$P_{allowed} \cdot SF = P_{cr}$

## Chapter 4 Equations

Axial tensile stress

$$\sigma = P/A \tag{4.1}$$

Average direct shear stress

$$\tau = P/A \tag{4.2}$$

Torsional shear stress at radius $r$ (round bar)

$$\tau = Tr/J \tag{4.3}$$

Surface torsional shear stress for round bar of diameter $d$

$$\tau = 16T/\pi d^3 \tag{4.4}$$

Maximum torsional shear stress for rectangular bar

$$\tau_{max} = T(3a + 1.8b)/a^2b^2 \tag{4.5}$$

Normal stresses for beam loaded in bending

$$\sigma = My/I \tag{4.6}$$

Maximum normal stresses for beam loaded in bending (section modulus $Z$)

$$\sigma_{max} = M/Z \tag{4.7}$$

Maximum normal stresses for solid round bar loaded in bending

$$\sigma_{max} = 32M/\pi d^3 \tag{4.8}$$

Tensile and compressive stresses for curved beam in bending

$$\sigma_i = +\frac{Mc_i}{eAr_i} \quad \text{and} \quad \sigma_o = -\frac{Mc_o}{eAr_o} \tag{4.9}$$

Distance between the neutral axis and the centroidal axis

$$e = \bar{r} - \frac{A}{\int dA/\rho} \tag{4.10}$$

Tensile and compressive stresses for curved beam in bending

$$\sigma_i = K_iMc/I = K_iM/Z \quad \text{and} \quad \sigma_o = -K_oMc/I = -K_oM/Z \tag{4.11}$$

Transverse shear stress in beam

$$\tau = \frac{V}{Ib}\int_{y=y_0}^{y=c} y\,dA \qquad \frac{VQ}{It} \tag{4.12}$$

Maximum transverse shear stress in beam (solid round section)

$$\tau_{max} = \tfrac{4}{3}V/A \tag{4.13}$$

Maximum transverse shear stress in beam (solid rectangular section)

$$\tau_{max} = \tfrac{3}{2}V/A \tag{4.14}$$

Axial Tensile Strain

$$\varepsilon = \frac{\Delta L}{L}$$

Torsional Shear modulus

$$G = \frac{E}{2(1+\nu)}$$

Factor by which compressive strength is reduced because of buckling tendencies

Euler range

$$\alpha = \frac{S_y}{S_{cr}} = \frac{S_y (L_e/\rho)^2}{\pi^2 E}$$

Johnson Range

$$\alpha = \frac{S_y}{S_{cr}} = \frac{4\pi^2 E}{4\pi^2 E - S_y (L_e/\rho)^2}$$

Column buckling; elastic instability

Euler $$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$S_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/\rho)^2} \text{ or } \frac{S_{cr}}{E} = \frac{\pi^2}{(L_e/\rho)^2}$$

slenderness ratio; radius of gyration

rectangle $\rho = .289h$; round solid, $\rho = D/4$

$$I = A\rho^2$$

Johnson $$S_{cr} = \frac{P_{cr}}{A} = S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_e}{\rho}\right)^2$$

Stress concentrations

$$\sigma_{max} = K_t \sigma_{nom}$$

$F = ma$

$$W = \int_0^s F\,ds = F \cdot s = T\theta = \frac{T 2\pi n}{1000 \cdot 60} \text{ (kW)} \quad \text{spinning}$$

$$\dot{W} = F \cdot V \qquad 550 \frac{\text{ft}\cdot\text{lb}/\text{s}}{\text{hP}}$$

$$\dot{W} = T\omega$$

$$\omega = \frac{V}{R} \qquad \Delta = \frac{PL}{AE} \qquad \theta = \frac{TL}{JG}$$

$1\ \text{lbm} = \frac{1}{32.2}$ slug

$\tau = \frac{VQ}{It}$

**(Chapter 4 Equations Continued)**

Maximum transverse shear stress in beam (thin-wall tubing)

$$\tau_{max} = 2V/A \qquad \textbf{(4.15)}$$

Principal normal stresses → Loc

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} \qquad \textbf{(4.16)}$$

Principal directions—angle between the principal axes and the $x$ and $y$ axes (or the angle between the principal planes and the $x$ and $y$ planes)

$2\theta$ (Mohrs) ⇒ $\theta$ real world

$$2\phi = \tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \textbf{(4.17)}$$

Maximum shear stress

$$\tau_{max} = \pm\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} \qquad \textbf{(4.18)}$$

Normal stress on a plane oriented at angle $\phi$ from the #1 principal plane

$$\sigma_\phi = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\phi \qquad \textbf{(4.19)}$$

Shear stress on a plane oriented at angle $\phi$ from the #1 principal plane

$$\tau_\phi = \frac{\sigma_1 - \sigma_2}{2}\sin 2\phi \qquad \textbf{(4.20)}$$

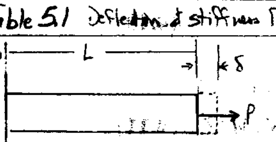
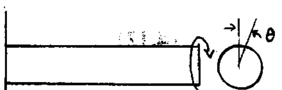
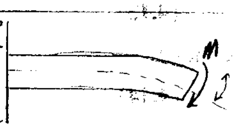
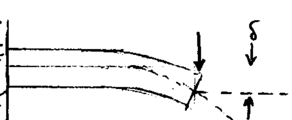
Maximum normal stress; maximum shear stress

$$\sigma_{max} = K_t\sigma_{nom} \quad \text{and} \quad \tau_{max} = K_t\tau_{nom} \qquad \textbf{(4.21)}$$

Strain for unrestrained homogeneous, isotropic body

$$\epsilon = \alpha\Delta T \qquad \textbf{(4.22)}$$

Table 5.1 Deflection & stiffness for straight bars (beams) of Uniform Area

| | | Deflection | Spring Rate |
|---|---|---|---|
| Tension or Compression |  A = cross sectional | $\delta = \frac{PL}{AE}$ | $k = \frac{P}{\delta} = \frac{AE}{L}$ |
| Torsion |  | $\theta = \frac{TL}{K'G}$ <br> solid round bar $\theta^\circ = \frac{584TL}{d^4G}$ | $K = \frac{T}{\theta} = \frac{K'G}{L}$ <br> $(K' = J = \frac{\pi}{32}d^4$ round bar) |
| Bending (Angular) |  | $\theta = \frac{ML}{EI}$ | $K = \frac{M}{\theta} = \frac{EI}{L}$ |
| Bending (linear) |  | $\delta = \frac{ML^2}{2EI}$ | $k = \frac{M}{\delta} = \frac{2EI}{L^2}$ |
| Cantilever beam loaded @ end |  | $\delta = \frac{PL^3}{3EI}$ | $k = \frac{P}{\delta} = \frac{3EI}{L^3}$ |

Static

$\Sigma F = 0 \quad \Sigma M = 0$

Dynamic

$\Sigma F = ma \quad \Sigma M = I\alpha$

reactions & expressions for moments

Castigliano's Method

1) identify sources of elastic nrg (axial, bending, torsion, transverse)
2) Apply dummy loads in direction of desired deflection if necessary
3) express, in general terms, internal loads over its whole length
4) Find all the partials (necessary) of internal loads
5) set dummy loads = to 0
6) Integrate