Efficient and Scalable Computation of the Energy and Makespan Pareto Front for Heterogeneous Computing Systems

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Problem Statement

- static scheduling
  - single bag-of-tasks
  - task assigned to only one machine
  - machine runs one task at a time
- heterogeneous tasks and machines
- desire to reduce energy consumption (operating cost) and makespan
- to aid decision makers
  - find high-quality schedules for both energy and makespan
  - desire computationally efficient algorithms to compute Pareto fronts

Outline

- linear programming lower bound
- recovering a feasible allocation to form an upper bound
- Pareto front generation techniques
- comparison with NSGA-II
- complexity and scalability

Preliminaries

- simplifying approx: **tasks are divisible among machines**
- T_i − number of tasks of type *i*
- M_j − number of machines of type *j*
- x_{ij} − number of tasks of type *i* assigned to machine type *j*
- ETC_{ij} − estimated time to compute for a task of type *i* running on a machine of type *j*
- finishing time of machine type *j* (lower bound):

$$F_j = \frac{1}{M_j} \sum_i x_{ij} ETC_{ij}$$

- APC_{ij} − average power consumption for a task of type *i* running on a machine of type *j*
- $APC_{\emptyset j}$ − idle power consumption for a machine of type *j*

Objective Functions

- makespan (lower bound)

$$MS_{LB} = \max_j F_j$$

- energy consumed by the bag-of-tasks (lower bound)

$$E_{LB} = \text{execution energy} + \text{idle energy}$$

$$= \sum_i \sum_j x_{ij} APC_{ij} ETC_{ij} + \sum_i \sum_j M_j APC_{\emptyset j} (MS_{LB} - F_j)$$

$$= \sum_i \sum_j x_{ij} ETC_{ij} (APC_{ij} - APC_{\emptyset j}) + \sum_j M_j APC_{\emptyset j} MS_{LB}$$

- note that energy is a function of makespan when we have non-zero idle power consumption

Linear Bi-Objective Optimization Problem

$$\underset{x_{ij},\, MS_{LB}}{\text{minimize}} \begin{pmatrix} E_{LB} \\ MS_{LB} \end{pmatrix}$$

subject to:

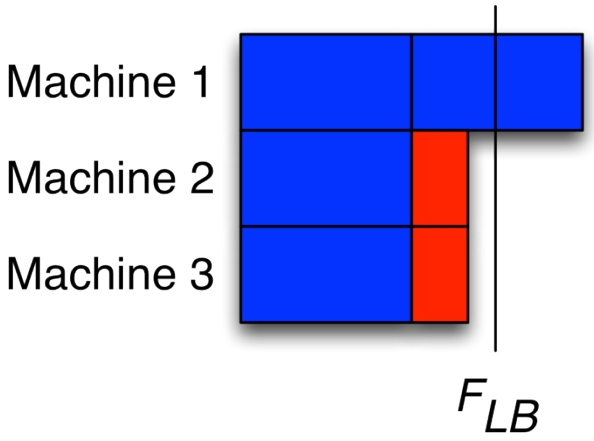
$\forall_i \quad \sum_j x_{ij} = T_i$ task constraint

$\forall_j \quad F_j \le MS_{LB}$ makespan constraint

$\forall_{ij} \quad x_{ij} \ge 0$ assignments must be non-negative

Linear Programming Lower Bound Generation

- solve for $x_{ij} \in \mathbb{R}$ to obtain a lower bound
  - generally infeasible solution to the actual scheduling problem
  - a lower bound for each objective function (tight in practice)
- solve for $x_{ij} \in \mathbb{Z}$ to obtain a tighter lower bound
  - requires branch and bound (or similar)
    - typically this is computationally prohibitive
  - still making the assumption that tasks are divisible among machines



Recovering a Feasible Allocation: Round Near

- find $\dot{x}_{ij} \in \mathbb{Z}$ that is "near" to x_{ij} while maintaining the task assignment constraint
- for task type (each row in x) do
  - let $n = T_i - \sum_j \text{floor}(x_{ij})$
  - let $f_j = \text{frac}(x_{ij})$ be the fractional part of x_{ij}
  - let set K be the indices of the n largest f_j
  - $\dot{x}_{ij} = \text{ceil}(x_{ij})$ if $j \in K$ else $\text{floor}(x_{ij})$

$x_i =$	3	0	6	5	0
$\dot{x}_i =$	3	0	6	5	0

$x_i =$	3	2.3	6.3	5.4	0
$\dot{x}_i =$	3	2	6	**6**	0

$x_i =$	3	0	6.6	5.4	0
$\dot{x}_i =$	3	0	**7**	5	0

$x_i =$	3.9	2.2	6.4	5.3	4.2
$\dot{x}_i =$	**4**	2	**7**	5	4

Recovering a Feasible Allocation: Local Assignment

- given integer number of tasks for each machine type
- assign tasks to actual machines within a homogeneous machine type via a greedy algorithm
- for each machine type (column in $\dot{x}$) do longest processing time algorithm
  - while any tasks are unassigned
    - assign longest task (irrevocably) to machine that has the earliest finish time
    - update machine finish time

