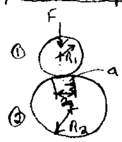
Chapter 9

Contact Stress

$R>0 \Rightarrow$ convex
$R<0 \Rightarrow$ concave
$R=\infty \Rightarrow$ flat surface

Sphere - sphere



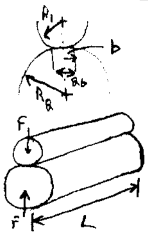
$\sigma_{z_{max}} = -P_a$

$P_o = 0.578\left[\dfrac{F\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2}{\Delta^2}\right]^{1/3}$

$\Delta = \dfrac{1-\nu_1^2}{E_1} + \dfrac{1-\nu_2^2}{E_2}$

$a = 0.908\left[\dfrac{F\Delta}{r}\right]^{1/3}$

Cylinder - Cylinder

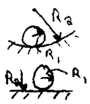


$\sigma_{z_{max}} = -P_o$

$P_o = 0.564\left[\dfrac{Fr}{L\Delta}\right]^{1/2}$

$r = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

$b = 1.13\left[\dfrac{F\Delta}{Lr}\right]^{1/2}$



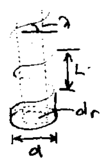
$r = \dfrac{1}{R_1} - \dfrac{1}{|R_2|}$

$r = \dfrac{1}{R_1}$

Chapter 10

Screws

d_r: root or minor dia.
d: major dia.
d_p or d_m: pitch or mean dia.
N: # of threads
p: pitch
TPI: $1/p$
L: lead $L = N \cdot p$
λ: lead angle



$\tan\lambda = \dfrac{L}{\pi d_p}$

$A_t = \dfrac{\pi}{4}\left(\dfrac{d_p + d_r}{2}\right)^2$

$\sigma = \dfrac{P}{A_t}$

Power screws

α: thread angle (axial plane) (Acme: 14.5°, square: 0°)
α_n: thread angle ⊥ to thread contact surface
$\alpha_n = \tan^{-1}\left[\tan\alpha \cdot \cos\lambda\right]$ λ = lead angle

$d_m = \dfrac{d + d_r}{2} = d - (\text{thread depth})$ acme 0.5p

$\lambda = \tan^{-1}\left(L/\pi d_m\right)$

$T_{\text{raise/lower}} = \dfrac{W d_m}{2}\,\dfrac{f\pi d_m \pm L\cos\alpha_n}{\pi d_m \cos\alpha_n \mp fL} + \dfrac{W f_c d_c}{2}$ (upper sign: raise, lower sign: lower)

f: friction between collar and screw
f_c: friction between collar and bearing

static (starting) $f + f_c \approx (1 + 1/3) \cdot$ Running friction (Dynamic)

$f \geq \dfrac{L\cos\alpha_n}{\pi d_m}$ self locking

e (efficiency) $= \dfrac{\text{work out}}{\text{work in}} = \dfrac{W \cdot L}{T \cdot 2\pi}$ (for 1 rev)

Sizing

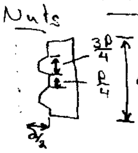
$\sigma = \dfrac{P}{A_t}$ $\quad P_{max_{bolt}} = A_t S_{y_{bolt}}$

$A_t \approx \dfrac{\pi}{4}(0.9d)^2$

$P_{max_{nut}} = A_{shear} \cdot S_{sy_{nut}} = A_{shear}(.58 S_{y_{nut}})$

$\tau_{shear} = \dfrac{P}{A_{shear}}$

Nuts



$A_{shear} \approx \pi d(.75t)$

$t = .47\dfrac{S_{y_{bolt}}}{S_{y_{nut}}} d$

$A_{t_{min}} = \dfrac{P}{\sigma_{axial}} = \dfrac{P}{S_{proof}}$

$A_t \geq SF \cdot A_{t_{min}}$

Finding CG

$\bar{x} = \dfrac{\sum_{i=1}^{N} x_i A_i}{A_{total}}$ $\quad \bar{y} = \dfrac{\sum_{i=1}^{N} y_i A_i}{A_{total}}$

$A_{total} = \sum_{i=1}^{N} A_i$

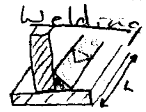
Multiple (no preload) fasteners!

