The Affecting Factors in Resource Optimization for Cooperative Communications: A Case Study

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***Abstract*—Cooperative networks provide enhanced system performance by exploiting spatial diversity in a distributed manner. Optimum resource allocation can help improve the performance of cooperative networks and increase the efficiency of resource usage. In the literature, various system performance and optimization results have been reported for different systems and with different optimization metrics. However, there lacks a unifying framework delineating the effects of different factors on resource optimization and the resultant benefit. In this paper, we investigate the relative effects of optimization metric (error rate versus outage probability), modulation type (coherent versus differential) and relaying protocol (amplify-and-forward (AF) versus decode-and-forward (DF)). To facilitate such a case study, we provide a comprehensive set of system performance for four commonly adopted cooperative systems: coherent amplify-and-forward (CAF), coherent decode-and-forward (CDF), differential amplify-and-forward (DAF), and differential decode-and-forward (DDF). A resource optimization problem that minimizes the total transmit energy is formulated. Since energy optimization has been intensively studied in the literature, location optimization will be investigated. The analyses and simulations suggest that: i) The error rate and outage probability metrics yield similar optimization results for AF relaying systems; ii) The relaying protocol determines the optimization results while the modulation type has no effect; and iii) The difference between different relaying protocols diminishes when the number of relays increases.**

***Index Terms*—Cooperative networks, resource optimization, error performance.**

I. Introduction

ORIGINATED from the study of relay transmission [8], [25], and inspired by the successful realization of spatial diversity in the multi-input multi-output (MIMO) communications, cooperative relay networks have attracted intensive research interests. In such networks, spatial diversity is achieved by deploying a number of geographically separated relay nodes between the source and the destination. It can enhance the system performance with improved transmission reliability and increased network coverage [16].

In the literature, relaying strategies can be mainly categorized into two groups: opportunistic relaying [3], [14], [22], [30] and fixed relaying [4], [10], [14], [15]. The former utilizes the instantaneous channel state information (CSI) to dynamically choose the best relay(s) and relaying protocol to forward, thus achieving the optimal performance. The latter assumes no CSI at the transmitter and adopts a pre-determined number of relays to assist the communication. Resource optimization has been intensively studied in fixed relaying systems to improve performance based on channel statistics. Such optimization has been studied based on different metrics, and for various relaying protocols and modulation types (see e.g., [2], [6], [7], [13], [15], [20], [26], [28]). From these diverse studies, one important question arises: how do these metrics, relaying protocols, and modulation types affect the optimization solution and the resultant performance? In this paper, we will investigate the relative effects of these factors by comparing several commonly adopted relay systems as a case study.

In cooperative relay networks, the system performance depends on both the transmit energy allocation among the transmitters and the relative location of the relays. Majority of existing work focuses only on the energy optimization [1], [2], [6], [13]. To avoid the possible high energy consumption at the relays, we recently investigated the relay location optimization, both individually and jointly with energy optimization [7]. In existing works, resource optimization is commonly adopted to minimize the error performance of a cooperative relay system with given total transmit energy. In practice, however, the energy saving is often of more interest so long as some preset error performance is satisfied. To facilitate the study on the affecting factors in resource optimization with these practical considerations, we formulate an optimization problem which minimizes the total transmit energy subject to some preset error performance. In addition, a quantitative measure of optimization gain is defined in terms of the energy saving percentage. Using this measure, the optimization solution and benefit will be compared among several relay systems with different optimization metrics.

Various relaying protocols and modulation types have been proposed and studied in the literature. To facilitate sufficient insights on the affecting factors analysis for resource optimization within the page limit, two prevailing relaying protocols and two fundamental modulation types are considered, which capture some common features of fixed relaying systems. Specifically, we consider amplify-and-forward (AF) and decode-and-forward (DF) relaying protocols. In AF, the relays amplify the received signal and forward to the destination, which enable fast and simple relay processing. To achieve this design, AF with fixed amplification (FA) [7], [12],

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[28], which obviates channel estimation and predetermine the amplification factor based on the channel statistics, is selected to represent the AF protocol. In DF, the relays decode the source signals and retransmit them, which facilitates reliable forwarding. By choosing only the correctly decoded signal to forward, DF with selective-relaying (SR) [23], which utilizes the decoding capability at the relays, is chosen to represent the DF protocol. Regarding the modulation type, coherent modulation [5], [20] and differential modulation [7], [28] are widely explored in the literature. Combing the two relaying protocols and the two modulation types, we will analyze and compare four different cooperative relay systems: coherent AF-FA (**CAF**), coherent DF-SR (**CDF**), differential AF-FA (**DAF**) and differential DF-SR (**DDF**). In terms of optimization, multiple optimization metrics have also been investigated in the literature from different aspects: network throughput optimization [9], [27], cross-layer compound utility function optimization [17], [24], and physical-layer error performance optimization. In this paper, we will focus on the physical-layer optimization. Symbol error rate (SER) with specific modulations [7], [13], [20] and outage probability (OP) without constraining the modulation format [2], [15] are two commonly adopted error performance metrics. Thus in this paper we will investigate the effect of these two metrics.

To facilitate the optimization and comparisons among all four systems, and with both SER and OP metrics, a complete set of error performance formulas will be provided. Some results in the existing literature can be directly adopted, which include the SER performance and location optimization result for the DAF system in [7], and the OP performance for DF systems in [2]. Besides these, we will derive the SER formulas for CAF, CDF and DDF, and the OP performance for AF systems. Since location optimization has not been thoroughly studied in the literature and for presentation brevity, we will only perform the location optimization based on these performance metrics. Analytical optimization results will show the effect of modulation type, and numerical results will reveal the impacts of optimization metric and relaying protocol.

The rest of this paper is organized as follows. The system model will be introduced in Section II. In Section III, the performance will be derived for all four relay systems. The location optimization and analytical results will be presented in Section IV. Following the numerical results and comparisons in Section V, summarizing remarks will be given in Section VI.

Notation: We will use $(\cdot)^*$ for conjugate, $\Re\{\cdot\}$ for the real part, and $\mathcal{CN}(\mu, \sigma^2)$ for the complex Gaussian distribution with mean μ and variance σ^2.

II. System Model

Consider a cooperative network setup with one source node s, L relay nodes $\{r_l\}_{l=1}^{L}$ and one destination node d. At the medium access control (MAC) layer, the time-division multiplexing (TDM) with half-duplex communications is assumed for presentation convenience. At the physical layer, we consider two relaying protocols: AF-FA and DF-SR, and two modulation types: coherent modulation and differential modulation.

A. TDM Transmission Scheme

With TDM, the end-to-end data transmission consists of two steps: source-to-relay transmission and relay-to-destination transmission.

In the first step, the source broadcasts a packet to all relays in the first time slot. For the nth transmitted symbol x_n^s, the corresponding received signal at each relay can be expressed as:

$$y_n^{r_l,s} = \sqrt{\mathcal{E}_s} h_n^{r_l,s} x_n^s + z_n^{r_l,s}, \quad l = 1, 2, ..., L, \tag{1}$$

where $\mathcal{E}_s$ is the transmit energy per symbol at the source, $h_n^{r_l,s}$ represents the fading coefficient between the source and the lth relay, and $z_n^{r_l,s}$ is the additive noise at the lth relay.

In the second step, after processing the received packet, each relay retransmits the packet to the destination in its distinct time slot. Depending on different relaying protocols (AF-FA or DF-SR), relay r_l generates the forwarding signal $x_n^{r_l}$ from the received signal $y_n^{r_l,s}$ correspondingly, which will be specified in the next subsection. At the destination, the received signal y_n^{d,r_l} can be represented as:

$$y_n^{d,r_l} = \sqrt{\mathcal{E}_{r_l}} h_n^{d,r_l} x_n^{r_l} + z_n^{d,r_l}, \; l = 1, 2, \ldots, L. \tag{2}$$

Similarly, $\mathcal{E}_{r_l}$ is the transmit energy per symbol at relay r_l, h_n^{d,r_l} represents the fading channel between the relay r_l and the destination, and z_n^{d,r_l} is the corresponding additive noise at the destination.

