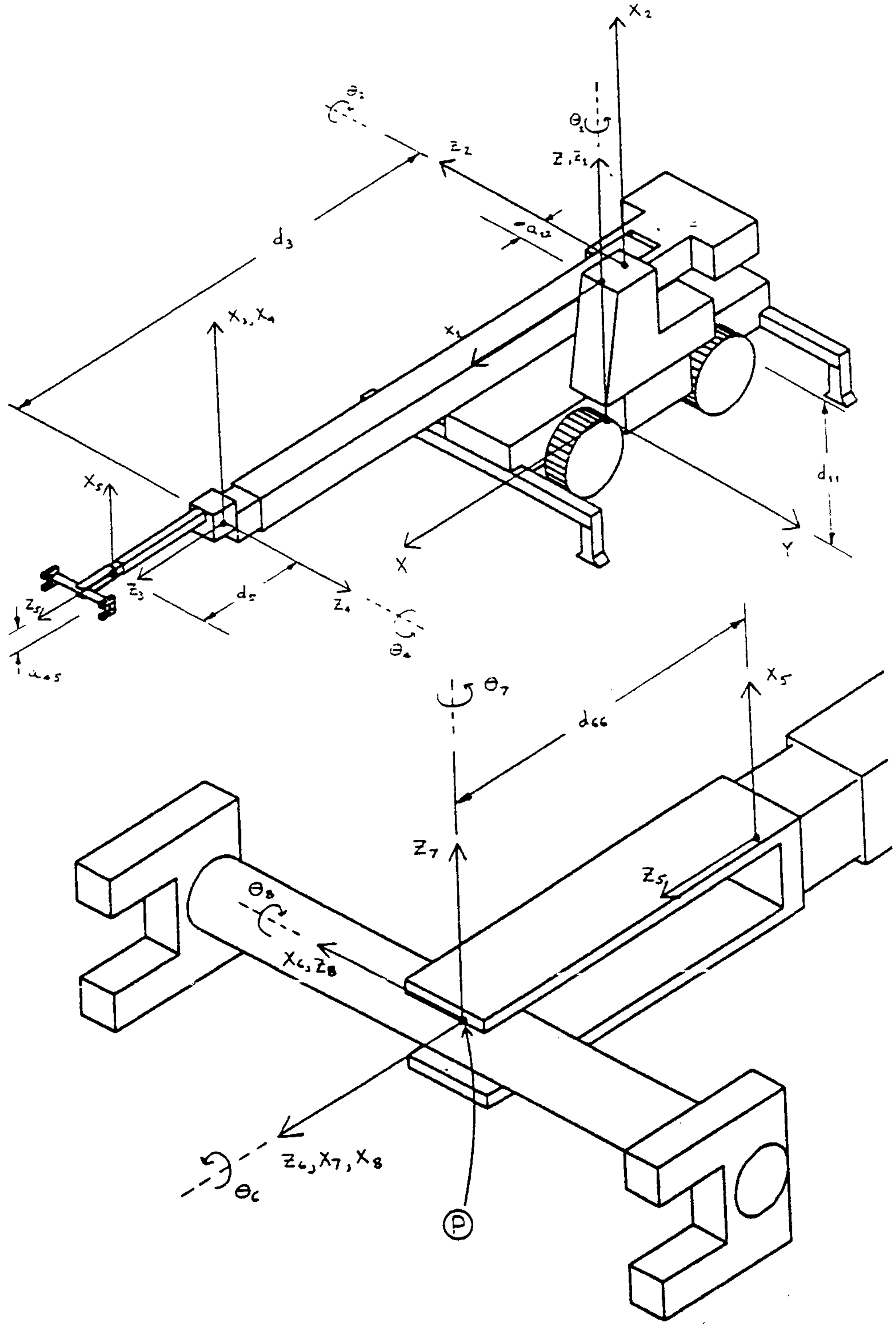

<u>Figure 3.1</u> Pipe Manipulator Reference Frames

Table 3.2 Link Parameter Table

dof (j)	α_{ij}	a_{ij}	d_j	θ_j
1	0°	0'	d_{11}	θ_1
2	90°	a_{12}	0'	θ_2
3	90°	0'	d_3	0°
4	90°	0'	0'	θ_4
5	-90°	a_{45}	d_5	0°
6	0°	0'	d_{66}	θ_6
7	90°	0'	0'	θ_7
8	90°	0'	0'	θ_8
Jaw	0°	a_{89}	0'	0°

Constant Parameters:

$a_{12} = 3.0'$ $d_{11} = 10.1'$ $a_{45} = 1.2'$

$d_{66} = 1.35'$ $a_{89} = f(jaw,D) \approx 0.0'$

$$^0T_8 = {}^0T_p\,{}^pT_8 = \begin{bmatrix} \mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