Pareto Front Generation Procedure

- **step 1** weighted sum scalarization
  - **step 1.1** find the utopia (ideal) and nadir (non-ideal) points
  - **step 1.2** sweep α between 0 and 1
    - at each step solve the linear programming problem

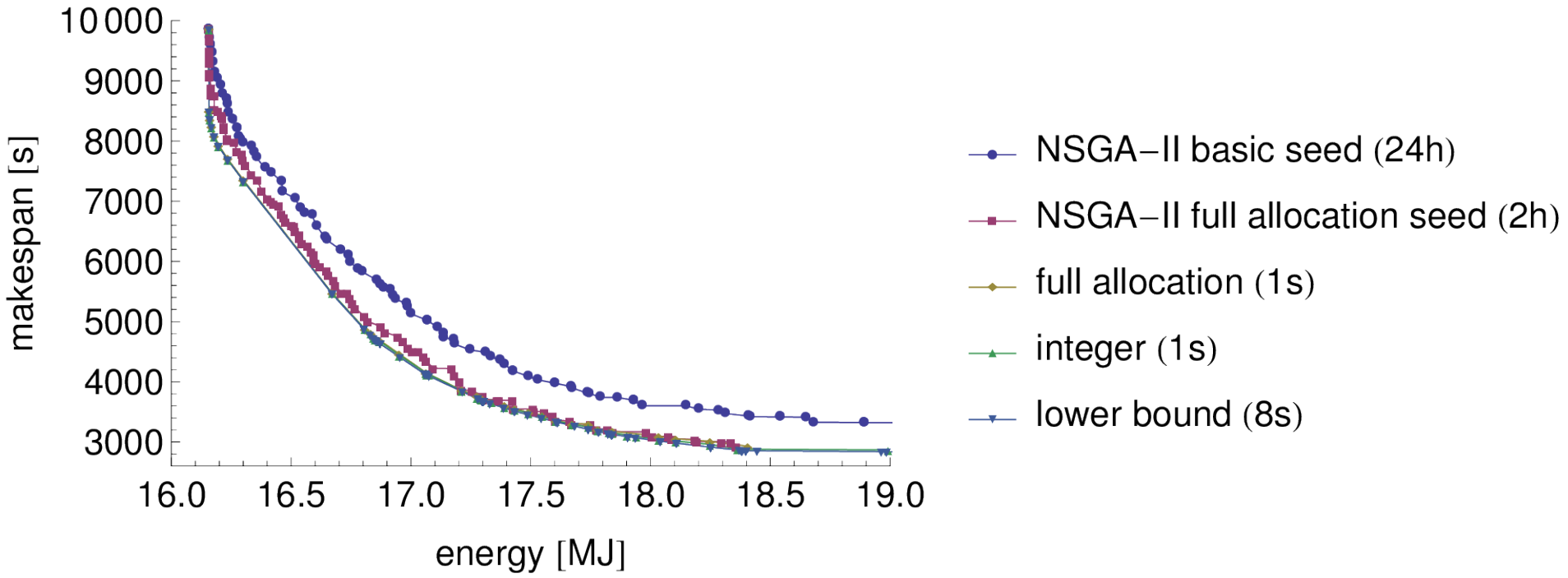
$$\min \frac{\alpha}{\Delta E_{LB}} E_{LB} + \frac{1-\alpha}{\Delta MS_{LB}} MS_{LB}$$

  - **step 1.3** remove duplicates
- linear objective functions and convex constraints
  - convex, lower bounds on Pareto front
- **step 2** round each solution
- **step 3** remove duplicates
- **step 4** locally assign each solution
- **step 5** remove duplicates and dominated solutions
- full allocation is an upper bound on the true Pareto front

Simulation Results

- simulation setup
  - ETC matrix derived from actual systems
  - 9 machine types, 36 machines, 4 machines per type
  - 30 task types, 1100 tasks, 11-75 tasks per type
- Non-dominated Sorting Genetic Algorithm II
  - NSGA-II is another algorithm for finding the Pareto front
  - an adaptation of classical genetic algorithms
  - basic seed
    - min energy, min-min completion time, and random
  - full allocation seed
    - all solutions from upper bound Pareto front

