3.7 *Workspace Definition*

The goal of this section is to define the reachable and dextrous workspace [Craig, 1986] of the Pipe Manipulator. The workspace describes the working volume of the manipulator which basically describes what positions the manipulator can and cannot reach in space. The reachable workspace describes the volume in space within which the manipulator end-effector center point can reach. The dextrous workspace describes a subset of the reachable workspace which also considers orientational reachability of the end-effector. The analytical description of the workspace boundaries was a fundamental step in defining some of the path planning heuristics used by the Heuristic Application-Specific Path Planner (HASPP) to be discussed in Chapter 5. Also presented in this section is a description of the self-motion envelope of the manipulator. This envelope shows how the redundancy can be used to vary the configuration of the RPRP manipulator boom and defines the range of this variance.

There are many methods available for determining the workspace boundaries of a manipulator. Many analytical methods have been proposed [Kohli and Spanos, 1984; Chen, 1986; Li et. al., 1988]. Iterative, numerical, and statistical methods have also been developed [Kumar and Waldron, 1981; Lee and Yang, 1982; Rastegar and Perel, 1988]. For the Pipe Manipulator, the joints are constrained to relatively small ranges of motion and the motion is well-behaved so that none of these methods are required. The workspace and its bounding surfaces are intuitively obvious by analyzing various combinations of the joint limits.