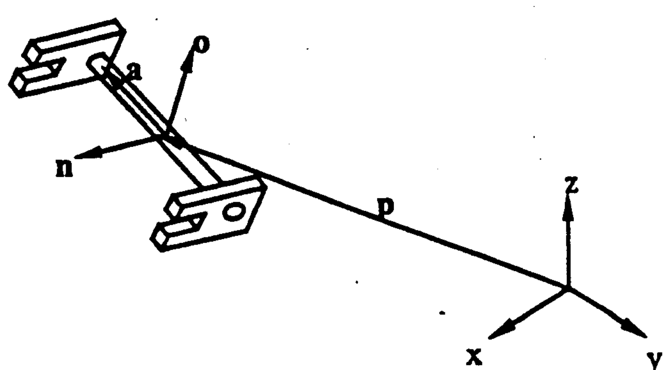

Figure 3.3 Jaw Vectors

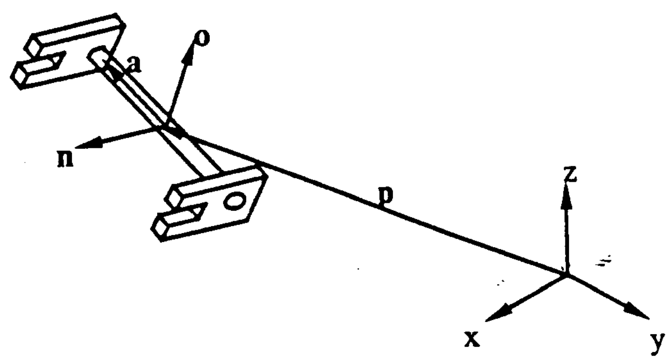

Figure 3.3 Jaw Vectors

A physical interpretation of the ${}^0T_8$ column vectors is illustrated in Figure 3.3. These four vectors both position and orient the manipulator jaws with respect to the base frame fully describing the location of a grasped pipe. The vectors may be described as:

n: jaw approach vector
o: jaw shift vector
a: pipe axis vector
p: jaw position vector

The expressions for each of the elements of ${}^0T_8$ follow:

$$\begin{aligned}
\mathbf{n}_x &= c_1\left[c_{2\text{-}4}\left(c_6c_7c_8 + s_6s_8\right) + s_{2\text{-}4}s_7c_8\right] + s_1\left(s_6c_7c_8 - c_6s_8\right) \\
&= c_1\left(c_{2\text{-}4}A + s_{2\text{-}4}s_7c_8\right) + s_1B \\
&= c_1I + s_1B \\
\mathbf{n}_y &= s_1\left[c_{2\text{-}4}\left(c_6c_7c_8 + s_6s_8\right) + s_{2\text{-}4}s_7c_8\right] - c_1\left(s_6c_7c_8 - c_6s_8\right) \\
&= s_1\left(c_{2\text{-}4}A + s_{2\text{-}4}s_7c_8\right) - c_1B \\
&= s_1I - c_1B \\
\mathbf{n}_z &= s_{2\text{-}4}\left(c_6c_7c_8 + s_6s_8\right) - c_{2\text{-}4}s_7c_8 \\
&= s_{2\text{-}4}A - c_{2\text{-}4}s_7c_8 \\
&= J
\end{aligned} \tag{3.16}$$

$$
\begin{aligned}
o_x &= c_1\left[c_{2\text{-}4}\left(-c_6c_7s_8 + s_6c_8\right) - s_{2\text{-}4}s_7s_8\right] - s_1\left(s_6c_7s_8 + c_6c_8\right)\\
&= c_1\left(c_{2\text{-}4}C - s_{2\text{-}4}s_7s_8\right) - s_1D\\
&= c_1K - s_1D\\
o_y &= s_1\left[c_{2\text{-}4}\left(-c_6c_7s_8 + s_6c_8\right) - s_{2\text{-}4}s_7s_8\right] + c_1\left(s_6c_7s_8 + c_6c_8\right)\\
&= s_1\left(c_{2\text{-}4}C - s_{2\text{-}4}s_7s_8\right) + c_1D\\
&= s_1K + c_1D\\
o_z &= s_{2\text{-}4}\left(-c_6c_7s_8 + s_6c_8\right) + c_{2\text{-}4}s_7s_8\\
&= s_{2\text{-}4}C + c_{2\text{-}4}s_7s_8\\
&= L
\end{aligned}
\tag{3.17}
$$

$$
\begin{aligned}
a_x &= c_1\left(c_{2\text{-}4}c_6s_7 - s_{2\text{-}4}c_7\right) + s_1s_6s_7\\
&= c_1E + s_1M\\
a_y &= s_1\left(c_{2\text{-}4}c_6s_7 - s_{2\text{-}4}c_7\right) - c_1s_6s_7\\
&= s_1E - c_1M\\
a_z &= s_{2\text{-}4}c_6s_7 + c_{2\text{-}4}c_7\\
&= F
\end{aligned}
\tag{3.18}
$$