In this paper, all channels are assumed to be independent Rayleigh fading, i.e. $h_n^{i,j} \sim \mathcal{CN}(0, \sigma_{i,j}^2)$, $i, j \in \{s, r_l, d\}$. In addition, all noise components are independent and identically complex Gaussian distributed, $z_n^{i,j} \sim \mathcal{CN}(0, \mathcal{N}_0)$. Accordingly, we can compute the instantaneous received signal-to-noise ratio (SNR) at the receiver i from transmitter j as: $\gamma_{i,j} = |h_n^{i,j}|^2 \mathcal{E}_j / \mathcal{N}_0, \quad i, j \in \{s, r_l, d\}$. And the corresponding average received SNR is

$$\bar{\gamma}_{i,j} = \frac{\sigma_{i,j}^2 \mathcal{E}_j}{\mathcal{N}_0}. \tag{3}$$

B. Relaying Protocols

For different system performance and relay processing complexity requirements, various relaying protocols are designed in the literature. In this paper, two popular relaying protocols are considered: AF-FA and DF-SR.

1) AF-FA: With AF protocol, each relay r_l generates the forwarding signal $x_n^{r_l}$ by simply amplifying the signal received from the source with a factor A_{r_l},

$$x_n^{r_l} = A_{r_l} y_n^{r_l,s}, \quad l = 1, 2, ..., L. \tag{4}$$

Thus there is no demodulation required in AF. We consider a fixed amplification scheme as in [12] for simple relay implementation. Specifically, the amplification factor is a constant determined by the channel and noise statistics, and the value is calculated to maintain a statistically fixed transmit energy at each relay. For relay r_l, the amplification factor is computed as:

$$A_{r_l} = \sqrt{\frac{1}{\sigma_{r_l,s}^2 \mathcal{E}_s + \mathcal{N}_0}}, \quad l = 1, 2, ..., L. \tag{5}$$

Notice that, with the fixed amplification, the relays can bypass instantaneous channel estimation, which renders energy-efficient relay implementation.

In the designated time slot for the relay r_l, the amplified signal $x_n^{r_l}$ is forwarded to the destination. At the destination, the signals from all relays arrive at distinct time slots. With appropriate diversity combining techniques described in next subsection, the total SNR at the destination can be computed as:

$$\gamma_d = \sum_{l=1}^{L} \gamma_{eq,r_l}, \tag{6}$$

where γ_{eq,r_l} is the equivalent received SNR corresponds to each source-relay-destination (S-R-D) link [29],

$$\gamma_{eq,r_l} = \frac{\gamma_{r_l,s}\gamma_{d,r_l}}{\bar{\gamma}_{r_l,s} + \gamma_{d,r_l} + 1}. \tag{7}$$

2) DF-SR: With DF-SR relaying protocol, each relay node first decodes the information x_n^s from the received signal $y_n^{r_l,s}$. As in [23], every relay can determine the correctness of each decoded symbol. Then only the relays that correctly decode the symbol will forward the symbol to the destination, while the relays with erroneous decoding will discard the symbol. Assume that the correctly decoding relay set is $\mathcal{D}$, then each relay $r_l \in \mathcal{D}$ will re-modulate x_n^s and transmit it to the destination with energy $\mathcal{E}_{r_l}$ in its designated time slot.

At the destination, after combining all received signals, the total SNR can be evaluated as:

$$\gamma_d = \sum_{r_l \in \mathcal{D}} \gamma_{d,r_l}. \tag{8}$$

C. Modulation Types

Two modulation types are commonly adopted in the literature. For better performance, coherent (de-)modulation is used with the knowledge of instantaneous CSI. On the other hand, differential (de-)modulation that bypasses channel estimation can simplify receiver design. We will briefly introduce each modulation type with phase shift keying (PSK) modulation format. At the source, the nth PSK symbol is represented as $s_n = e^{j2\pi m/M}$, $m \in \{0,\ 1,\ \cdots,\ M-1\}$ and M is the modulation size.

1) Coherent modulation/demodulation: At transmitter $i \in \{s,\ r_l\}$, the nth transmitted symbol is modulated as $x_n^i = \sqrt{\mathcal{E}_i} s_n$. To decode the information, the receiver node j keeps tracking the instantaneous channel gain $h_n^{j,i}$. Assume the CSI is perfectly known at each receiver. At relay r_l, the decoding of the nth symbol $\hat{s}_n^{r_l}$ is determined as follows:

$$\hat{s}_n^{r_l} = s_n(\hat{m}) : \hat{m} = \arg\min_{m \in \{1,2,\cdots,M-1\}} \left\| y_n^{r_l,s} - h_n^{r_l,s}\sqrt{\mathcal{E}_s} s_n(m) \right\|. \tag{9}$$

At the destination, the decoder adds up the signals from all the relays with maximum ratio combing (MRC). And the nth symbol is estimated as:

$$\hat{s}_n^d = s_n(\hat{m}) : \hat{m} = \arg\min_{m \in \{1,2,\cdots,M-1\}} \left\| \frac{\sum_{l \in \mathcal{F}} \left(w_n^{d,r_l}\right)^* y_n^{d,r_l}}{\sum_{l \in \mathcal{F}} \left(w_n^{d,r_l}\right)^* g_n^{d,r_l}} - s_n(m) \right\|, \tag{10}$$

where $\mathcal{F}$ is the forwarding relay set and w_n^{d,r_l} is the combining weight depending on the relaying protocol: $w_n^{d,r_l} = \frac{g_n^{d,r_l}}{|h_n^{d,r_l}|^2 \mathcal{E}_{r_l} A_{r_l}^2 + 1}$ for AF protocol and $w_n^{d,r_l} = h_n^{d,r_l}\sqrt{\mathcal{E}_{r_l}}$ for DF protocol. g_n^{d,r_l} is the channel and energy gain observed at the destination with $g_n^{d,r_l} = h_n^{d,r_l}\sqrt{\mathcal{E}_{r_l}} A_{r_l} h_n^{r_l,s}\sqrt{\mathcal{E}_s}$ for AF protocol and $g_n^{d,r_l} = h_n^{d,r_l}\sqrt{\mathcal{E}_{r_l}}$ for DF protocol.

2) Differential modulation/demodulation: At transmitter i, the nth symbol is modulated as $x_n^i = x_{n-1}^i s_n$ with initial value $x_0^i = \sqrt{\mathcal{E}_i}$. Assume that the channel is slowly time-varying such that the fading coefficient remains invariant over two consecutive symbols. For DF protocol, the differential decoder at the lth relay is given by

$$\hat{s}_n^{r_l} = s_n(\hat{m}) : \hat{m} = \arg\max_{m \in \{1,2,\cdots,M-1\}} \Re\left\{ \left(y_n^{r_l,s}\right)^* y_{n-1}^{r_l,s} s_n(m) \right\}. \tag{11}$$

To maximize the receive SNR at the destination, the decoder sums up all signals received from multiple relays with weights w_{d,r_l} and estimates the transmitted symbol as:

$$\hat{s}_n^d = s_n(\hat{m}) : \hat{m} = \arg\max_{m \in \{1,2,\cdots,M-1\}} \sum_{l \in \mathcal{F}} w_{d,r_l} \Re\left\{ \left(y_n^{d,r_l}\right)^* y_{n-1}^{d,r_l} s_n(m) \right\}, \tag{12}$$

where $w_{d,r_l} = \frac{1+\bar{\gamma}_{s,r_l}}{1+\bar{\gamma}_{s,r_l}+\bar{\gamma}_{r_l,d}}$ [7] for AF protocol and $w_{d,r_l} = 1$ [6] for DF protocol. No channel information is required at either the relays or the destination node.

III. System Performance Measures

Two commonly adopted performance measures in cooperative relay networks are the symbol error rate (SER) and the outage probability (OP). While the SER captures the system performance in terms of the probability of erroneous decoding, outage probability reveals the system capacity characteristics by the probability that the capacity falls below some threshold. In this section, we present both the SER and OP performance metrics for the relay systems described in the preceding section. Existing results are summarized, and derivations and proofs are provided for cases that have not been studied in the literature. To facilitate the comparison, we consider relay systems without a direct link.

A. Symbol Error Rate

In this paper, binary phase shift keying (BPSK) is chosen for SER analyses due to its popularity and mathematical tractability [7], [20], [28].

Lemma 1 (coherent AF: SER) *At high SNR, the average SER at the destination for an $L-$relay cooperative system with coherent BPSK (de-)modulation and AF-FA relaying protocol can be approximated as:*

$$\bar{P}_{ser}^{CAF} \approx \binom{2L-1}{L} \prod_{l=1}^{L} \left[\frac{1}{4\bar{\gamma}_{r_l,s}} + \frac{1}{4\bar{\gamma}_{d,r_l}} \ln(\bar{\gamma}_{d,r_l}) \right], \tag{13}$$

where $\bar{\gamma}_{r_l,s}$ and $\bar{\gamma}_{d,r_l}$ are the average SNR at the lth relay and at the destination corresponding to the lth relay.

