}\right) \tag{23}$$

where $\gamma_{b,n} = |h_{b,n}|^2$, $b \in [1, B]$, $n \in [0, L-1]$, $A_{b,n}$ represents the PDP effect of the nth channel tap in the bth frequency band, $\bar{\gamma}$ is the average SNR and $\Gamma(x)$ is the Gamma function.

For correlated Nakagami-m distributed random variables $\gamma_{b,n}$, a simple form PDF can not be found for $\lambda_l = \sum_{b=1}^{B}\sum_{n=0}^{l-1}\gamma_{b,n}$ [11], but the Laplace transform of λ_l can be written as [12] where $\rho_{i,j}$ is the power correlation between γ_{b_1,n_1} and γ_{b_2,n_2} with $i = (b_1-1)l+n_1$ and $j = (b_2-1)l+n_2$. Using property of the Laplace transform, we can obtain the pairwise mistiming probability P_l [cf. (21) and (22)]

$$P_l = \sum_{k=0}^{Bl-1}(-1)^k c_{B,l,k}F_\Lambda^{(k)}\left(\frac{1}{2}\right) \tag{25}$$

where $c_{B,l,k}$ is defined after Eq. (21).

At high SNR as $\bar{\gamma} \to \infty$, the Laplace transform $F_\Lambda(s)$ can be approximated by [12]

$$F_\Lambda(s) \approx (\mathcal{D}_{B,l})^{-m}\prod_{b=1}^{B}\prod_{n=0}^{l-1}\left(\frac{sA_{b,n}\bar{\gamma}}{m}\right)^{-m} \tag{26}$$

where $\mathcal{D}_{B,l}$ denotes the determinant term in (24) when $\bar{\gamma} \to \infty$. Then P_l can be approximated at high SNR by

$$\begin{aligned}P_l =& \left(\frac{\bar{\gamma}}{2m}\right)^{-mBl}(\mathcal{D}_{B,l})^{-m}\prod_{b=1}^{B}\prod_{n=0}^{l-1}A_{b,n}^{-m} \\ &\times\sum_{k=0}^{Bl-1}c_{B,l,k}(-2)^k\frac{(mBl)!}{(mBl-k)!}\ .\end{aligned} \tag{27}$$

$$F_{\Lambda}(s)=\prod_{b=1}^{B}\prod_{n=0}^{l-1}\left(1+\frac{sA_{b,n}\bar{\gamma}}{m}\right)^{-m}$$
$$\left(\det\begin{bmatrix} 1 & \sqrt{\rho_{0,1}}\left(1+\frac{m}{sA_{1,1}\bar{\gamma}}\right)^{-1} & \dots & \sqrt{\rho_{0,Bl-1}}\left(1+\frac{m}{sA_{B,l-1}\bar{\gamma}}\right)^{-1} \\ \sqrt{\rho_{0,1}}\left(1+\frac{m}{sA_{1,0}\bar{\gamma}}\right)^{-1} & 1 & \dots & \sqrt{\rho_{1,Bl-1}}\left(1+\frac{m}{sA_{B,l-1}\bar{\gamma}}\right)^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{\rho_{0,Bl-1}}\left(1+\frac{m}{sA_{1,0}\bar{\gamma}}\right)^{-1} & \sqrt{\rho_{1,Bl-1}}\left(1+\frac{m}{sA_{1,1}\bar{\gamma}}\right)^{-1} & \dots & 1 \end{bmatrix}\right)^{-m} \tag{24}$$

From (27), the pairwise probability of mistiming by l taps can be approximated at high SNR by

$$P_l = C_{B,m,l}\left(\bar{\gamma}\right)^{-mBl},\ l\in[1,L_2] \tag{28}$$

where $C_{B,m,l} = (2m)^{mBl}\left(\mathcal{D}_{B,l}\right)^{-m}\prod_{b=1}^{B}\prod_{n=0}^{l-1}A_{b,n}^{-m}\cdot\sum_{k=0}^{Bl-1}c_{B,l,k}(-2)^k\frac{(mBl)!}{(mBl-k)!}$ is a constant independent of $\bar{\gamma}$. For $l\in[-L_1,-1]$, the pairwise probability of mistiming can be derived in the same way which has the form of $P_l = C_{B,m,l}\left(\bar{\gamma}\right)^{mBl},\ l\in[-L_1,-1]$.