Pareto Fronts



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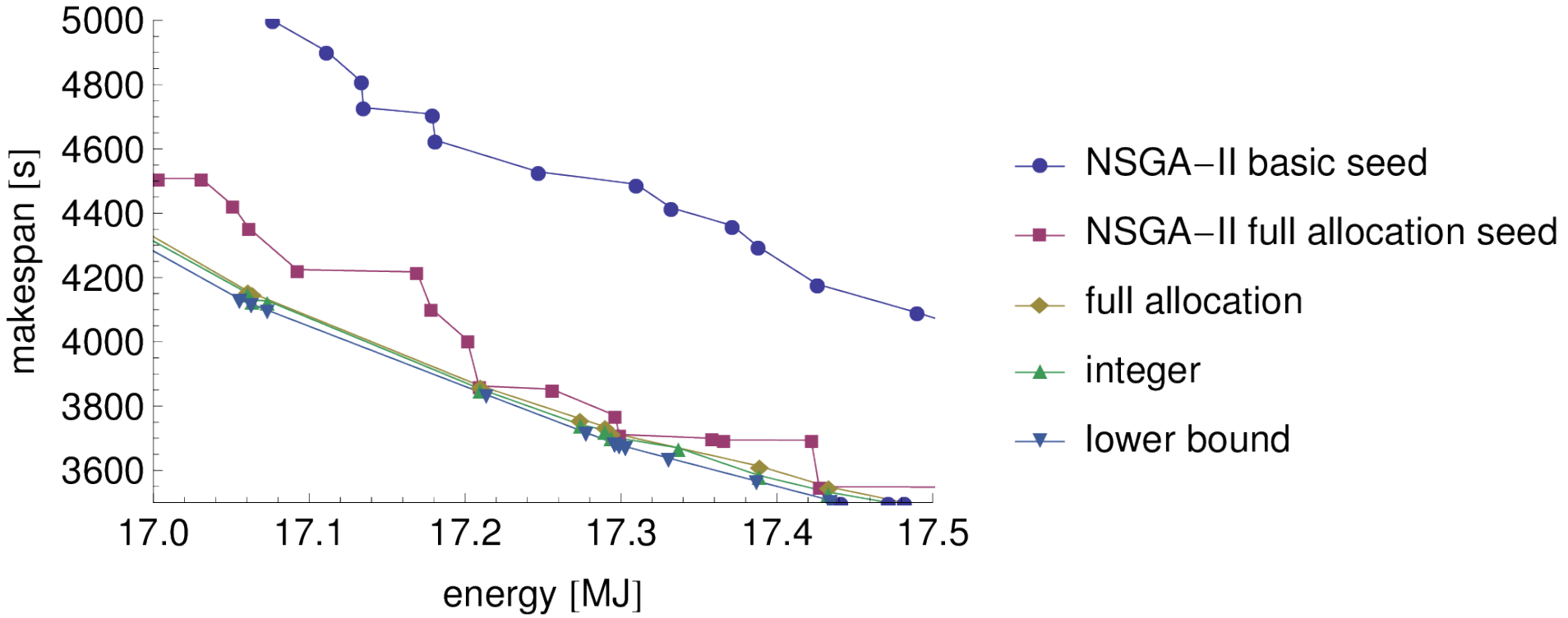
Pareto Fronts (Zoomed)



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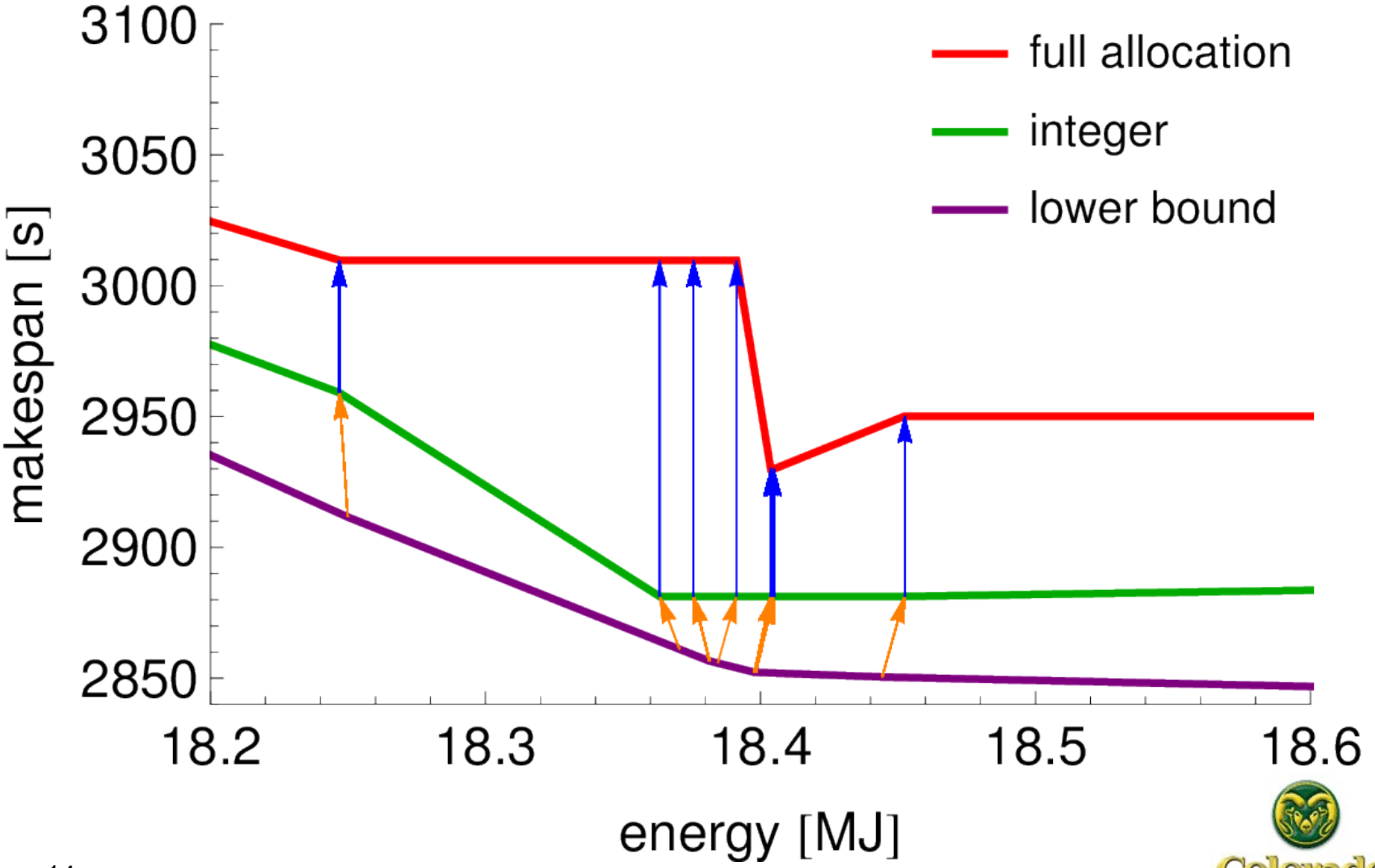
Lower Bound to Full Allocation, No Idle Power