Proof: See Appendix I. ■

Lemma 2 (coherent DF: SER) *At high SNR, the average SER at the destination for an $L-$relay cooperative system with coherent BPSK (de-)modulation and DF-SR relaying protocol can be approximated as:*

$$\bar{P}_{ser}^{CDF} \approx \sum_{\mathcal{D}\notin\phi} C(|\mathcal{D}|) \prod_{l\in\mathcal{D}} \frac{1}{4\bar{\gamma}_{d,r_l}} \prod_{k\notin\mathcal{D}} \frac{1}{4\bar{\gamma}_{r_k,s}}, \tag{14}$$

where

$$C(|\mathcal{D}|) = \begin{cases} \binom{2|\mathcal{D}|-1}{|\mathcal{D}|}, & |\mathcal{D}| \neq 0, \\ 1/2, & |\mathcal{D}| = 0. \end{cases} \tag{15}$$

$\bar{\gamma}_{r_l,s}$, $\bar{\gamma}_{r_k,s}$ and $\bar{\gamma}_{d,r_l}$ are the average SNRs at the relays and destination, and $\mathcal{D}$ is the correctly decoding relay set.

Proof: See Appendix II. ■

Lemma 3 (differential AF: SER)[7] [1] *At high SNR, the average SER for an $L-$relay cooperative system using differential BPSK (de-)modulation and AF-FA relaying protocol can be computed as:*

$$\bar{P}_{ser}^{DAF} \approx \binom{2L-1}{L} \prod_{l=1}^{L} \left[\frac{1}{2\bar{\gamma}_{r_l,s}} + \frac{1}{2\bar{\gamma}_{d,r_l}} \ln(\bar{\gamma}_{d,r_l}) \right]. \tag{16}$$

Lemma 4 (differential DF: SER) *At high SNR, the average SER for an $L-$relay cooperative system using differential BPSK (de-)modulation and DF-SR relaying protocol can be computed as:*

$$\bar{P}_{ser}^{DDF} \approx \sum_{\mathcal{D}} C(|\mathcal{D}|) \prod_{l\in\mathcal{D}} \frac{1}{2\bar{\gamma}_{d,r_l}} \prod_{k\notin\mathcal{D}} \frac{1}{2\bar{\gamma}_{r_k,s}}. \tag{17}$$

Proof: If the correctly decoding set $\mathcal{D}$ is empty, then no relay will forward the information to the destination. Thus the instantaneous SER $P_{ser} = 1/2$.

Otherwise, for nonempty set $\mathcal{D}$, the instantaneous SER at the destination for differential BPSK modulation is [18, Chap. 11]:

$$P_{ser|\gamma_d} = \frac{1}{2^{2L-1}} e^{-\gamma_d} \sum_{n=0}^{L-1} c_n \gamma_d^n,$$

where $c_n = \frac{1}{n!}\sum_{k=0}^{L-1-n}\binom{2L-1}{k}$. Adopting the same polynomial expansion technique in [7, Appendix I], the average SER at the destination with decoding set $\mathcal{D}$ can be computed as:

$$\bar{P}_{ser,\mathcal{D}}^{DDF} \approx B(|\mathcal{D}|) \prod_{l\in\mathcal{D}} \frac{1}{\bar{\gamma}_{d,r_l}},$$

where

$$B(|\mathcal{D}|) = \frac{1}{2^{2|\mathcal{D}|-1}} \sum_{n=0}^{|\mathcal{D}|-1} \binom{n+|\mathcal{D}|-1}{|\mathcal{D}|-1} \sum_{k=0}^{|\mathcal{D}|-1-n} \binom{2|\mathcal{D}|-1}{k}.$$

By swapping the summation order and make use of the polynomial property:

$$\sum_{n=0}^{|\mathcal{D}|-1-k} \binom{n+|\mathcal{D}|-1}{|\mathcal{D}|-1} = \binom{2|\mathcal{D}|-1-k}{|\mathcal{D}|},$$

[1]The coefficient $C(L)$ in [7] is simplified here using the technique in the proof of Lemma 4.

$B(|\mathcal{D}|)$ can be simplified as:

$$B(|\mathcal{D}|) = \binom{2|\mathcal{D}|-1}{|\mathcal{D}|} 2^{-|\mathcal{D}|}.$$

With this neat result, the average SER with decoding set $\mathcal{D}$ can be reexpressed as:

$$\bar{P}_{ser,\mathcal{D}}^{DDF} \approx \binom{2|\mathcal{D}|-1}{|\mathcal{D}|} \prod_{l\in\mathcal{D}} \frac{1}{2\bar{\gamma}_{d,r_l}},$$

By summing up all possible decoding set $\mathcal{D}$ with probability:

$$P(\mathcal{D}) = \prod_{l\in\mathcal{D}} \left(1 - \frac{1}{2\bar{\gamma}_{r_l,s}}\right) \prod_{k\notin\mathcal{D}} \frac{1}{2\bar{\gamma}_{r_k,s}},$$

we can achieve the average SER for DDF system in Eq. (17). ■

B. Outage Probability

In a block fading environment, outage probability is defined as the probability of occurrence that the channel capacity (C) is below some threshold R, $P_{out} = \text{Prob}(C \leq R)$. For an $L-$relay cooperative system, the outage probability can be calculated as [2]:

$$P_{out} = \text{Prob}\left(\frac{1}{L+1}\log_2(1+\gamma) \leq R\right) = \text{Prob}\left(\gamma \leq 2^{(L+1)R} - 1\right), \tag{18}$$

where γ is the SNR at the receiver. Thus the OP can also be computed as the probability that the received SNR is below some threshold. Since OP is independent of the modulation types, we will compute the OPs for relay systems with different relaying protocols (AF-FA and DF-SR) in the following.

Lemma 5 (Amplify-and-Forward: OP) *At high SNR, the outage probability for an $L = 1$ cooperative relay system with AF-FA relaying protocol can be evaluated as follows:*

$$P_{out,L=1}^{AF} \approx 1 + \frac{\bar{\gamma}_{r,d}}{\bar{\gamma}_{s,r}} - 2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_{r,d}}} K_1\left(2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_{r,d}}}\right) - 2\frac{\gamma_{th}}{\bar{\gamma}_{s,r}} K_2\left(2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_{r,d}}}\right), \tag{19}$$

where $\gamma_{th} = 2^{2R} - 1$, R is the capacity threshold, and K_α, $\alpha \in 0, 1, 2\cdots$ are the modified bessel functions of the second kind.

Proof: Notice from Eq. (18) that the outage probability is only related to the distribution of SNR at the destination. The SNR statistics for a single-relay AF system is given in [29], thus we can compute outage probability for an AF system as follows:

$$P_{out}^{AF} = \int_0^{\gamma_{th}} p(\gamma) d\gamma = \int_0^{\gamma_{th}} \left[\frac{\beta^2}{2} e^{\frac{-\gamma}{\bar{\gamma}_{s,r}}} K_0(\beta\sqrt{\gamma}) + \frac{\beta\sqrt{\gamma}}{\bar{\gamma}_{s,r}} e^{\frac{-\gamma}{\bar{\gamma}_{s,r}}} K_1(\beta\sqrt{\gamma})\right] d\gamma.$$

After change of variables: $x = \sqrt{\frac{\gamma}{\gamma_{th}}}$, $\alpha = \frac{\gamma_{th}}{\bar{\gamma}_{s,r}}$ and $\theta = \beta\sqrt{\gamma_{th}} = 2\sqrt{\frac{\gamma_{th}}{\bar{\gamma}_{s,r}}}$, the above equation can be rewritten as:

$$P_{out}^{AF} = \theta^2 \int_0^1 x e^{-\alpha x^2} K_0(\theta x) dx + 2\alpha\theta \int_0^1 x^2 e^{-\alpha x^2} K_1(\theta x) dx. \tag{20}$$

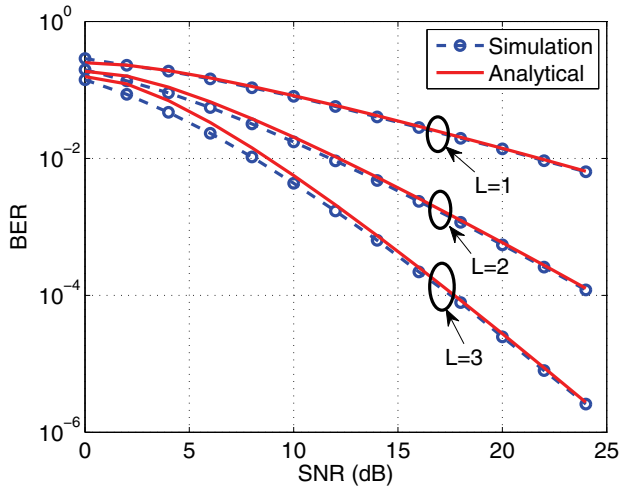

Fig. 1. Analyses vs. simulations: the SER performance of coherent AF relay systems with different number of relays. The system parameters are: $\bar{\gamma}_{r,s} = \bar{\gamma}_{d,r}$, $D = 1.2$, $\lambda_D = 0.5$ and $\lambda_{\mathcal{E}} = 1/(L+1)$.