From the union bound, the probability of mistiming P_{mt} is upper bounded by the summation of the pairwise probability of mistiming. This means that

$$P_{mt} < \sum_{l=1}^{L_2}C_{B,m,l}\left(\bar{\gamma}\right)^{-mBl}+\sum_{l=-L_1}^{-1}C_{B,m,l}\left(\bar{\gamma}\right)^{mBl}\ . \tag{29}$$

When the SNR $\bar{\gamma}$ is high, the right hand side of (29) approximates $\left(C_{B,m,1}+C_{B,m,-1}\right)\left(\bar{\gamma}\right)^{-mB}$. As a result, the slope of the probability of mistiming curves is $G_d = mB$ in log-log scale. In the field of communications, this slope is often defined as the diversity of the system error performance. Eq. (29) implies that the diversity gain of the estimator is $G_d = mB$ which increases with both the subband number and the Nakagami-m parameter. This shows that by use of the noncoherent combining, the energy detection based ToA algorithm can achieve a higher diversity gain, proportional to the number of subbands and the diversity achieved in each channel.

IV. Simulations

Simulations are first carried out for the discrete time channel with $L = 12$ independent Nakagami-m distributed taps. The channel has an exponentially decaying PDP with the last tap being 20 dB weaker than the first tap. The average power of the first channel tap is normalized. Assume that due to the lack of timing information, $L_1 = L_2 = 5$ pure noise terms are involved in the estimated channel at the front and rear parts. The probability of mistiming is simulated for the energy detection based ToA estimator when the channel is independent for different subbands.

Analysis has shown that by using B subbands, the energy detection based ToA estimation can obtain a diversity gain of mB in the Nakagami-m channel. This is verified by simulations, as the probability of mistiming curves with the same mB are roughly parallel to each other (see Fig. 2). In addition,

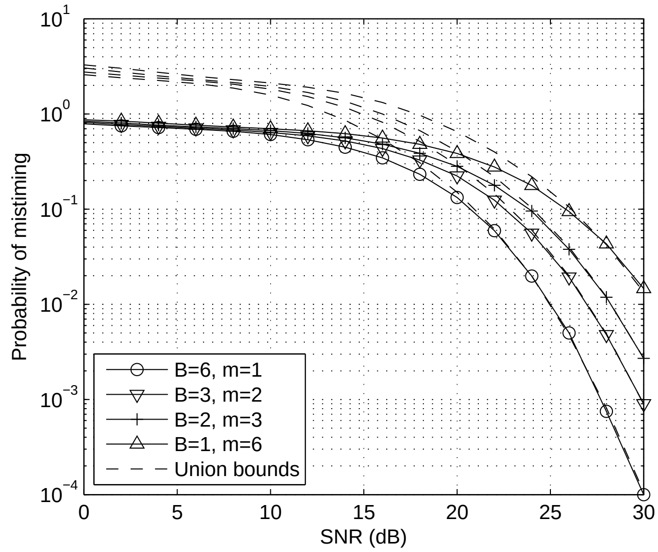

Fig. 2. The probability of mistiming with energy detection for $Bm = 6$. The parameter B is the number of subbands and m is the Nakagami-m fading parameter.

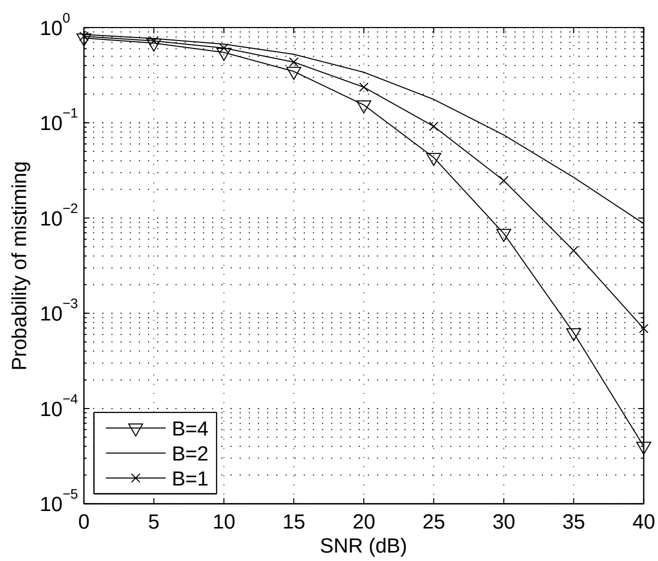

Fig. 3. The probability of mistiming with energy detection for Saleh-Valenzuela channels. The parameter B is the number of subbands.

given mB, the parallel curves have different horizontal shifts. In the literature of communications, this shift is defined as the *coding gain* of the probability of mistiming curves. The simulation result is reasonable since a higher coding gain can be obtained when more signal energy is collected from more subbands. Union bounds are also plotted in Fig. 2 by using Eq. (29), which confirms that these bounds are tight at high SNR.