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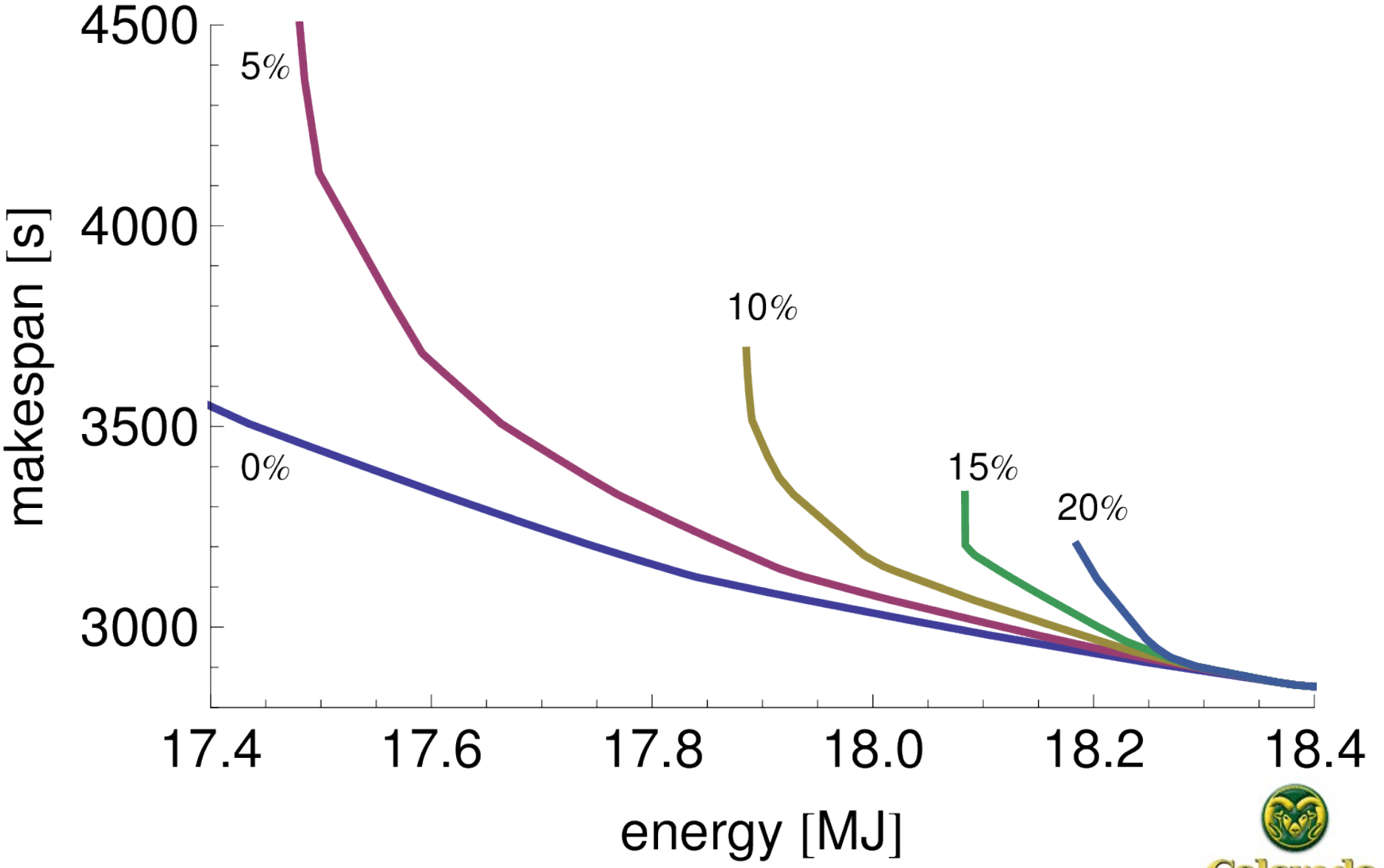
Effect of Idle Power