$$
\begin{aligned}
p_x &= c_1\left[d_{566}s_{2\text{-}4} + a_{45}c_{2\text{-}4} + d_3s_2 + a_{12}\right]\\
&= c_1G''\\
p_y &= s_1\left[d_{566}s_{2\text{-}4} + a_{45}c_{2\text{-}4} + d_3s_2 + a_{12}\right]\\
&= s_1G''\\
p_z &= -d_{566}c_{2\text{-}4} + a_{45}s_{2\text{-}4} - d_3c_2 + d_{11}\\
&= -H' + d_{11}
\end{aligned}
\tag{3.19}
$$

where:

$$
\begin{aligned}
A &= Rc_8 + P & D &= Sc_8 - Q\\
B &= Sc_8 - Q & E &= Ts_7 - s_{2\text{-}4}c_7\\
C &= -Rs_8 + P & F &= Us_7 + c_{2\text{-}4}c_7
\end{aligned}
$$

$$
\begin{aligned}
G &= d_{566}s_{2\text{-}4} + d_3s_2 & H &= d_{566}c_{2\text{-}4} + d_3c_2\\
G' &= G + X & H' &= H - Y\\
G'' &= G' - a_{12}
\end{aligned}
$$

$$\begin{aligned} I &= c_{2\text{-}4}A + s_{2\text{-}4}N \\ J &= s_{2\text{-}4}A - c_{2\text{-}4}N \\ K &= c_{2\text{-}4}C - s_{2\text{-}4}O \\ L &= s_{2\text{-}4}C + c_{2\text{-}4}O \end{aligned} \qquad \begin{aligned} M &= s_6 s_7 \\ N &= s_7 c_8 \\ O &= s_7 s_8 \\ P &= s_6 c_8 \\ Q &= c_6 c_8 \\ R &= c_6 c_7 \\ S &= s_6 c_7 \end{aligned} \qquad \begin{aligned} T &= c_{2\text{-}4} c_6 \\ U &= s_{2\text{-}4} c_6 \\ V &= s_{2\text{-}4} s_7 \\ W &= c_{2\text{-}4} s_6 \\ X &= a_{45} c_{2\text{-}4} \\ Y &= a_{45} s_{2\text{-}4} \end{aligned}$$

$$d_{566} = d_5 + d_{66}. \tag{3.20}$$

The parameters and substitutions described in Equations 3.20 result in numerically concise expressions which simplify some of the subsequent analyses.

3.2.1 *Generalized Coordinates*

Six independent parameters are required to specify the position of the jaw arm arbitrarily. In cartesian space, the parameters to be used are:

$$\mathbf{x} = \begin{pmatrix} \mathbf{p} \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} p_x \\ p_y \\ p_z \\ \hline a_x \\ a_y \\ a_z \end{pmatrix}$$

where:

$$\mathbf{p} = (p_x, p_y, p_z) = \text{jaw pivot point location}$$

and

$$\mathbf{a} = (a_x, a_y, a_z) = \text{jaw arm (and pipe axis) orientation.}$$

Six parameters are used in this assignment, but a_x, a_y, and a_z are not independent since they are constrained by: $a_x^2 + a_y^2 + a_z^2 = 1$. Therefore, five degrees of freedom are specified by these parameters. The sixth degree of freedom represents rotation about the pipe axis **a**. This rotation has no effect on the manipulation of straight pipes.

In joint space, the following parameters will be used:

$$\phi = \left(\theta_1\ \theta_2\ d_3\ \theta_6\ \theta_7\ \theta_8\right)^T$$

This joint space assignment assumes that the auxiliary lift and extend (θ_4 and d_5) have already been chosen to resolve the redundancy (as discussed in Section 3.1.2). As will be discussed in Section 3.7.1, the Pipe Manipulator's redundancy is constrained to the RPRP boom plane and since the range of motion provided by θ_4 and d_5 is small compared to that of θ_2 and d_3, θ_4 and d_5 were the natural choices for the redundant parameters. θ_2 and d_3 will provide large scale positioning of the boom plane and the redundant parameters θ_4 and d_5 will provide the self motion necessary for small adjustments in avoiding obstacles and/or joint limits. The joint space assignment is further reduced to five degrees of freedom by eliminating θ_8 (rotation about the jaw arm axis). θ_8 has no effect on **a** since it represents rotation about this vector. One possible choice for θ_8 is the value which orients the jaws vertically (i.e., $\mathbf{n} = -\hat{\mathbf{k}}$ when $a_z = 0$). This can be accomplished by setting $\theta_8 = \theta_4 - \theta_2$.