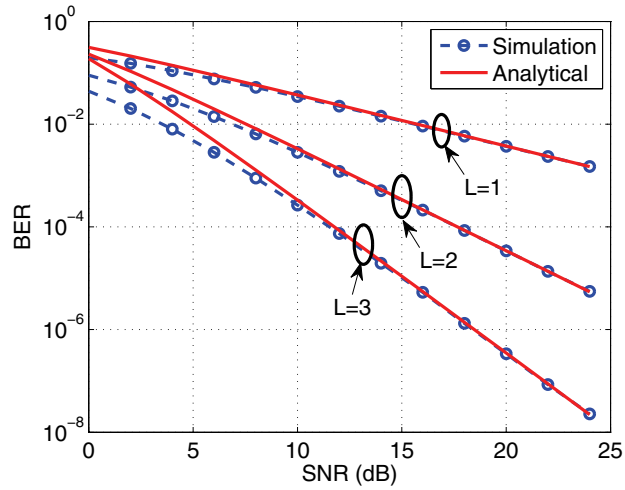

Fig. 2. Analyses vs. simulations: the SER performance of coherent DF relay systems with different number of relays. The system parameters are: $\bar{\gamma}_{r,s} = \bar{\gamma}_{d,r}$, $D = 1.2$, $\lambda_D = 0.5$ and $\lambda_{\mathcal{E}} = 1/(L+1)$.

With high SNR assumption, we have $\alpha x^2 = \frac{\gamma}{\bar{\gamma}_{s,r}} \approx 0$ for $\gamma \in [0, \gamma_{th}]$. Then Eq. (20) can be approximated as:

$$P_{out}^{AF} \approx \theta^2 \int_0^1 x K_0(\theta x) dx + 2\alpha\theta \int_0^1 x^2 K_1(\theta x) dx.$$

Furthermore, by using the integration formulas for modified Bessel functions [11, Eq. (6.561.8)], we can obtain the outage probability for an AF relay system in Eq. (19). ■

Lemma 6 (Decode-and-Forward: OP)[2] *At high SNR, the outage probability for the cooperative relay system with DF-SR relaying protocol can be computed as:*

$$P_{out}^{DF} \approx C_{out}(L) \sum_{\mathcal{D}} \frac{1}{|\mathcal{D}|!} \left[\prod_{k \notin \mathcal{D}} \frac{1}{\bar{\gamma}_{r_k,s}} \right] \left[\prod_{l \in \mathcal{D}} \frac{1}{\bar{\gamma}_{d,r_l}} \right], \quad (21)$$

where

$$C_{out}(L) = [(2^{(L+1)R} - 1)]^L.$$

C. Tightness of Performance Results

Notice that the analytical results above are approximations at high SNR. In the literature, the tightness of the formulas for the SER of DAF systems and the OP of DF systems has been verified in [7] and [2], respectively. To demonstrate the validity of the high-SNR approximation of the remaining performance results, we simulate the SERs for CAF, CDF and DDF relay systems, and the OP for AF systems with a single relay. In all simulations, we assume that the relay is at the same distance to the source and destination with sum distance $D = 1.2$, and $\bar{\gamma}_{r_l,s} = \bar{\gamma}_{d,r_l}$. The simulated performance is plotted in Figs. 1, 2, 3 and 4 for the SERs of the CAF, CDF and DDF systems, and the OP of the AF system, respectively. The corresponding analytical results are also shown in the corresponding figures. The comparisons clearly confirm that all four analytical performance results are tight to the simulations at high SNR.

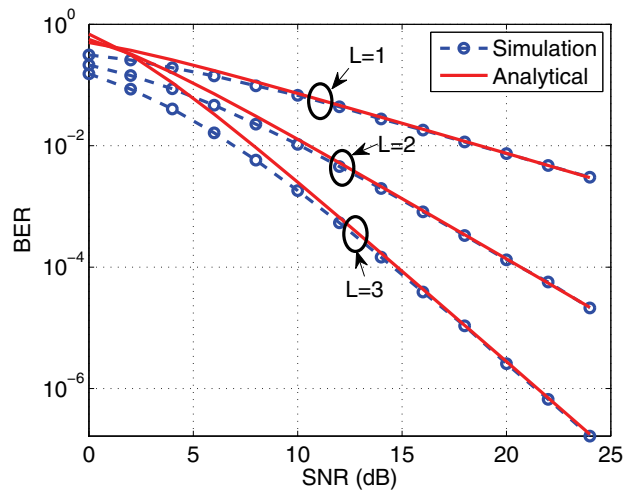

Fig. 3. Analyses vs. simulations: the SER performance of differential DF relay systems with different number of relays. The system parameters are: $\bar{\gamma}_{r,s} = \bar{\gamma}_{d,r}$, $D = 1.2$, $\lambda_D = 0.5$ and $\lambda_{\mathcal{E}} = 1/(L+1)$.

IV. Resource Optimization

In this section, resource optimization is investigated for the four relay systems. For practical considerations, an optimization problem is formulated to minimize the total transmit energy. Since the energy allocation optimization has been intensively studied in the literature, we will focus on the location optimization. Analytical optimization results based on the SER performance are derived and compared to show the effect of the modulation type.

A. Problem Formulation

Notice from the analytical results in the preceding section, the relay system performance is determine by the average SNR $\bar{\gamma}_{i,j}$ on each link. Eq. (3) indicates that $\bar{\gamma}_{i,j}$ is related to the transmit energy $\mathcal{E}_j$ and channel fading statistics $\sigma_{i,j}^2$. The latter can be represented as $\sigma_{i,j}^2 = G \cdot D_{i,j}^{-\nu}, \quad i,j \in \{s, r_l, d\}$ [19, Sec. 4.9], where $D_{i,j}$ represents the distance between

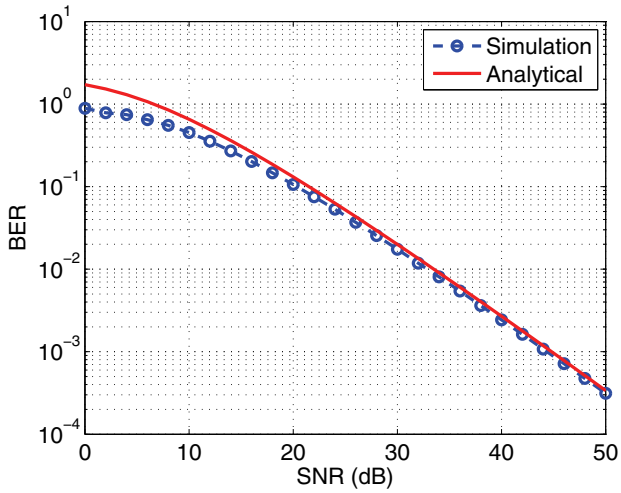

Fig. 4. Analyses vs. simulations: the OP performance of AF relay systems with a single relay. The system parameters are: $\bar{\gamma}_{r,s} = \bar{\gamma}_{d,r}$, $D = 1.2$, $\lambda_D = 0.5$ and $\lambda_{\mathcal{E}} = 0.5$.

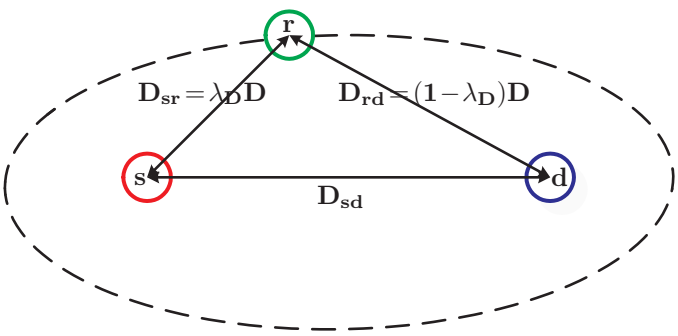

Fig. 5. The ellipse topology with source s, relay r and destination d; and the S-R distance is denoted as $D_{sr} = \lambda_D D$, R-D distance is $D_{rd} = (1-\lambda_D)D$ and S-D distance is D_{sd}. The relay can move on the dashed ellipse defined by the two focuses s, d, and sum distance $D = D_{sr} + D_{rd}$.

nodes i and j, and ν is the path loss exponent of the wireless environment. G is a parameter which can incorporate the effect of shadowing. In a uniform transmission environment, we can set $G = 1$ without loss of generality. Then the average SNR in Eq. (3) can be re-expressed as $\bar{\gamma}_{i,j} = D_{i,j}^{-\nu}\mathcal{E}_j/\mathcal{N}_0$. Therefore, the relay system performance can be optimized by adjusting the transmit energy allocation and the relay location.