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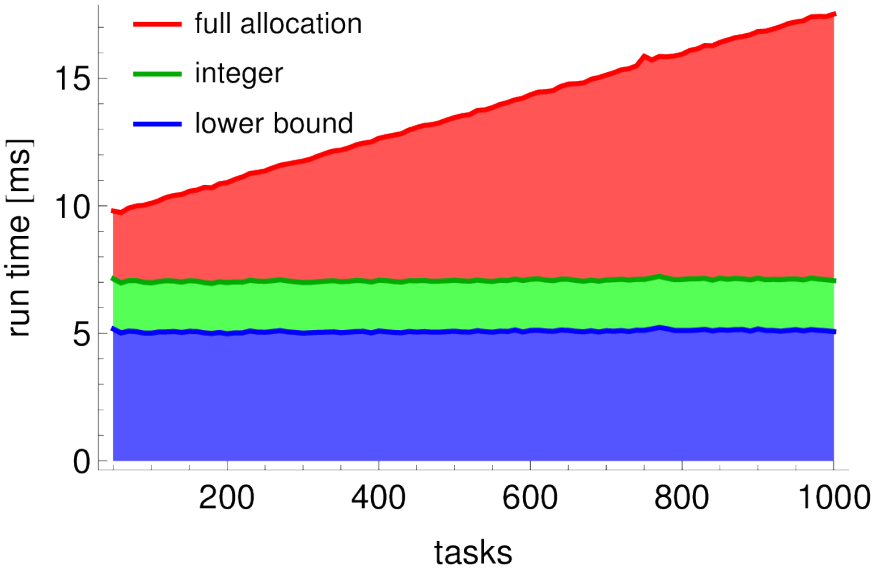
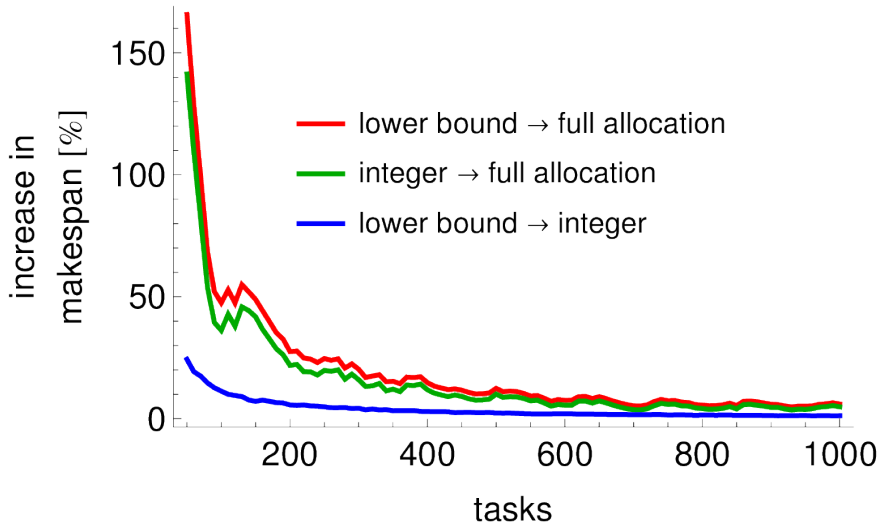
Complexity

- let the *ETC* matrix be $T \times M$
- linear programming lower bound
  - average complexity $(T+M)^2\ (TM+1)$
- rounding step: $T(M \log M)$
- local assignment step:
  - number of tasks for machine type j is $n_j = \sum_i x_{ij}$
  - worst cast complexity is $M \max_j (n_j \log n_j + n_j \log M_j)$
- complexity of all steps is dominated by either
  - linear programming solver
  - local assignment
- complexity of linear programming solver is **independent** of the number tasks and machines
  - depends number of task types and machine types