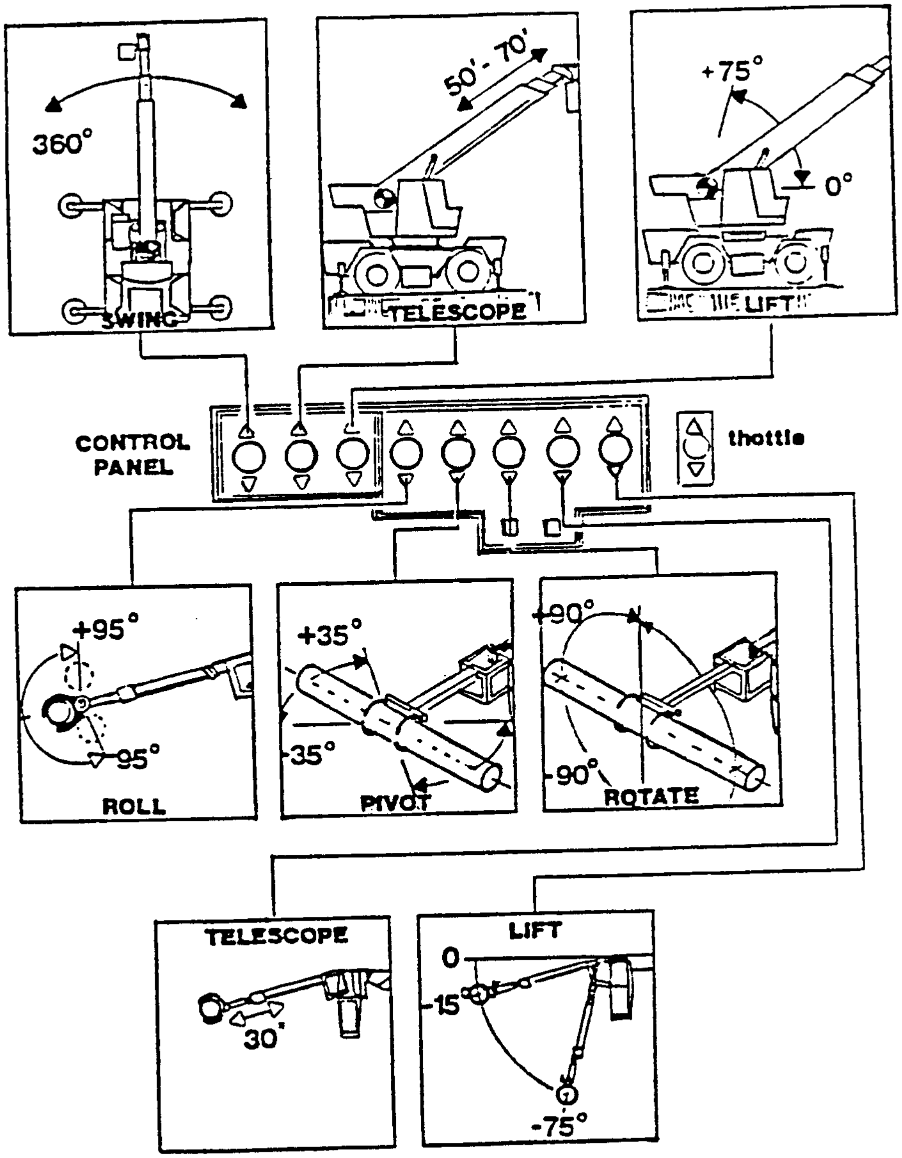


Figure 1.2 The Pipe Manipulator Degrees of Freedom

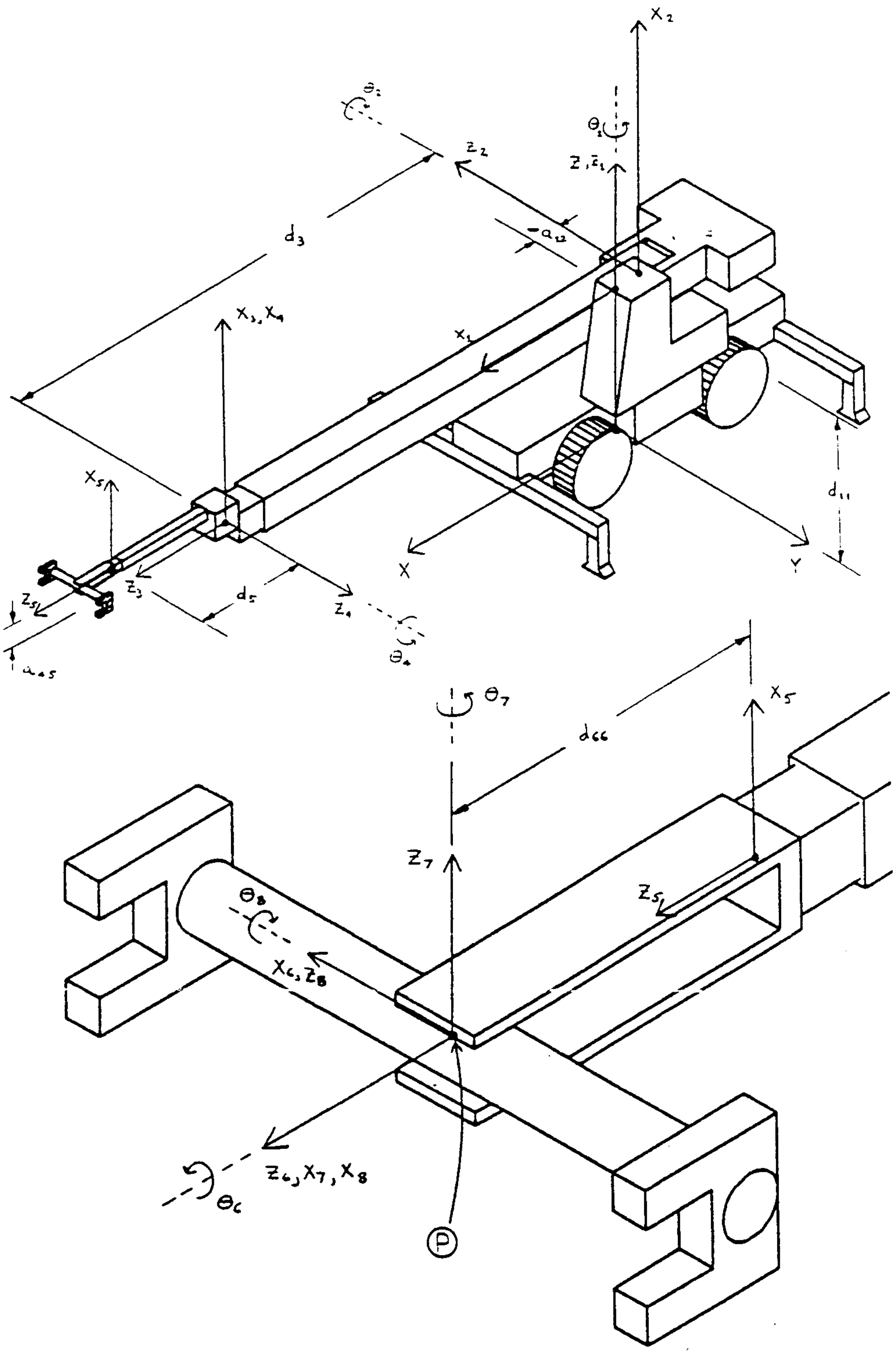


Figure 3.1 Pipe Manipulator Reference Frames

The reachable workspace is determined by the motion of the boom plane (see Figure 3.6a). By varying the degrees of freedom in this plane, the end-effector point "P" can be moved to occupy any position shown in the workspace profile in Figure 3.6b. Table 3.3 lists the exterior points in this profile with the combination of joint limits used to reach these points. The coordinates of any point in the profile are given by:

$$
\begin{aligned}
r_p^{xy} &= d_{566}s_{2\text{-}4} + d_3 s_2 + a_{45}c_{2\text{-}4} + a_{12} \\
p_z &= -d_{566}c_{2\text{-}4} - d_3 c_2 + a_{45}s_{2\text{-}4} + d_{11}
\end{aligned}
\qquad (3.80)
$$

