

MECH564 – Introduction to Robotics  
EXAM I Handout

*USEFUL (but not necessarily required) FORMULAS*

Inverse of a Homogeneous Transformation:

$$T = \begin{bmatrix} [R] & \bar{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} & \bar{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} [R]^T & -[R]^T \bar{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \hat{x}^T & -\hat{x} \cdot \bar{p} \\ \hat{y}^T & -\hat{y} \cdot \bar{p} \\ \hat{z}^T & -\hat{z} \cdot \bar{p} \\ 0 & 1 \end{bmatrix}$$

Link Transformation Matrix:

$${}^{i-1}T_i = \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & a_{i-1} \\ s_{\theta_i} c_{\alpha_{i-1}} & c_{\theta_i} c_{\alpha_{i-1}} & -s_{\alpha_{i-1}} & -s_{\alpha_{i-1}} d_i \\ s_{\theta_i} s_{\alpha_{i-1}} & c_{\theta_i} s_{\alpha_{i-1}} & c_{\alpha_{i-1}} & c_{\alpha_{i-1}} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Basic Homogeneous Transformation Matrices:

Translation:

$$D_{xyz}(\Delta x, \Delta y, \Delta z) = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotations:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\theta & -s_\theta & 0 \\ 0 & s_\theta & c_\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} c_\theta & 0 & s_\theta & 0 \\ 0 & 1 & 0 & 0 \\ -s_\theta & 0 & c_\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_z(\theta) = \begin{bmatrix} c_\theta & -s_\theta & 0 & 0 \\ s_\theta & c_\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Common Trigonometric Function Values:

$$c_{60^\circ} = s_{30^\circ} = 1/2 \quad c_{30^\circ} = s_{60^\circ} = \sqrt{3}/2 \quad c_{45^\circ} = s_{45^\circ} = \sqrt{2}/2$$