We then simulate the ToA estimator in the more realistic Saleh-Valenzuela (S-V) channel model [13]. The channel has a cluster structure with the cluster arrival rate of $\Lambda = 1/200$ ns^{-1} and the ray arrival rate of $\lambda = 1/20$ ns^{-1}. The clusters and rays inside each cluster decay exponentially with time constants $\Gamma = 60$ ns and $\gamma = 20$ ns, respectively. The channel rays are generated as random variables with independent Rayleigh distributed amplitudes and independent uniformly distributed phases. Note that in addition to the random and irregular PDP, closely spaced multipath arrivals will also give rise to correlations among the discrete time equivalent channel taps.

MB-OFDM signals with the bandwidth of 100 MHz are modulated to different subbands around the 1.5 GHz center frequency and passed through the channel. For each subband, the discrete time channel is generated by sampling the subband channel at the rate of $1/T_s = 100$ MHz. Since each subband only contains part of the entire channel information, the equivalent discrete time channels differ across subbands but are correlated. Numerical analysis shows that given these channel parameters, the correlation coefficient between subbands can be up to about 0.5. As shown by Fig. 3, given the Nakagami-m parameter ($m = 1$), the diversity gain increases linearly with the number of subbands available (B). This again verifies the diversity gain analysis of the energy detection based ToA estimator.

V. Conclusions

In this letter, we analyzed the probability of mistiming performance for an energy detection based ToA estimator for MB-OFDM systems in Nakagami-m channels. Theoretical analysis shows that a higher diversity gain can be achieved by using more subbands in channels with a lower amount of fading (larger m). Simulations in both the Nakagami-m discrete time channel and the S-V channel model have corroborated our theoretical analysis.

References

[1] *IEEE 802.15 WPAN High Rate Alternative PHY Task Group 3a (TG3a).*
[2] *ECMA-368: High rate ultra wideband PHY and MAC standard, 1st edition*, December 2005.
[3] E. Saberinia and A. H. Tewfik, "Ranging in multiband ultrawideband communication systems," in *IEEE Trans. Vehicular Technology*, vol. 57, no. 4, pp. 2523–2530, July 2008.
[4] H. Xu, C.-C. Chong, I. Guvenc, and L. Yang, "High-resolution TOA estimation with multi-band OFDM UWB signals," in *Proc. of International Conf. on Communications*, Beijing, China, May 19-23, 2008.
[5] C. R. Berger, S. Zhou, Z. Tian, and P. Willett, "Performance analysis on an MAP fine timing algorithm in UWB multiband OFDM," *IEEE Trans. on Communications*, vol. 56, no. 10, pp. 1606–1611, October 2008.
[6] W. Q. Malik, D. J. Edwards, and C. J. Stevens, "Frequency dependence of fading statistics for ultrawideband systems," *IEEE Trans. on Wireless Communications*, vol. 6, no. 3, pp. 800–804, March 2007.
[7] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. on Communications*, vol. 25, no. 7, pp. 673–679, July 1977.
[8] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. John Wiley & Sons, 2000.
[9] J. N. Pierce, "Theoretical diversity improvement in frequency-shift keying," *Proc. IRE*, vol. 46, no. 5, pp. 903–910, May 1958.
[10] W. P. Johnson, "The curious history of Faà di Bruno's Formula," *American Mathematical Monthly*, pp. 217–234, March 2002.
[11] N. C. Beaulieu and A. A. Abu-Dayya, "Analysis of equal gain diversity on Nakagami fading channels," *IEEE Trans. on Communications*, vol. 39, pp. 225–234, February 1991.
[12] Y.-C. Ko, M.-S. Alouini, and M. K. Simon, "Outage probability of diversity systems over generalized fading channels," *IEEE Trans. on Communications*, vol. 48, no. 11, pp. 1783–1787, November 2000.
[13] A. Saleh and R. Valenzuela, "A statistical model for indoor multipath propagation," *IEEE J. Select. Areas Commun.*, vol. 2, no. 5, pp. 128–137, February 1987.