$P_{i_{max}} = A_i(.58 S_y)$

$SF_i = \dfrac{P_{i_{max}}}{P_i}$ $\quad SF = \min(SF_i)$

shear load $S_i = \left(\dfrac{A_i}{A}\right)P$ $\quad$ If $A_1 = A_2 \dots = A_i$: $S_i = \dfrac{P}{N}$ ← # of A_i's

Welding



h ← leg length

t: throat length

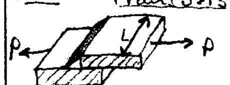
$t = h\sin 45° = \dfrac{\sqrt{2}}{2}h = .707h$

throat area $A = tL$

$S_y = S_u - 12\text{ksi}$

butt $\sigma = \dfrac{P}{A} = \dfrac{P}{hL}$ $\quad SF = \dfrac{S_y}{\sigma}$ ← $\min(S_{y_{weld}}, S_{y_{plate}})$

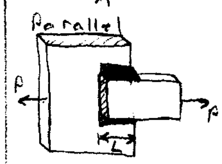
Fillet transverse



one weld
$A = tL = .707hL$

$\tau = \dfrac{P}{A}$ $\quad SF = \dfrac{S_{sy}}{\tau} = \dfrac{.58 S_y}{\tau}$

Parallel



$A = 2tL = 2(.707)hL$

$SF = \dfrac{0.58 S_y}{\tau}$

$A_i = t_i L_i = (.707 h_i)L_i$

$A = \sum_{i=1}^{N} A_i$

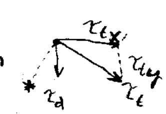
$r_i^2 = (x_i - \bar{x})^2 + (y_i - \bar{y})^2$

$J_i = \bar{J}_i + A_i r_i^2$ (weld i)

$\bar{J}_i = I_{x_i} + I_{y_i}$, $I_{y_i} = \frac{1}{12} t_i L_i^3$, $I_{x_i} = \frac{1}{12} t_i L_i^3$

$J = \sum_{i=1}^{N} J_i$ $\quad \tau_x = \dfrac{Tr}{J}$

resultant shear
$\tau = \sqrt{(\tau_{tx})^2 + (\tau_d + \tau_{ty})^2}$



$SF = \dfrac{.58 S_y}{\tau}$

Preloaded joint (Bolts)

$F_e = F_b - F_c = \delta_e(k_b + k_c)$

$\delta_e = \dfrac{F_e}{k_b + k_c}$

$r = \dfrac{k_c}{k_b}$ (usually $r > 1$)

$R_b = \dfrac{1}{1+r}$

$R_c = \dfrac{r}{1+r}$

$F_b = F_i + R_b F_e$ $\quad F_i = F_b = F_c$ @ $F_e = 0$

$F_c = F_i - R_c F_e$

r: joint stiffness ratio
F_e: Total applied seperating force
F_c: Total joint clamping force
F_b: Total bolt force $(F_e + F_c)$

$\Delta F_b = k_b \delta_e$ $\quad \Delta F_c = -k_c \delta_e$

Fatigue

$\sigma_{max} = k_f \dfrac{F_{b_{max}}}{A_t}$ (k_f: Table 10.6)

$P_{max} = f F_c \rightarrow SF = \dfrac{P_{max}}{P}$

No preload

$\tau = \dfrac{P}{A}$ ← total shear area
$= .58 S_y = S_{sy}$

Rivet

$A_r = \left(\dfrac{\pi d^2}{4}\right) A \cdot N$

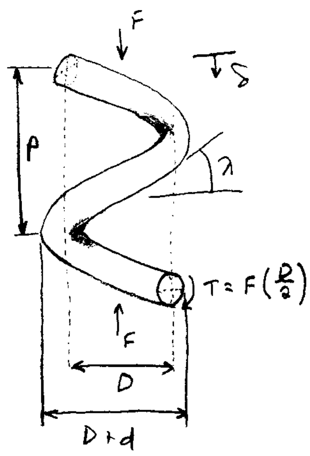
$P_{max} = A(.58 S_y)$

Double shear
$A = 2A_t$

Compression Springs

- λ : helix angle
- d : diameter of spring wire
- D : mean coil dia.
- p : Pitch
- $C = D/d$ spring index
- N_t : total # of turns
- N : # of active turns
- δ_s : "Solid" deflection (max)

