

## RPR Redundant Manipulator

Geometry:

$$L_1 = L_3 = 10$$

Joint Positions:

$$\theta_1 = 45^\circ \quad d_2 = 10 \quad \theta_3 = 60^\circ$$

Jacobian:

$$J = \begin{bmatrix} -(L_1 + d_2)s_1 - L_3s_{13} & c_1 & -L_3s_{13} \\ (L_1 + d_2)c_1 + L_3c_{13} & s_1 & L_3c_{13} \end{bmatrix} = \begin{bmatrix} -11.554 & 0.707 & 2.588 \\ 23.801 & 0.707 & 9.659 \end{bmatrix}$$

Joint Speeds:

$$\dot{\phi} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Forward Kinematics:

$$\dot{x} = J\dot{\phi} = \begin{bmatrix} -8.259 \\ 34.168 \end{bmatrix}$$

Pseudo-Inverse:

$$J^+ = J^T(JJ^T)^{-1} = \begin{bmatrix} -0.055 & 0.015 \\ 0.021 & 0.009 \\ 0.134 & 0.065 \end{bmatrix}$$

Particular Solution (minimum joint speed Euclidean norm):

$$\dot{\phi}_p = J^+\dot{x} = \begin{bmatrix} 0.975 \\ 0.134 \\ 1.125 \end{bmatrix} \quad \dot{x}_p = J\dot{\phi}_p = \begin{bmatrix} -8.259 \\ 34.168 \end{bmatrix}$$

General Solution:

$$\dot{\phi}_k = J^+\dot{x} + (I - J^+J)k\dot{x}_z = J\dot{\phi}_z$$

$$k_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \dot{\phi}_{k_1} = \begin{bmatrix} 1.020 \\ 1.696 \\ 0.900 \end{bmatrix} \quad \dot{x}_{k_1} = \begin{bmatrix} -8.259 \\ 34.168 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix} \quad \dot{\phi}_{k_2} = \begin{bmatrix} 1.095 \\ 4.304 \\ 0.523 \end{bmatrix} \quad \dot{x}_{k_2} = \begin{bmatrix} -8.259 \\ 34.168 \end{bmatrix}$$