Consider the location optimization in a two-dimension (2D) plane (Fig. 5). Extending from the line topology with $D_{s,r} + D_{r,d} = D_{s,d}$, two parameters can be used to represent one position on a 2D plane: the sum distance $D = D_{s,r} + D_{r,d} \geq D_{s,d}$ and the source-relay (S-R) distance ratio $\lambda_D = D_{s,r}/D$. Thus the location optimization is a 2D problem. Intuitively, for fixed λ_D, the optimum system performance is achieved with the smallest D, and $D = D_{s,d}$ if there is no restriction on the relay placement. However, in practice, the geographical constraints (mountain, lake and etc.) may not allow the relays to be placed on the source-destination (S-D) line. One intuitive solution is to sweep all feasible sum distance D and find the optimal location for each D. We will focus on the optimization of λ_D for a given $D \geq D_{s,d}$, which is shown in Fig. 5. Furthermore, in a multi-relay system, the performance optimality will be achieved if all relays are located at the optimum position, namely $D_{s,r_l} = D_{s,r}$ and $D_{r_l,d} = D_{r,d}$, $\forall l \in \{1, 2, \cdots, L\}$, which results in $\bar{\gamma}_{s,r_l} = \bar{\gamma}_{s,r}$ and $\bar{\gamma}_{r_l,d} = \bar{\gamma}_{r,d}$. Thus, we will consider this uniform relay setup in our location optimization. To simplify the presentation, we define the energy allocation ratio $\lambda_{\mathcal{E}} = \mathcal{E}_s/\mathcal{E}_t$ as the portion of the total transmit energy $\mathcal{E}_t$ allocated to the source.

In wireless communication systems, energy consumption is a critical factor for battery-powered mobile devices. From a practical point of view, the system designer is interested in minimizing system energy while satisfying some target system performance. Thus we define our location optimization as follows.

Problem Statement: *Consider an L-relay system in a wireless environment with path loss exponent ν. For any given system energy allocation ratio $\lambda_{\mathcal{E}}$, and S-R-D distance D, we will determine the optimum relay location ($D^o_{s,r}$), namely, λ^o_D, such that some preset system performance P_{target} can be achieved with the minimum system energy. This can be mathematically expressed as:*

$$\begin{aligned} &\textit{minimize } \mathcal{E}_t = \mathcal{E}_s + L\mathcal{E}_r, \\ &\textit{subject to } \lambda_D \in \{0, 1\} \textit{ and } \bar{P} \leq P_{target}, \end{aligned} \tag{22}$$

where $\bar{P}$ can be either $\bar{P}_{ser}$ or P_{out} depending on which optimization metric is used.

In addition, the location optimization gain is evaluated in terms of energy saving. Compared to an unoptimized relay system with all relays placed at $\lambda_D = 1/2$, the energy-saving gain is computed as:

$$\mathrm{G}_{opt} = \frac{\mathcal{E}^u_t - \mathcal{E}^o_t}{\mathcal{E}^u_t} \times 100\%, \tag{23}$$

where $\mathcal{E}^u_t$ and $\mathcal{E}^o_t$ are the minimum energy required for the unoptimized and optimized systems to achieve P_{target}.

B. Analytical Optimization Results

For analytical tractability, we will only show the optimization results using the SER performance metric for all four relay systems. Comparisons will reveal the effect of modulation types.

1) Coherent AF & differential AF: With the uniform relay setup, the SERs of CAF and DAF systems in Eqs. (13) and (16) are simplified to:

$$\bar{P}^{CAF}_{ser} \approx \left(\frac{1}{4}\right)^L \binom{2L-1}{L} \left[\frac{1}{\bar{\gamma}_{r,s}} + \frac{1}{\bar{\gamma}_{d,r}} \ln(\bar{\gamma}_{d,r})\right]^L, \tag{24}$$

$$\bar{P}^{DAF}_{ser} \approx \left(\frac{1}{2}\right)^L \binom{2L-1}{L} \left[\frac{1}{\bar{\gamma}_{r,s}} + \frac{1}{\bar{\gamma}_{d,r}} \ln(\bar{\gamma}_{d,r})\right]^L. \tag{25}$$

Observe that the SERs for coherent AF and differential AF relay systems are almost the same except for a constant term, which reflects the coding gain difference between these two modulation types. Thus with the same total transmit energy, the optimum relay location and optimization gain are the same for both systems. On the other hand, to achieve the same error performance, the optimization results can be found in the following proposition.

Proposition 1 *For any given system parameters L, $\lambda_{\mathcal{E}}$, D, P_{target}, and the channel parameter ν, the unique optimum relay location of CAF and DAF systems can be found from the following equation:*

$$\ln\left[\frac{1-\lambda_{\mathcal{E}}}{L\lambda_{\mathcal{E}}}\left(\frac{\lambda_D^o}{1-\lambda_D^o}\right)^{\nu}\frac{1}{\lambda_D^o}+1\right]+\ln\kappa-1=\frac{(1-\lambda_{\mathcal{E}})}{\lambda_{\mathcal{E}}L}\left(\frac{\lambda_D^o}{1-\lambda_D^o}\right)^{\nu-1}, \tag{26}$$

and the corresponding minimum system energy is calculated as:

$$\mathcal{E}_t^o=\kappa\left(\frac{D^{\nu}}{\lambda_{\mathcal{E}}}\right)\left[{\lambda_D^o}^{\nu-1}+(1-\lambda_D^o)^{\nu}\frac{\lambda_{\mathcal{E}}L}{1-\lambda_{\mathcal{E}}}\right], \tag{27}$$

where κ is a system constant,

$$\kappa=\left(\frac{\binom{2L-1}{L}}{g^L P_{target}}\right)^{\frac{1}{L}},$$

and g is the coding gain coefficient with $g=4,\ 2$ for the CAF and DAF systems respectively.

Proof: To find the optimum relay location, we take the first order derivative of Eqs. (24) and (25) with respect to λ_D, and the optimum relay location ratio λ_D^o can be obtained from the following equation:

$$\ln\left[\frac{\mathcal{E}_t^o(1-\lambda_{\mathcal{E}})}{LD^{\nu}(1-\lambda_D^o)^{\nu}}\right]=\frac{(1-\lambda_{\mathcal{E}})}{\lambda_{\mathcal{E}}L}\left(\frac{\lambda_D^o}{1-\lambda_D^o}\right)^{\nu-1}+1. \tag{28}$$

By rearranging Eqs. (24) and (25), we can find the required system energy as:

$$\mathcal{E}_t=\kappa\frac{(D\lambda_D)^{\nu}}{\lambda_{\mathcal{E}}}\left[1+\left(\frac{1-\lambda_D}{\lambda_D}\right)^{\nu}\frac{\lambda_{\mathcal{E}}L}{1-\lambda_{\mathcal{E}}}\ln\left(\frac{\mathcal{E}_t(1-\lambda_{\mathcal{E}})}{LD^{\nu}(1-\lambda_D)^{\nu}}\right)\right]. \tag{29}$$

Substituting the optimal location relationship in Eq. (28), we can obtain the expression for $\mathcal{E}_t^o$ in Eq. (27). After plugging $\mathcal{E}_t^o$ back into Eq. (28), the optimum relay location can be computed as in Eq. (26).

The concavity of $\bar{P}_{ser}^{DAF}$ over λ_D is proven in [7]. Due to the similarity of $\bar{P}_{ser}^{CAF}$, its concavity can be readily proven. Thus the optimal location result obtained from Eq. (26) is unique. ■

To compute the optimization gain, we also need to find the energy consumption $\mathcal{E}_t^u$ for the unoptimized system. With $\lambda_D=1/2$, $\mathcal{E}_t^u$ can be determined from Eq. (29) as follows:

$$\left(\frac{1-\lambda_{\mathcal{E}}}{\kappa(D/2)^{\nu}L}\right)\mathcal{E}_t^u=\ln\mathcal{E}_t^u+\ln\frac{1-\lambda_{\mathcal{E}}}{L(D/2)^{\nu}}+\frac{1-\lambda_{\mathcal{E}}}{\lambda_{\mathcal{E}}L}. \tag{30}$$

Then the optimization gain can be calculated correspondingly.