$\frac{\delta_s}{N}$ (deflection per coil) $= p - d$



$N_t = N + 2$

$$L_s = \begin{cases}(N_t+1)d & \text{ends not ground}\\ N_t d & \text{ends ground}\end{cases}$$

$$k = \frac{F}{\delta} = \frac{d^4 G}{8D^3 N} = \frac{dG}{8NC^3}$$

$$F = k\delta = \frac{d^4 G}{8D^3}\left(\frac{\delta}{N}\right) = \frac{d^4 G(p-d)}{8D^3}$$

$$\tau_{\substack{\text{torsion}\\ \text{only}}} = \frac{T(d/2)}{J} = \frac{8FD}{\pi d^3} = \frac{8F}{\pi d^2} C$$

$T = F\frac{D}{2}$ $\quad J = \frac{\pi d^4}{32}$

$\tau_{total} = K \, \tau_{torsion\ only}$ (K: Wahl corr. fac.) $\quad k_s$: static, k_w : fatigue

$a = .1\delta_s$ $\quad F_s = F_{max} + ka$

$F_s = \frac{F_{max}}{0.9}$ $\quad F_s \approx 1.1 F_{max}$

$L_f = L_s + \delta_s = L_s + \frac{F_s}{k} = L_s + a + \frac{F_{max}}{k}$

$$\tau_s \le \begin{cases} .45\, S_u & \text{No preset}\\ .65\, S_y & \text{w/ preset}\end{cases}$$

Fatigue

$$\tau = \frac{8FD}{\pi d^3} k_w = \frac{8F}{\pi d} C k_w$$

$$k_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

static

$$\tau = \frac{8FD}{\pi d^3} k_s = \frac{8F}{\pi d^2}(C k_s) \qquad k_s = 1 + \frac{0.5}{C}$$

buckling

$\frac{\delta}{L_f}$ $\quad \frac{L_f}{D}$

Gear Drives

- ω : angular speed (rad/s)
- n : rpm
- $\omega = \frac{\pi}{30} n$
- $\dot{W} = T_p \omega_p$ $\quad \dot{W} = hp\left(550 \frac{ft\cdot lb/s}{hp}\right)$
- $\dot{W}_g = e \dot{W}_p$
- $T_g = e\, \omega_r T_p$
- N = # of teeth
- d : Pitch diameter
- P : diametral pitch
- a : radial height of teeth above pitch circle (Addendum)
  - involute : $a = \frac{1}{P}(in) = m\ (mm)$
  - 20° stub : $a = \frac{.8}{P}$

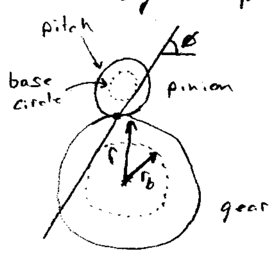
Gear ratio

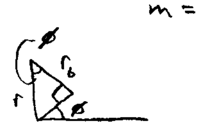
$$\omega_r = \frac{\omega_p}{\omega_g} = \frac{n_p}{n_g} = \frac{d_g}{d_p} = \frac{r_g}{r_p} = \underbrace{\left[\frac{T_g}{T_r}\right]}_{\text{when } e=1} = \frac{N_g}{N_p}$$

$$\frac{N_g}{d_g} = \frac{N_p}{d_p} = P$$

SI: $m = \frac{d}{N}$ $\qquad m = \frac{1}{P} 25.4 \frac{mm}{in.}$

standard $\phi = 20°$ (SI + English)





$r_b = r\cos\phi$

$$r_{a_{max}} = \sqrt{r_b^2 + (c\sin\phi)^2}$$

$r_a = r + a$

$$\omega_r = \frac{\omega_{out}}{\omega_{in}} = \frac{\text{Product of driving teeth}}{\text{Product of driven teeth}}$$