Impact of Number of Tasks





number of task types, machines, and machine types held constant

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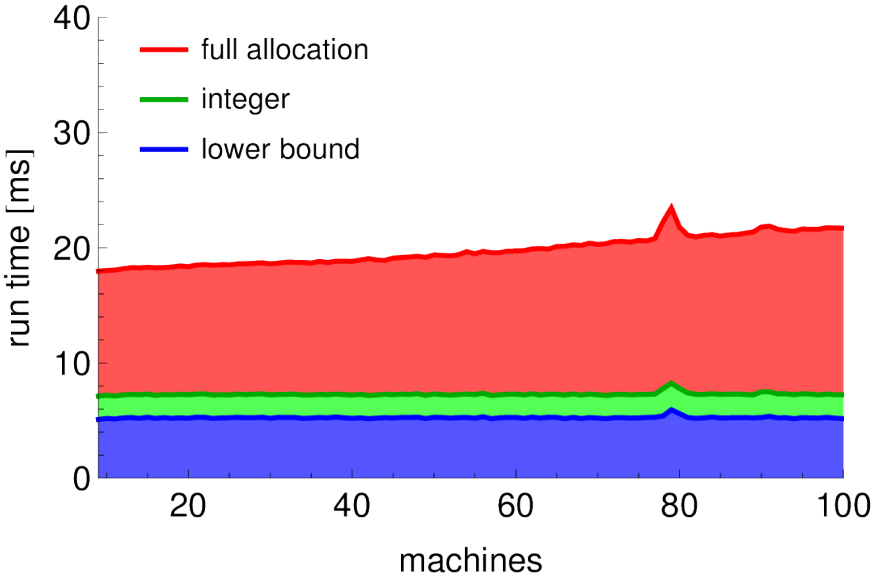
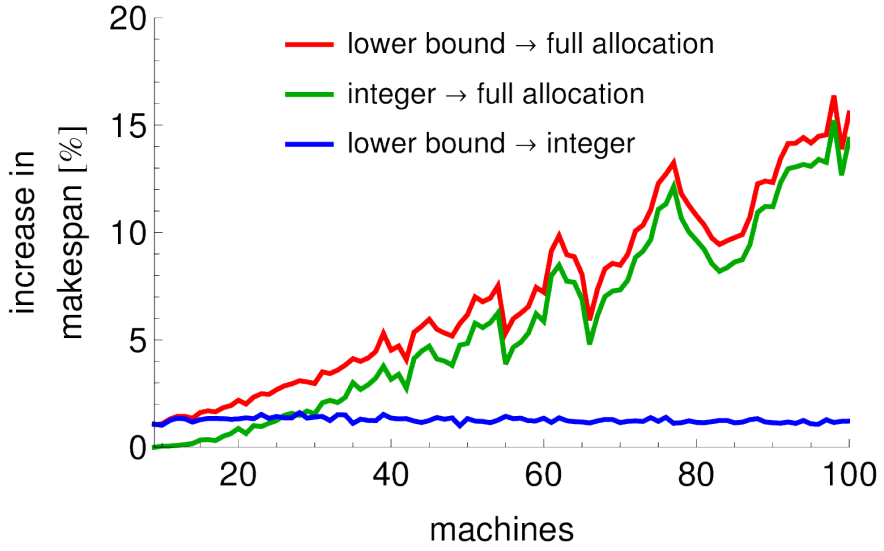
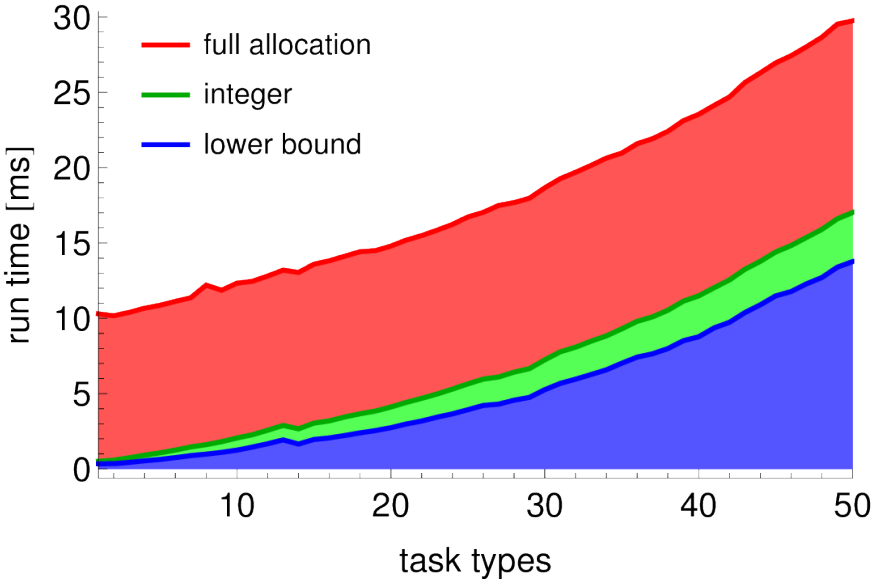
Impact of Number of Machines