In Proposition 1, we notice that the optimum location results for CAF and DAF systems are different due to the g value, which affects the constant term $\ln\kappa$ in Eq. (26):

$$\ln\kappa=\frac{1}{L}\ln\left(\frac{1}{P_{target}}\binom{2L-1}{L}\right)-\ln g.$$

For $g=2$ and 4, the difference is only $\ln(2)$ for CAF and DAF systems. With small P_{target}, this difference will have negligible effect on $\ln\kappa$. Thus the CAF and DAF systems will have similar location optimization results. This indicates that the modulation type has limited impact on the optimization for AF systems using SER as the performance metric.

2) Coherent DF & differential DF: For the DF relay systems with the uniform setup, all $\binom{L}{l}$ forwarding relay sets with the same number of relays l will result in the same error rate. By adding up all possible l, we can re-express the SER formulas in Eqs. (14) and (17) as follows:

$$\bar{P}_{ser}^{CDF}\approx\left[\frac{1}{4\bar{\gamma}_{r,s}}\right]^L\left[\frac{1}{2}+\sum_{l=1}^{L}\binom{L}{l}\binom{2l-1}{l}\left[\frac{\bar{\gamma}_{r,s}}{\bar{\gamma}_{d,r}}\right]^l\right], \tag{31}$$

$$\bar{P}_{ser}^{DDF}\approx\left[\frac{1}{2\bar{\gamma}_{r,s}}\right]^L\left[\frac{1}{2}+\sum_{l=1}^{L}\binom{L}{l}\binom{2l-1}{l}\left[\frac{\bar{\gamma}_{r,s}}{\bar{\gamma}_{d,r}}\right]^l\right]. \tag{32}$$

These two equations show that the SER of the coherent DF relay system is very similar to that of the differential DF system, except for a difference in coding gain. However, compared to the SER formulas for the AF systems, Eqs. (31) and (32) have only polynomial terms of the SNR ratios in the summation. Thus, by setting the λ_D-derivative of the SERs to zero, the optimum location will not be affected by the coding gain difference and the target error performance P_{target}. This is verified by the following proposition.

Proposition 2 *For any given system parameters L, $\lambda_{\mathcal{E}}$, D, P_{target}, and the channel parameter ν, the optimum relay location of CDF and DDF systems can be found in the following equation:*

$$\sum_{l=1}^{L}\binom{L}{l}\binom{2l-1}{l}\left[\frac{L\lambda_{\mathcal{E}}}{1-\lambda_{\mathcal{E}}}\left(\frac{1-\lambda_D^o}{\lambda_D^o}\right)^{\nu}\right]^l[l-L(1-\lambda_D^o)]=\frac{1}{2}L(1-\lambda_D^o), \tag{33}$$

and the corresponding minimum system energy is:

$$\mathcal{E}_t^o=\eta\frac{(\lambda_D^o)^{\nu}}{\lambda_{\mathcal{E}}}\left[\frac{1}{2}+\sum_{l=1}^{L}\binom{L}{l}\binom{2l-1}{l}\left[\frac{\lambda_{\mathcal{E}}}{1-\lambda_{\mathcal{E}}}\left(\frac{\lambda_D^o}{1-\lambda_D^o}\right)^{\nu}\right]^l\right], \tag{34}$$

where $\eta=D^{\nu}/(gP_{target}^{1/L})$ is a constant.

The result indicates that the optimum location is exactly the same for CDF and DDF systems. In addition, to compute the optimization gain, we can obtain the required energy $\mathcal{E}_t^u$ for the unoptimized system by plugging $\lambda_D^o=1/2$ into Eq. (34). Denote that the constant term η will not affect the optimization gain. Thus the optimization gain for CDF and DDF systems is also irrelevant to the target error performance.

Therefore, from the analyses above, we can conclude that the coherent and differential modulations have negligible effect on resource optimization for relay systems with AF or DF relaying protocols. This suggests that the optimization results obtained for a coherent relay system can also be applied to a differential one.

V. Numerical Results and Comparisons

In this section, we will present the numerical results of location optimization for all four systems. Through comparisons, the effects of optimization metrics and relaying protocols will be illustrated. In all numerical results, we consider relay systems with a target error probability of $P_{target}=10^{-3}$, and

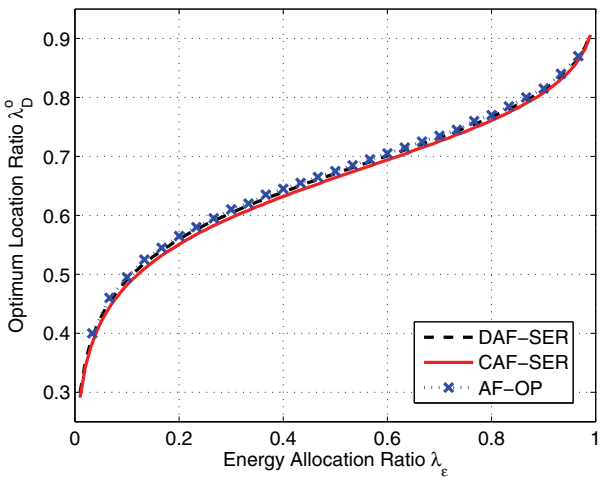

Fig. 6. SER vs. OP: The optimum relay location of AF relay systems. The target performance is $P_{target} = 10^{-3}$, the rate threshold is $R = 1$ for OP metrics, and $\nu = 4$.

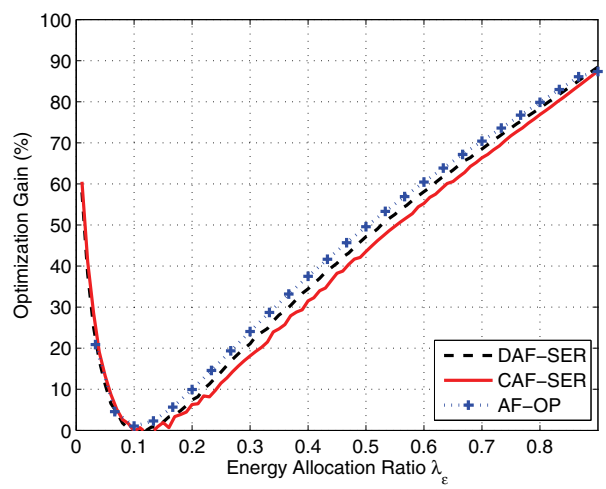

Fig. 7. SER vs. OP: The optimization gain of AF relay systems. The target performance is $P_{target} = 10^{-3}$, the rate threshold is $R = 1$ for OP metrics, and $\nu = 4$.

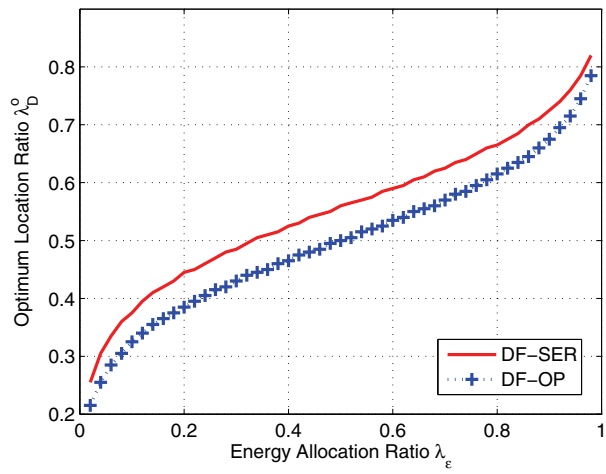

Fig. 8. SER vs. OP: The optimum relay location of DF relay systems with $L = 1$. The target performance is $P_{target} = 10^{-3}$, the rate threshold is $R = 1$ for OP metrics, and $\nu = 4$.