The values in Table 3.3 were calculated from the joint limit values provided in the Link Parameter Table (Table 3.2). The closed form expressions for the workspace profile boundaries can be easily derived as equations of circular arcs and straight lines:

$$
r_{min} = \begin{cases}
B_r + (z - B_z)\dfrac{A_r - B_r}{A_z - B_z} & ; B_z \leq z \leq A_z \\
a_{12} + \left\{\left[(A_z - d_{11})^2 + (A_r - a_{12})^2\right] - (z - d_{11})\right\}^{1/2} & ; A_z \leq z \leq H_z \\
O_{GHr} + \left\{\left[(H_z - O_{GHz})^2 + (H_r - O_{GHr})^2\right] - (z - O_{GHz})\right\}^{1/2} & ; H_z \leq z \leq G_z \\
G_r + (z - G_z)\dfrac{F_r - G_r}{F_z - G_z} & ; G_z \leq z \leq F_z \\
F_r + (z - F_z)\dfrac{E_r - F_r}{E_z - F_z} & ; F_z \leq z \leq E_z
\end{cases}
$$

$$
r_{max} = \begin{cases}
O_{CDr} + \left\{\left[(A_z - O_{CDz})^2 + (A_r + O_{CDr})^2\right] - (z - O_{CDz})\right\}^{1/2} & ; C_z \leq z \leq D_z \\
a_{12} + \left\{\left[(E_z - d_{11})^2 + (E_r - a_{12})^2\right] - (z - d_{11})\right\}^{1/2} & ; D_z \leq z \leq E_z
\end{cases}
$$

(3.81)

