Ex.) FIND EFFECT OF TIME CONSTANT ON THE SENSITIVITY OF A CLOSED-LOOP SYSTEM FOR A 1st ORDER SYSTEM UNDER PROP. CONTROL

$$G_c = K_p \quad G_p = \frac{1}{\tau s + 1}, \quad H = 1, \quad b = \tau$$

$$G = G_c G_p = \frac{K_p}{\tau s + 1}$$

(CLOSED LOOP TRANSFER FUNC.)

$$T = \frac{G}{1+G} = \frac{K_p/\tau s+1}{1+K_p/\tau s+1} = \frac{K_p}{\tau s + 1 + K_p}$$

$$S_\tau^T = \frac{\partial T}{\partial \tau} \cdot \frac{\tau}{T}$$

$$= \frac{-K_p \cdot s}{(\tau s + 1 + K_p)^2} \cdot \frac{\tau}{K_p/(\tau s + 1 + K_p)}$$

$$= \frac{-\tau s}{\tau s + 1 + K_p}$$

$$s \to j\omega \Rightarrow \quad S_\tau^T(\omega) = \frac{-\tau j\omega}{\tau j\omega + 1 + K_p} = \frac{N_s}{D_s}$$

For very low freq ($\omega \approx 0$), $S_\tau^T = 0$ (T DOES NOT DEPEND ON τ)
$\Rightarrow$ dc

For very high freq ($\omega \approx \infty$), $S_\tau^T = -1$ (% INCREASE IN τ RESULTS IN = % DECREASE IN T)

For medium frequencies, INCREASING K_p, DECREASES SENSITIVITY

(mag in MATLAB)

$$\left| S_\tau^T(\omega) \right| = \frac{|N_s|}{|D_s|} = \frac{\tau\omega}{[(1+K_p)^2 + (\tau\omega)^2]^{1/2}}$$