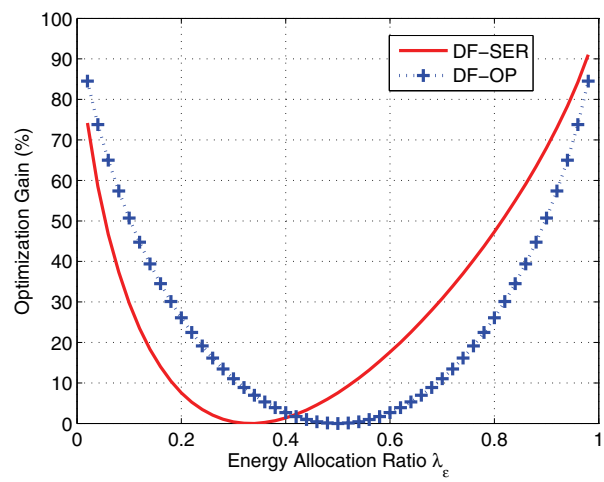

Fig. 9. SER vs. OP: Optimization gain of DF relay systems with $L = 1$. The target performance is $P_{target} = 10^{-3}$, the rate threshold is $R = 1$ for OP metrics, and $\nu = 4$.

capacity threshold $R = 1$, a pathloss factor of $\nu = 4$, and an ellipse topology with S-D distance $D_{s,d} = 1$ and S-R-D sum distance $D = 1.2$.

A. Optimization Metric: SER vs. OP

For AF relay systems with $L = 1$, the SERs for both coherent and differential modulations are computed as:

$$\bar{P}^{AF}_{ser,L=1} = \frac{1}{g}\left[\frac{1}{\bar{\gamma}_{r,s}} + \frac{1}{\bar{\gamma}_{d,r}}\ln(\bar{\gamma}_{d,r})\right]. \tag{35}$$

Compared to the outage probability of an AF relay system in Eq. (19), the above SER expression appears to be very different mathematically. To reveal the relationship of their location optimization results, the optimum relay location and optimization gain using both SER and OP metric are shown in Figs. 6 and 7, respectively. Interestingly, the SER and OP curves match very well, which means that the SER and OP performance metric are equivalent for AF relay systems from the perspective of location optimization. In addition, the figures verify that the optimization results of the DAF and CAF systems have very small difference.

On the other hand, for DF relay systems with $L = 1$, the outage probability of DF relay system is:

$$P^{DF}_{out,L=1} = \left(2^{2R} - 1\right)\left[\frac{1}{\bar{\gamma}_{r,s}} + \frac{1}{\bar{\gamma}_{d,r}}\right]. \tag{36}$$

While the SERs for CDF and DDF relay systems with $L = 1$ are:

$$\bar{P}^{DF}_{ser,L=1} = \frac{1}{g}\left[\frac{1}{2\bar{\gamma}_{r,s}} + \frac{1}{\bar{\gamma}_{d,r}}\right]. \tag{37}$$

The OP and SER formulas have different coefficients of the first term in the bracket. This difference comes from the fact that when the relay detects an error or outage and does not forward the information, the average SER at the destination is $1/2$, however the corresponding OP is 1. The resultant

optimum locations using these two metrics are compared in Fig. 8. We observe that the OP-based optimal location ratio is smaller than the SER-based result. Since the OP is larger than SER for the case when the relay is not forwarding, the OP of a relay system relies more on the source-to-relay link, thus the corresponding optimal relay location needs to be closer to the source. The plot of optimization gain in Fig. 9 confirms this observation.

In summary, from the above comparisons of optimization results using both OP and SER, we notice that, even though SER and OP evaluate the system performance from different aspects, surprisingly, the SER behaves exactly the same as OP in location optimization for $L = 1$ AF relay systems. This indicates that the SER-optimized relay system is also OP-optimal. On the other hand, for DF relay systems, SER and OP generate different optimization results due to the performance difference in the relay non-forwarding scenario. Since SER can be formulated in closed form for all four relay systems with arbitrary number of relays, while OP is only available for single relay systems for AF protocol. Therefore, we adopt SER to further study the effect of the relaying protocols on location optimization.

B. Relaying Protocol: AF vs. DF

In Fig. 10, the optimum relay location ratios (λ_D^o) are plotted with respect to different energy allocation ratios ($\lambda_{\mathcal{E}}$) for all four relay systems with $L = 1,\ 2,$ and 3. The corresponding optimization benefits are shown in Fig. 11. In all figures, the $L = 1,\ 2,$ and 3 systems are represented by the solid, dashed and dotted curves, respectively. The curves for DF systems are illustrated with circle markers.

Comparing these curves for AF and DF relay systems, we observe that the optimum relay locations are distinct for AF and DF relay systems. At any energy allocation ratio, compared to the AF relay system, the DF relay system always has a larger optimum relay location ratio. The observation means that the DF system requires the relays to stay closer to the source to achieve the optimum system performance. This is induced by the different performance dependence on the S-R link and relay-destination (R-D) link between AF and DF protocols. In DF-SR, only correctly decoded relay(s) will forward the signal to the destination, thus the end-to-end performance has higher dependence on the S-R link. On the other hand, in AF-FA, all signals are retransmitted to the destination, resulting in balanced dependence on both links. Thus compared to the optimum AF systems, the optimum DF systems require the relay(s) to move closer to the source.

Furthermore, we observe that the difference between AF and DF relay systems diminishes as L increases, which indicates that as more relays are involved, AF and DF systems have more similar optimum resource allocation. This is due to the change of the performance balance between the S-R link and R-D link. For a DF relay system, the scenario with most relays decoding in error will generate a large BER. Particularly, the BER is 50% if all relays decode in error. As L increases, the probability of all relays in error decreases exponentially as $\bar{\gamma}_{r,s}^{-L}$, thus the dependence on the S-R link is lessened, i.e. the S-R link and the R-D link are more balanced.

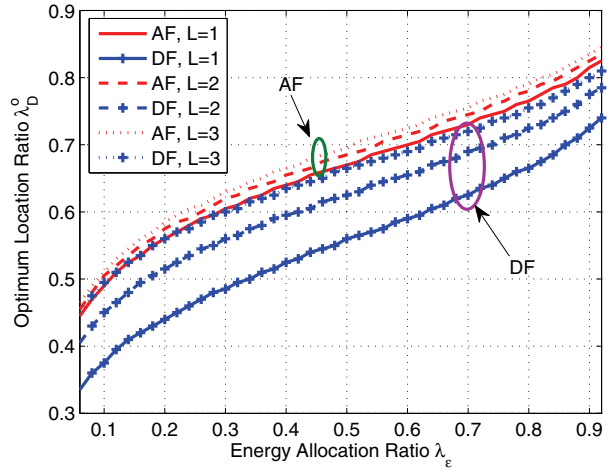

Fig. 10. AF vs. DF: The optimum relay location with $\nu = 4$ and $P_{target} = 10^{-3}$.

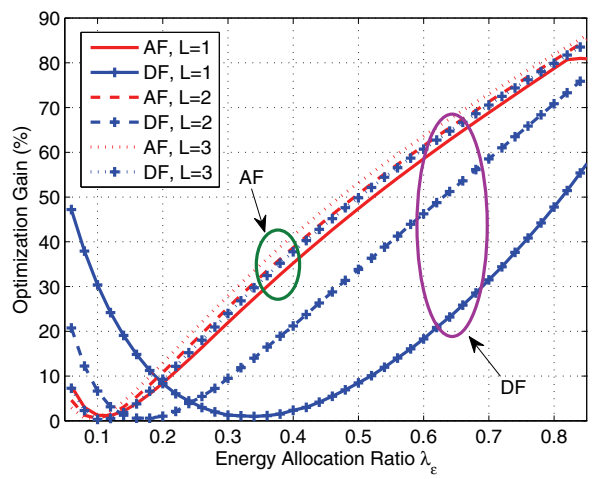

Fig. 11. AF vs. DF: The optimization gain with $\nu = 4$ and $P_{target} = 10^{-3}$.

Hence, the optimum location moves towards the destination. On the other hand, for AF systems, the performance balance on both links does not vary much with L. Therefore, the difference between the AF and DF systems reduces in terms of optimum location. Besides, the optimum relay location of both the AF and DF relay systems share some similarity: the more energy is allocated to the source, the closer the optimum relay needs to move to the destination.

The optimization gain curves in Fig. 11 confirm the difference between AF and DF systems. Since the unoptimized relay system has a relay location ratio of $\lambda_D = 1/2$, thus the energy allocation corresponding to '0' percent optimization gain is the $\lambda_{\mathcal{E}}$ for an optimum relay system with $\lambda_D^o = 1/2$. Fig. 11 shows that larger $\lambda_{\mathcal{E}}$ is needed for a DF relay system than an AF relay system with the same L. This restates that the S-R link is more critical for a DF relay system. However, as L increases, the optimization gain curves of DF and AF relay systems merge together, which means that the difference between DF and AF relay systems shrinks for large L.