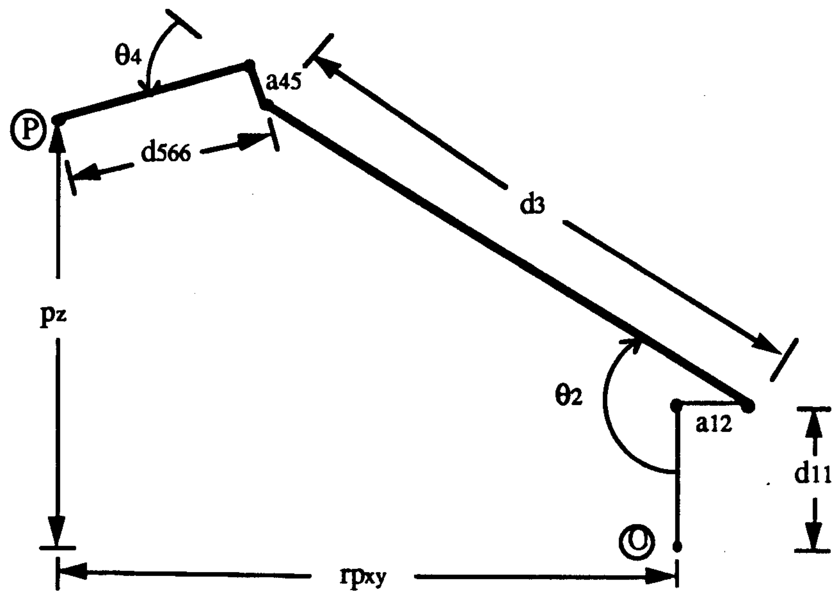
where r_{min} and r_{max} are the minimum and maximum radii reachable at a given height z. If $z < Bz$ or $z > Ez$ or $r < r_{min}$ or $r > r_{max}$ then the point in question is not within the workspace and cannot be reached.

Because joint one ("swing") is allowed unlimited rotation, the reachable workspace is simply the volume of revolution of the profile as shown in Figure 3.7 (note - only half of the volume is shown since it is symmetrical). This figure was generated with CATIA (discussed in Chapter 2) using the volume of revolution feature.

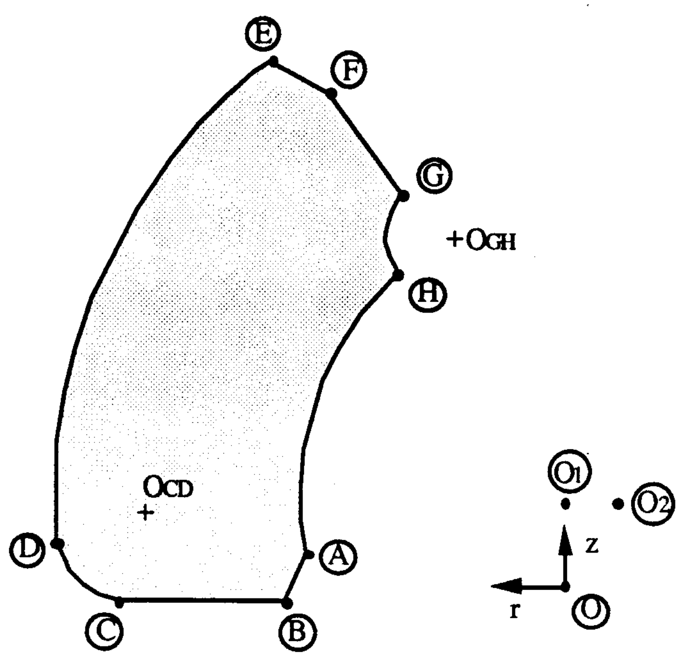
Table 3.3 Reachable Workspace Profile Points

Point	θ_2	d_3	θ_4	d_5	r^{xy}_p(ft)	p_z(ft)
A	min	min	max	min	30.468	2.538
B	min	min	max	max	31.116	0.123
C	min	max	max	max	61.116	0.123
D	min	max	min	max	67.798	8.503
E	max	max	min	max	16.867	78.072
F	max	max	min	min	15.617	75.907
G	max	min	min	min	7.852	46.929
H	max	min	max	min	12.966	40.471

O_{CD}: (57.200, 10.100) O_{GH}: (4.816, 39.217)