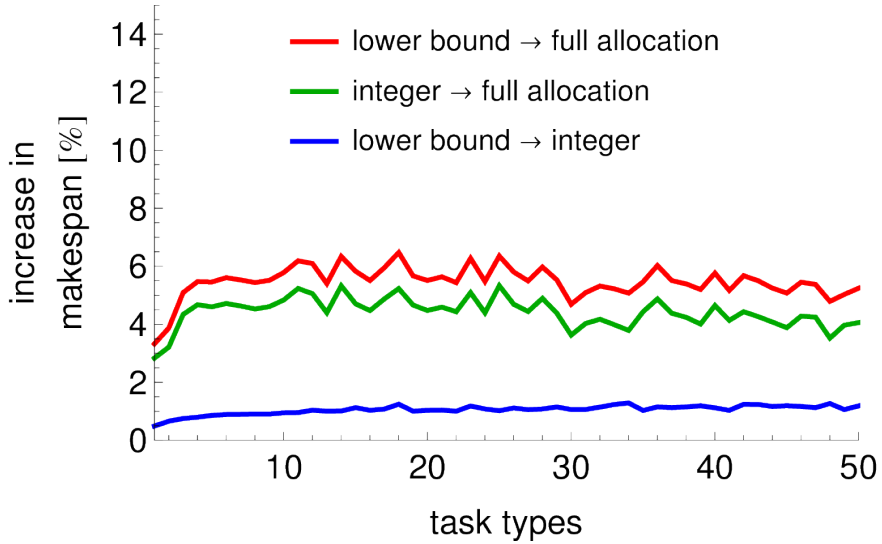




number of tasks, task types, and machine types held constant

Impact of Number of Task Types





number of tasks, machines, and machine types held constant

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Contributions

- algorithms to compute
  - tight lower bounds on the energy and makespan
  - quickly recovers near optimal feasible solutions
  - high quality bi-objective Pareto fronts
- evaluation of the scaling properties of the proposed algorithms

Conclusions

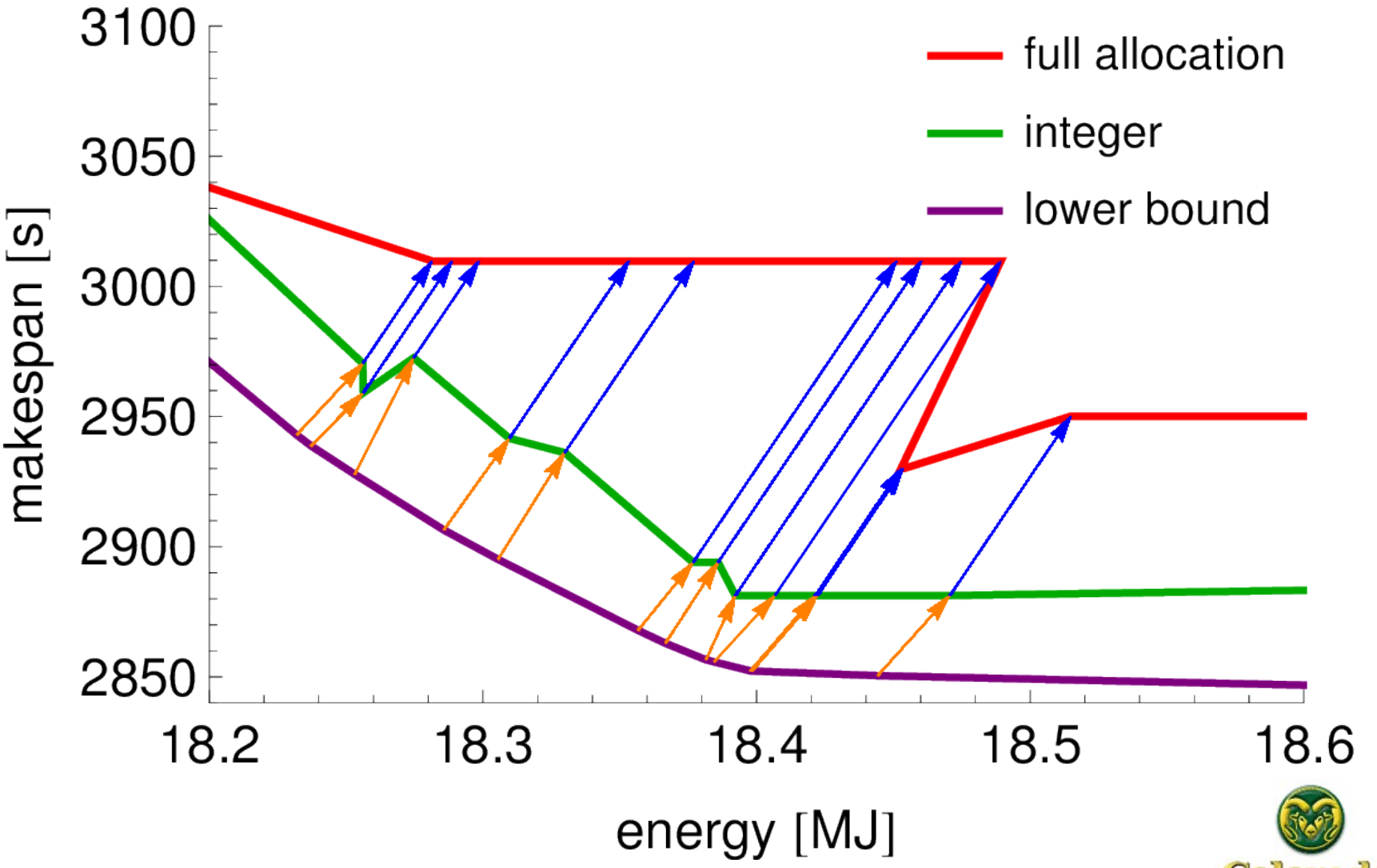
- this linear programming approach to Pareto front generation is efficient, accurate, and practical