C. Discussions

Putting all four systems together, the effects of optimization metric, modulation type and relaying protocol on resource optimization be can summarized as: (1) The SER and OP optimization metrics provide different performance representations from different aspects. However, they suggest the same resource allocation for $L = 1$ AF relay systems; (2) Although the coherent and differential modulation types render different coding gains, they have negligible effect on the optimum resource allocation; (3) The AF and DF relaying protocols have prominent impact on the optimum resource allocation. But their difference diminishes as the number of relays L increases.

VI. Conclusions

In this paper, we investigated the affecting factors of resource optimization in cooperative relay networks. Several commonly adopted performance metrics and relay systems are chosen for a case study. To facilitate this study, we derived and summarized the analytical SER and OP results for four commonly adopted relay systems: CAF, DAF, CDF and DDF. In addition, a practical location optimization problem was formulated, which minimizes the total transmit energy subject to some preset SER or OP performance requirement. Then we performed location optimization on all four systems based on both the SER and OP metrics. To reveal the relative effects of different factors on the location optimization: optimization metric (SER vs. OP), modulation type (coherent vs. differential) and relaying protocol (AF vs. DF), comparative analyses and numerical results were presented. The results show that relaying protocol is the major factor dictating the optimization results, whereas all optimization results become similar as the number of relays increases.

Appendix I: Proof of Average SER for Coherent AF System

For CAF relay systems, the received SNR at the destination is given in Eq. (6). According to Eq. (41), the average SER of a CAF system is calculated as:

$$\bar{P}_{ser}^{CAF} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} d\theta \prod_{l=1}^{L}\int_0^{\infty} e^{-\frac{\gamma_{eq,r_l}}{\sin^2(\theta)}} p(\gamma_{eq,r_l}) d\gamma_{eq,r_l}.$$

The distribution of the equivalent SNR γ_{eq,r_l} on each relay link in an AF system is [28]:

$$\begin{aligned} p(\gamma_{eq,r_l}) = & \frac{\beta^2}{2} e^{-\frac{\gamma_{eq,r_l}}{\bar{\gamma}_{r_l,s}}} K_0(\beta\sqrt{\gamma_{eq,r_l}}) \\ & + \frac{\beta}{\bar{\gamma}_{r_l,s}} e^{-\frac{\gamma_{eq,r_l}}{\bar{\gamma}_{r_l,s}}} \sqrt{\gamma}_{eq,r_l} K_1(\beta\sqrt{\gamma_{eq,r_l}}), \end{aligned} \tag{38}$$

where $\beta = 2\sqrt{(1+1/\bar{\gamma}_{r_l,s})/\bar{\gamma}_{d,r_l}}$. With the same technique used in [7, Appendix I], we can compute:

$$\int e^{-\frac{\gamma_{eq,r_l}}{\sin^2(\theta)}} p(\gamma_{eq,r_l}) d\gamma_{eq,r_l} \approx \sin^2(\theta)\left[\frac{1}{\bar{\gamma}_{r_l,s}} + \frac{1}{\bar{\gamma}_{d,r_l}} \ln\left(\frac{\bar{\gamma}_{d,r_l}}{\sin^2(\theta)}\right)\right].$$

It can be reexpressed by expanding the product,

$$\begin{aligned} \bar{P}_{ser}^{CAF} \approx & \sum_{\mathcal{A}\subseteq\mathcal{R}} \left\{\prod_{i\in\mathcal{A}} \frac{1}{\bar{\gamma}_{r_i,s}}\right\} \cdot \\ & \cdot \left\{\prod_{j\notin\mathcal{A}} \frac{1}{\bar{\gamma}_{d,r_j}} \sum_{\mathcal{B}\subseteq\bar{\mathcal{A}}} \left[\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{2L}(\theta)}{\left(\ln\frac{1}{\sin^2(\theta)}\right)^{|\mathcal{A}|+|\mathcal{B}|-L}} d\theta\right] \prod_{k\in\mathcal{B}} \ln\bar{\gamma}_{d,r_k}\right\}, \end{aligned} \tag{39}$$

where $\mathcal{A}$ is any subset of L relay set $\mathcal{R}$, and $\bar{\mathcal{A}}$ is the complementary set of $\mathcal{A}$. $\mathcal{B}$ is any subset of $\bar{\mathcal{A}}$. Denote that,

$$J(\mu) := \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin^{2L}(\theta)(\ln\frac{1}{\sin^2(\theta)})^{\mu} d\theta, \qquad \mu \in \{0, \cdots, L\},$$

$J(0)$ is much larger than $J(\mu),\ \mu \neq 0$. Combining with the high SNR assumption, where $\ln(\bar{\gamma}_{d,r_k}) \gg 1$, we can neglect the term for $\mu \neq 0$, thus Eq. (39) can be approximated as:

$$\bar{P}_{ser}^{CAF} \approx \left(\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin^{2L}(\theta) d\theta\right) \prod_{l=1}^{L}\left[\frac{1}{\bar{\gamma}_{r_l,s}} + \frac{1}{\bar{\gamma}_{d,r_l}} \ln\bar{\gamma}_{d,r_l}\right].$$

With the trigonometric result in Eq. (43), the average SER for CAF systems can be expressed as in Eq. (13).

Appendix II: Proof of Average SER for Coherent DF System

For the coherent BPSK system, the corresponding SER at the destination conditioned on the instantaneous SNR γ_d can be calculated as $P_{ser|\gamma_d} = Q(\sqrt{2\gamma_d})$, [18, Chap. 11]. The alternative representation of Q function is given in [21, Chap. 4],

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} e^{-\frac{x^2}{2\sin^2(\theta)}} d\theta. \tag{40}$$

Thus the average SER can be calculated by averaging over the distribution of the SNR on each link as follows.

$$\bar{P}_{ser} = \int P_{ser|\gamma_d} p(\gamma_d) d\gamma_d = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} d\theta \int_0^{\infty} e^{-\frac{\gamma_d}{\sin^2(\theta)}} p(\gamma_d) d\gamma_d. \tag{41}$$

Given γ_d of DF systems in Eq. (8), the average SER for a CDF system conditioned on the correct decoding set $\mathcal{D}$ is computed as,

$$\bar{P}_{ser|\mathcal{D}}^{CDF} = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} d\theta \prod_{l\in\mathcal{D}} \int_0^{\infty} e^{-\frac{\gamma_{d,r_l}}{\sin^2(\theta)}} p(\gamma_{d,r_l}) d\gamma_{d,r_l}. \tag{42}$$

Under the assumption of Rayleigh fading channel, the distribution of the received SNR is:

$$p(\gamma_{i,j}) = \frac{1}{\bar{\gamma}_{i,j}} e^{-\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}}.$$

Then each integration term in Eq. (42) can be approximated as follows with high average SNR,

$$\int e^{-\frac{\gamma_{i,j}}{\sin^2(\theta)}} p(\gamma_{i,j}) d\gamma_{i,j} \approx \frac{\sin^2(\theta)}{\bar{\gamma}_{i,j}}.$$

In addition, with the trigonometric integration result:

$$\frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin^{2K}(\theta) d\theta = \frac{\binom{2K-1}{K}}{4^K},$$

the conditional average SER in Eq. (42) can be calculated as:

$$\bar{P}_{ser|\mathcal{D}}^{CDF} \approx \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \sin^{2|\mathcal{D}|}(\theta)d\theta \prod_{l\in\mathcal{D}} \frac{1}{\bar{\gamma}_{d,r_l}} = \frac{\binom{2|\mathcal{D}|-1}{|\mathcal{D}|}}{4^{|\mathcal{D}|}} \prod_{l\in\mathcal{D}} \frac{1}{\bar{\gamma}_{d,r_l}}.$$

Notice from the above equation that the average SER for each link as: $\bar{P}_{i,j} = \frac{1}{4\bar{\gamma}_{i,j}},\ i,j \in \{s, r_1 \cdots r_L, d\}$. Thus the probability of the correctly decoding set $\mathcal{D}$ is computed as:

$$P(\mathcal{D}) = \prod_{l\in\mathcal{D}} \left(1 - \frac{1}{4\bar{\gamma}_{r_l,s}}\right) \prod_{k\notin\mathcal{D}} \frac{1}{4\bar{\gamma}_{r_k,s}}.$$

By summing up all possible decoding set $\mathcal{D}$ with the corresponding probability, we can compute the average SER of the CDF relay system as in Eq. (14).

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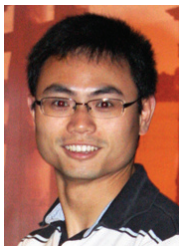

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