(a) boom plane



(b) workspace profile

Figure 3.6 Reachable Workspace Profile

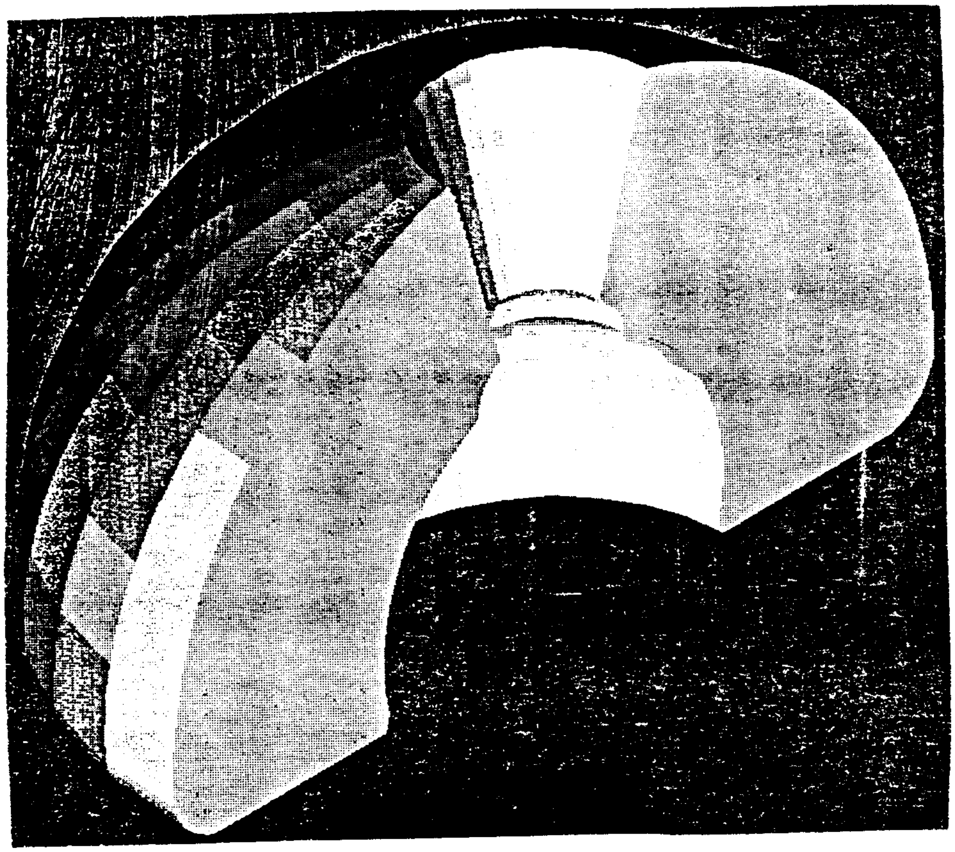


Figure 3.7 Reachable Workspace

The dextrous workspace is defined as the volume representing allowable orientations at a reachable point. Figure 3.8 shows the shape of the dextrous workspace profile at an arbitrary point "P". The profile is determined by the range of self-motion of the auxiliary boom angle β and by rotations of the wrist. Except at the outer limits of the reachable workspace, the range of β can be approximated by:

$$\Delta\beta = \theta_{4max} - \theta_{4min} \tag{3.82}$$

since $\Delta\theta_2$ is small. Given this, the angular width of the profile is:

$$\Delta\gamma = \Delta\beta + \left(\theta_{7max} - \theta_{7min}\right) \tag{3.83}$$

where θ_7 is the wrist "pivot" motion.