- bounds are tight when
  - a small percentage of tasks are divided
  - a large number of tasks assigned to each machine type and individual machine
- asymptotic solution quality and runtime are very reasonable

QUESTIONS?

BACKUP SLIDES

Lower Bound to Full Allocation, 10% Idle Power



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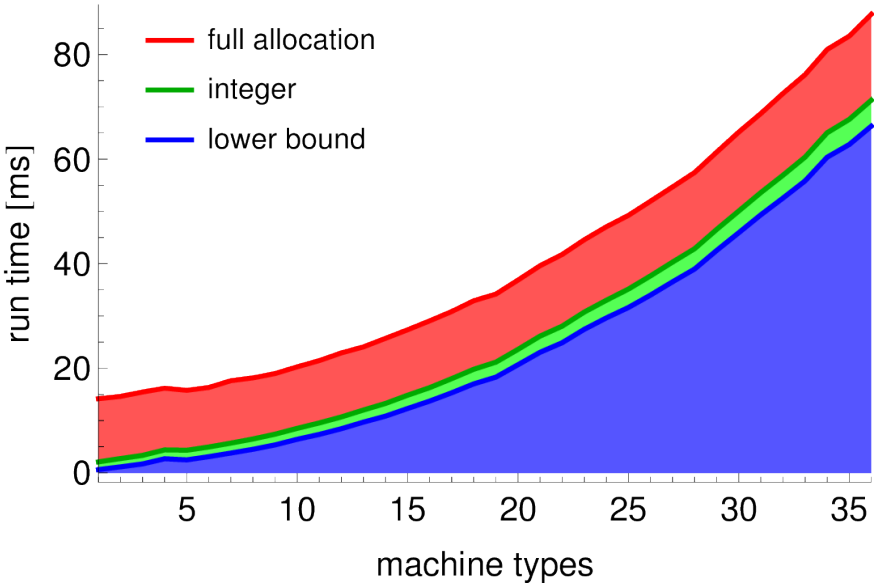
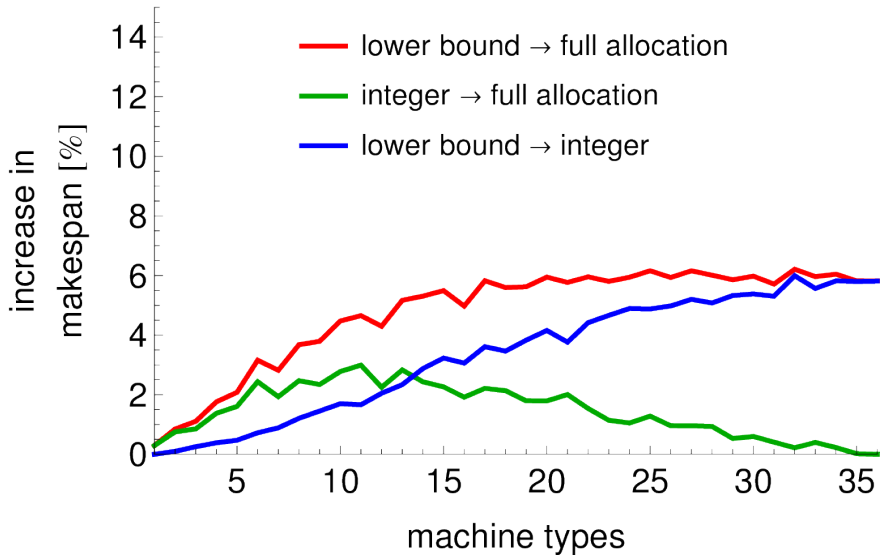
Impact of Number of Machine Types





number of tasks, task types, and machines held constant

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