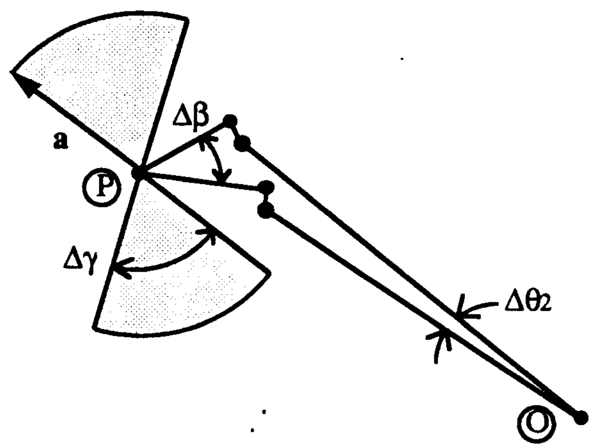


Figure 3.8 Dextrous Workspace Profile

The resulting dextrous workspace volume at point "P" is then the volume of revolution of the dextrous workspace profile about the line of action of the medium auxiliary boom angle β (since joint six can rotate $\pm 90°$). Half of this symmetrical volume is shown in Figure 3.9.

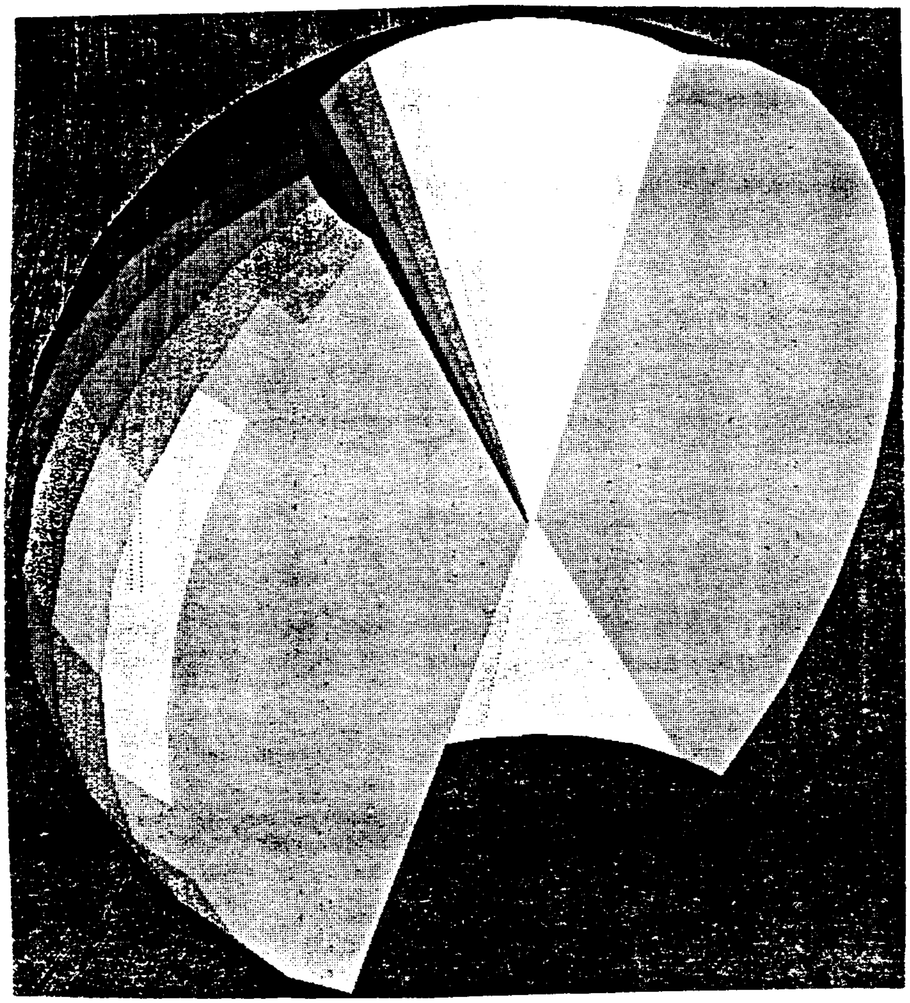


Figure 3.9 Dextrous Workspace